Exercise 1.24

a. From JMP:
   Mean age = \( \bar{y} = 48.2 \) years.
   Std. dev. of age = \( s = 6.0 \) years.

b. Histogram suggests that it might be reasonable to assume normality for the age of female executive in America. If so, then:

\[
\Pr \left( \mu - 2s < \bar{y} < \mu + 2s \right) = 0.95
\]

From sample data then, we can say that a randomly chosen female executive in the US has approx. a 95% chance of being between 36 and 60 years of age, because estimated interval is:

\[
\bar{y} \pm 2s = 48.2 \pm 2 \times 6
\]

(Note that we wish to construct a confidence interval for an observation not for a mean thus the use of \( s \) rather than \( s/\sqrt{n} \).

Exercise 1.27

If "likely" is interpreted as 95%:

a. \( 19 \pm 2 \times 6.5 = (-410, 149) \)

b. \( 7 \pm 2 \times 49 = (-91, 105) \)

c. The Math SAT is the more likely to have the 140 point increase. Note that:
Math: \( \frac{140 - 19}{65} = 1.86 \) meaning that the score of 140 in the Math test is 1.86 standard deviations away from the mean.

So the same score of 140 is now 2.71 std. deviations away from the mean.

The value 140 is much more unlikely for Verbal than for Math.

**Exercise 1.29**

\[ y \sim N(100, 64) \]

(a) \( \Pr \left\{ \mu - 2\sigma \leq y \leq \mu + 2\sigma \right\} = 0.95 \)

(b) \( \Pr \left\{ y > 108 \right\} = \Pr \left\{ \frac{y - \mu}{\sigma} > \frac{108 - \mu}{\sigma} \right\} \)

\[ = \Pr \left\{ z > \frac{108 - 100}{8} \right\} = \Pr \left\{ z > 1 \right\} = 1 - \Pr \left\{ z < 1 \right\} \]

\[ = 1 - 0.8413 = 0.1587. \]
c) \( \Pr \{ y < 92 \} = \Pr \{ z < \frac{92 - 100}{8} \} = \Pr \{ z < -1 \} \).

But \( \Pr \{ z < -1 \} = \Pr \{ z > 1 \} \), so answer is 0.1587.

d) \( \Pr \{ 92 \leq y \leq 116 \} = \Pr \{-1 \leq z \leq 2\} \).

In parts:
\( \Pr \{ z \leq 2 \} = 0.9772 \)
\( \Pr \{ z \geq -1 \} = 1 - \Pr \{ z < -1 \} = 0.8413 \)

Then \( \Pr \{-1 \leq z \leq 2\} = 0.9772 - (1 - 0.8413) = 0.82 \)

e) \( \Pr \{ 92 \leq y \leq 96 \} = \Pr \{-1 \leq z \leq -0.5\} \).

\( = \Pr \{ z \leq -0.5 \} - \Pr \{ z \leq -1 \} \)
\( = 0.1498 \)

f) \( \Pr \{ 76 \leq y \leq 124 \} = \Pr \{-3 \leq z \leq 3\} = 0.99. \)

Exercise 1.32

a) \( \Pr \{ y \geq 70 \} = \Pr \{ z \geq \frac{50 - 19}{6} \} = \Pr \{ z \geq 0.48 \} \).

\( = 1 - \Pr \{ z \leq 0.48 \} = 1 - 0.1844 = 82\% \).

b) \( \Pr \{ y \geq 70 \} = \Pr \{ z \geq \frac{\sqrt{9} - 7}{49} \} = \Pr \{ z \geq 0.88 \} \).

\( = 1 - \Pr \{ z \leq 0.88 \} = 1 - 0.3106 = 0.69. \)
Exercise 1.44

Note that sample size are small (n=11 and n=14), so the appropriate intervals would use a t rather than a z score.

a) 95% CI for mean FNE score for bulimic students:

\[
\bar{y} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} = 17.82 \pm t_{0.025, 10} \frac{4.92}{\sqrt{11}} = \]
\[
= 17.82 \pm 2.228 \frac{4.92}{\sqrt{11}} = (14.5, 21.1)
\]

b) Non-bulimic

\[
\bar{y} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} = 14.14 \pm t_{0.025, 13} \frac{5.29}{\sqrt{14}}
\]
\[
= 14.14 \pm 2.16 \frac{5.29}{\sqrt{14}} = (11.1, 17.2)
\]

Interpretation: With 95% confidence the intervals cover the true mean FNE scores for bulimic and non-bulimic students, respectively.

c) Assumptions:
- Distribution of FNE scores in each group of students is approximately normal. (Note that this would not be required if n was large because of CLT theorem)
- Student in each group were randomly sampled from the two populations.