Solving a Plane Truss

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Introduction

In this short paper, a method for solving plane trusses in presented. The method will give you a taste of how the finite-element-method (FEM) works for the solution of problems in solid mechanics. You have learned how to solve a truss problem by using the methods of joints and sections. The goal there was to calculate the forces on each member and the support reactions. The present method will give you not only the forces but also the displacement of each pin, the stress and strain in each member.

The method has six essential steps.

- Numbering convention for the pins, members, forces and displacements at the pins.
- Global to local-axial coordinate transformation
- Mechanics of Materials for axial loading
- Construction of stiffness matrix
- Insertion of the boundary conditions
- Solution
- Recovering derived quantities, such as stress and strain

Two example problems are solved to demonstrate the functioning of the method.

Numbering Convention

A truss is a complex structure where many members are joined at the pins. In order to store the arrangement of the members and pins in a computer program, we need a suitable numbering scheme. This numbering scheme also helps in the bookkeeping during the writing of the program. We demonstrate the numbering and data input for the truss of Figure-1.

Numbering

- Number the members/elements as 1,2,3,...
- Number the pins as 1,2,3,...
- Put forces and displacements at each pin and number them. The pattern of this numbering is as follows:
  - At pin #1: x-direction is #1 [force f(1), displacement u(1)] and y-direction is #2 [force f(2), displacement u(2)]
  - At pin #2: x-direction is #3 [force f(3), displacement u(3)]and y-direction is #4 [force f(4), displacement u(4)].
  - In general, at pin #k: x-direction is #(2k-1) [force f(2k-1), displacement u(2k-1)] and y-direction is #2k [force f(2k), displacement u(2k)].

The information about the arrangement of the members is stored in a matrix called the ”Connectivity Matrix”. For each member, we identify the two pins at the two ends. We call one pin as the beginning pin ”b” and the other pin as the ending pin ”e”. Exactly which end is beginning and which is ending are unimportant. Referring to Figure-1, for member #1, $b = 1$ and $e = 2$; for member #2, $b = 2$ and $e = 3$; for member #3, $b = 3$ and $e = 1$. For member #3, we could have reversed the numbering and said $b = 1$ and $e = 3$ without making any error in the programing or calculation.

This end-pin information of a member is stored in the member's connectivity matrix $IJK(index1, index2)$. For the $j-th$ member, the number of the beginning pin is stored in $IJK(j,1) = b$ and the number of the ending pin is stored in $IJK(j,2) = e$.

For the truss in Figure-1,

- Member #1: begins at $IJK(1,1) = 1$, ends at $IJK(1,2) = 2$
- Member #2: begins at $IJK(2,1) = 2$, ends at $IJK(2,2) = 3$
- Member #3: begins at $IJK(3,1) = 3$, ends at $IJK(3,2) = 1$

Data Input

- Line 1: Number of elements *nele*.
- Line 2: Number of pins *npin*.
- *npin* number of lines: Each line contains the *x* and *y* coordinates of a pin.
- *nele* number of lines: Each line contains the $IJK(*,*)$ of a member.
- *nele* number of lines: Each line contains the cross-sectional area $area(*)$ and Young's Modulus $young(*)$ of a member.

