Hypothesis testing - Steps

- Steps to do a *two-tailed test* of the hypothesis that $\beta_1 \neq 0$:
  1. Set up the hypotheses:
  $$H_0 : \beta_1 = 0$$
  $$H_a : \beta_1 \neq 0.$$
  2. Compute the test statistic:
  $$t = \frac{b_1 - 0}{\text{Std. error of } b_1} = \frac{b_1}{\hat{\sigma}_{b_1}}$$
  3. Read critical value off the $t-$table: $t_{\alpha/2,n-2}$
  4. Reach your conclusion: If $|t| >$ critical value conclude H_a. If $|t| \leq$ critical value, conclude H_0 or *fail to reject* H_0 at level α.

One-tailed hypothesis tests

- We can also test *one-sided* or *one-tailed* hypothesis.
- A one-sided hypotheses would be:

$$
\begin{aligned}
H_0 &: \beta_1 = 0 \\
H_a &: \beta_1 > 0,
\end{aligned}
$$

  or $H_a : \beta_1 < 0$.

- Steps are exactly the same, but now the critical value is $t_{\alpha,n-2}$ (we do not divide α into 2).
- If the test statistic $|t|$ is larger than the critical value off the table and we conclude H_a, we would be concluding that β_1 is larger (or smaller, depending on the test we set up) than zero.

Hypothesis testing (cont'd)

- If we wish to test a two-sided hypothesis about β_1 at level α we can also use the $100(1-\alpha)\%$ confidence interval to do so.
- We will conclude H_a whenever the CI does not include the hypothesized value for β_1 (the value in H_0).
- If the null hypothesis is $H_0 : \beta_1 = 0$, then we will reject the null at level α when the CI does not contain the value zero.
- Why?

Hypothesis testing (cont'd)

- Note that the test of hypothesis (two-tailed) says: fail to reject $H_0 : \beta_1 = 0$ if

$$\frac{b_1 - 0}{\hat{\sigma}_{b_1}} > t_{\frac{\alpha}{2},n-2} \text{ or } \frac{b_1 - 0}{\hat{\sigma}_{b_1}} < -t_{\frac{\alpha}{2},n-2}.$$

- Equivalently, by multiplying both sides of expressions above by $\hat{\sigma}_{b_1}$ and subtracting b_1 from both sides also, note that we fail to reject when

$$0 > b_1 - t_{\frac{\alpha}{2},n-2}\hat{\sigma}_{b_1} \text{ and } 0 < b_1 + t_{\frac{\alpha}{2},n-2}\hat{\sigma}_{b_1}$$

  in other words, when 0 is inside the $100(1-\alpha)\%$ CI for β_1.

- This applies only to two-sided tests.

Hypothesis testing (cont'd)

- JMP and SAS produce a $p-$ value.
- The $p-$ value is the smallest $\alpha-$ level that still leads to rejecting H_0.
- A $p-$value of 0.001 means that if I conclude H_a I only have a 1 in a thousand chance of reaching the wrong conclusion.
- A $p-$ value of 0.3 means that if I conclude H_a I have a 1 in 3 chances of reaching the wrong conclusion.
- Typically we choose H_a only if the chance of making the wrong choice is small, say below 5%. Thus, $p-$values of 0.05 or smaller lead to H_a.
- The $p-$ values produced by computer programs the test $H_0 : \beta_1 = 0$ versus $H_a : \beta_1 \neq 0$.

Hypothesis testing (cont'd)

- We can test any hypothesis that might seem appropriate for the application at hand. For example, we might wish to test

$$H_0 \;:\; \beta_1 = 250$$
$$H_a \;:\; \beta_1 > 250,$$

  if that makes sense from a subject-matter point of view.

- The appropriate test statistic is

$$t = \frac{b_1 - 250}{\hat{\sigma}_{b_1}}$$

  and the rest is the same as in the one-tailed test described earlier.

Coefficient of correlation

- The **coefficient of correlation** r measures the linear association between two variables.

- r is defined to be between -1 and $+1$.

- Given n measurements x and y on a sample of units:

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

Coefficient of correlation (cont'd)

- Since $b_1 = SS_{xy}/SS_{xx}$, we can also compute r as

$$r = b_1 \sqrt{\frac{SS_{xx}}{SS_{yy}}}.$$

  so b_1 and r will always be of the same sign.

- A positive (negative) r means positive (negative) linear association between x and y. An r close to zero means no *linear* association (but there can be nonlinear association!).

- Note that the correlation between x and itself is equal to 1 because:

$$r_{xx} = \frac{SS_{xx}}{\sqrt{SS_{xx}SS_{xx}}} = \frac{SS_{xx}}{SS_{xx}} = 1.$$

Correlation coefficient - Example

- Consider the five stores on which me measured advertising expenses and units of a product sold.

- From earlier example we had: $SS_{xx} = 10$, $SS_{xy} = 8$, $SS_{yy} = 8.8$ and $b_1 = 0.8$. Then

$$r = \frac{8}{\sqrt{10 \times 8.8}} = \frac{8}{9.38} = 0.85.$$

- Using the other formula:

$$r = 0.8 \times \sqrt{\frac{10}{8.8}} = 0.85.$$

Correlation coefficient - Test of hypothesis

- The sample correlation r estimates the population correlation ρ.

- If r is close to zero, we tend to believe that the true population correlation is also zero.

- As in the case of β_1, we can *test hypotheses* about the true population correlation ρ.

- Steps are the same as before:

  1. Set up hypothesis (one or two tailed) H_0 and H_a and choose α
  2. Construct test statistic t and compare t to critical value from table.
  3. If statistic falls in critical region, reject H_0.

Correlation - Test of hypothesis (cont'd)

- For a two-tailed test, hypotheses are:

$$H_0 : \rho = 0 \text{ versus } H_a : \rho \neq 0.$$

- Test statistic is given by:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

- Test statistic is distributed as a t random variable with $n-2$ degrees of freedom. Thus, for a test with level α, the critical value is $t_{\alpha/2,n-2}$ from the t table.

Correlation - Test of hypothesis (cont'd)

- Decision is:
  - If $|t| > t_{\alpha/2,n-2}$: reject H_0.
  - If $|t| \leq t_{\alpha/2,n-2}$: fail to reject H_0.

- For a one-tailed test, the only change is in the formulation of H_a and in the critical value, that now is equal to $t_{\alpha,n-2}$.

- A test of hypothesis for ρ is equivalent to the test for β_1. If one test results in rejecting H_0, the other will too.

- Example in class.