Testing Nested Models

• Two models are *nested* if both contain the same terms and one has at least one additional term.

• Example:

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \epsilon
\]  
(1)

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2 + \epsilon
\]  
(2)

• Model (1) is *nested within* model (2).

• Model (1) is the **reduced** model and model (2) is the **full** model.
Testing Nested Models (cont’d)

- How do we decide whether the more complex (full) model contributes additional information about the association between $y$ and the predictors?

- In example above, this is equivalent to testing $H_0 : \beta_4 = \beta_5 = 0$ versus $H_a : \text{at least one } \beta \neq 0$.

- Test consists in comparing the SSE for the reduced model ($SSE_R$) and the SSE for the complete model ($SSE_C$).

- $SSE_R > SSE_C$ always so question is whether the drop in SSE from fitting the complete model is ‘large enough’.
Testing Nested Models (cont’d)

- We use an $F$–test to compare nested models, one with $k$ parameters (reduced) and another one with $k + p$ parameters (complete or full).

- Hypotheses: $H_0 : \beta_{k+1} = \beta_{k+2} = \ldots = \beta_{k+p} = 0$ versus $H_a : \text{At least one } \beta \neq 0$.

- Test statistic: $F = \frac{(SSE_R - SSE_C)}{\# \text{ of additional } \beta' s} \cdot \frac{SSE_C}{n - (k + p + 1)}$

- At level $\alpha$, we compare the $F$–statistic to an $F_{\nu_1, \nu_2}$ from table, where $\nu_1 = p$ and $\nu_2 = n - (k + p + 1)$.

- If $F \geq F_{\alpha, \nu_1, \nu_2}$, reject $H_0$. 
Testing Nested Models (cont’d)

• See Example 4.10 on page 233.

• Steps are:

  1. Fit complete model with \( k + p \) \( \beta \)'s and get \( SSE_C \).
  2. Fit reduced model with \( k \) \( \beta \)'s and get \( SSE_R \).
  3. Set up hypotheses and choose \( \alpha \) value.
  4. Compute \( F \)–statistic and compare to table \( F_{\alpha,\nu_1,\nu_2} \).

• If test leads to rejecting \( H_0 \), then at least one of the additional terms in the model contributes information about the response.
Testing Nested Models (cont’d)

• Parsimonious models are preferable to big models as long as both have similar predictive power.

• A parsimonious model is one with a small number of predictors.

• If models are not nested, cannot use the $F$-test above to choose between one and another. Must rely on other sample statistics such as $R^2_a$ and $RMSE$.

• In the end, choice of model is subjective.