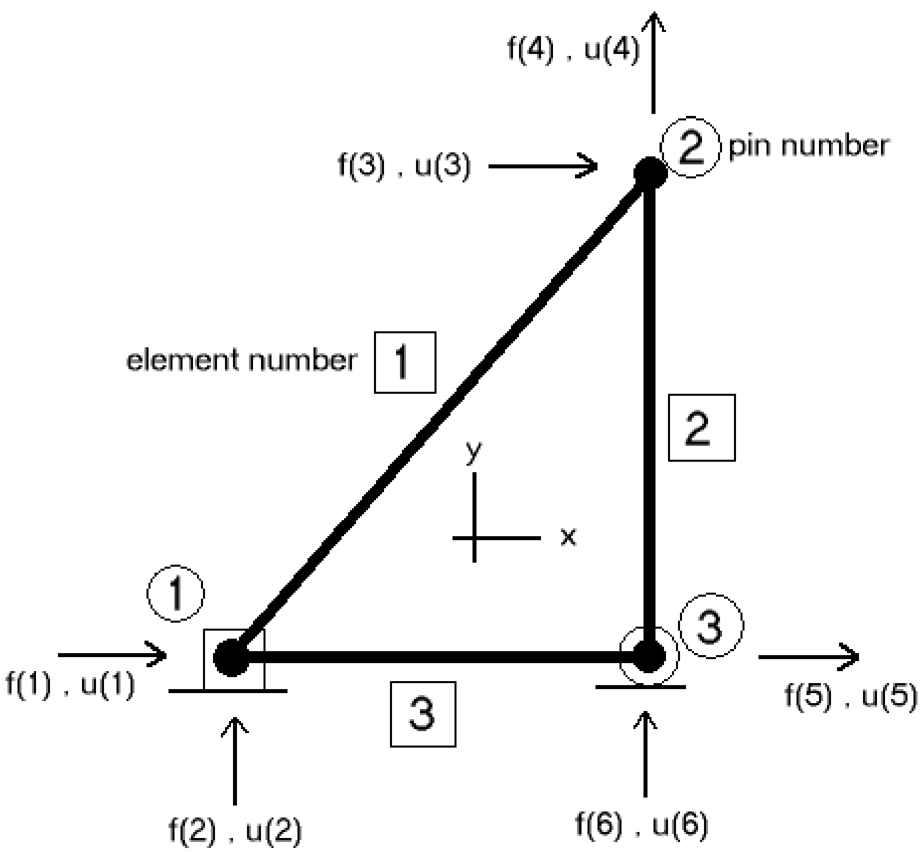

Figure-1: Numbering scheme for truss solution

Global to Local Coordinate Transformation

Consider the member #j with beginning pin at $IJK(j,1)$ and ending pin at $IJK(j,2)$. The coordinates of the end points of the member are:

$$x_b = x(IJK(j,1)), \qquad y_b = y(IJK(j,1))$$

$$x_e = x(IJK(j,2)), \qquad y_e = y(IJK(j,2))$$

The length of the element is

$$L(j) = \sqrt{(x_e - x_b)^2 + (y_e - y_b)^2}$$

The cos and sin of the angle this element makes with the $x - axis$ are given by

$$\cos\theta_j = \frac{x_e - x_b}{L(j)} = C(j)$$

$$\sin\theta_j = \frac{y_e - y_b}{L(j)} = S(j)$$

From our numbering scheme, we know that at the beginning pin, the x direction is numbered as $(2b - 1)$ and the y direction is numbered as $2b$. Similarly, at the ending pin, the x and y directions are numbered $(2e - 1)$ and $2e$, respectively. We use the symbols U for displacement and F for force at each pin. In Figure-2, the displacements U and forces F at the two ends of a member are shown.

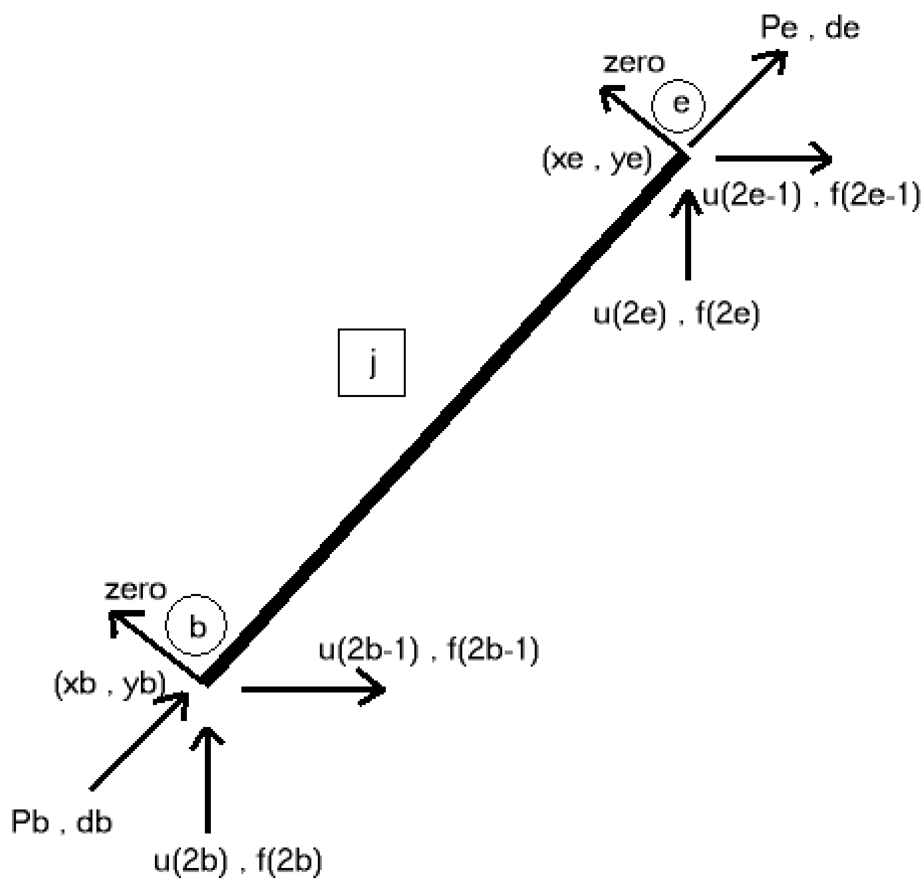

Figure-2: Displacements and forces on a member.

We know that all the members in a truss are two-force members where the forces at the two ends are aligned along the member and the elongations at the two ends are also aligned along the member. The displacement and the force at the beginning pin b are denoted by d_b and P_b, and displacement and force at the ending pin e are denoted by d_e and P_e. The displacements and forces perpendicular to the element are zero. Thus, we can write:

$$\begin{aligned}
d_b &= U_{2b-1}C(j) + U_{2b}S(j)\\
0 &= -U_{2b-1}S(j) + U_{2b}C(j)\\
d_e &= U_{2e-1}C(j) + U_{2e}S(j)\\
0 &= -U_{2e-1}S(j) + U_{2e}C(j)\\
P_b &= F_{2b-1}C(j) + F_{2b}S(j)\\
0 &= -F_{2b-1}S(j) + F_{2b}C(j)\\
P_e &= F_{2e-1}C(j) + F_{2e}S(j)\\
0 &= -F_{2e-1}S(j) + F_{2e}C(j)
\end{aligned}$$

In matrix notation, we can write these equations as:

$$\begin{Bmatrix} d_b \\ 0 \\ d_e \\ 0 \end{Bmatrix} = [T(j)] \begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} P_b \\ 0 \\ P_e \\ 0 \end{Bmatrix} = [T(j)] \begin{Bmatrix} F_{2b-1} \\ F_{2b} \\ F_{2e-1} \\ F_{2e} \end{Bmatrix} \tag{2}$$

where

$$[T(j)] = \begin{bmatrix} C(j) & S(j) & 0 & 0 \\ -S(j) & C(j) & 0 & 0 \\ 0 & 0 & C(j) & S(j) \\ 0 & 0 & -S(j) & C(j) \end{bmatrix} \tag{3}$$

Mechanics of Materials

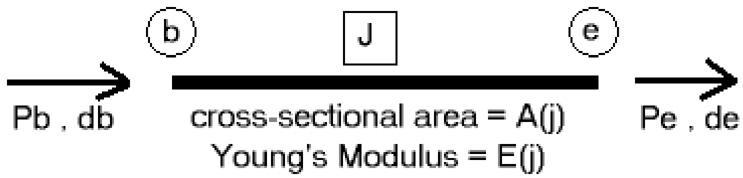

Figure-3: Equilibrium of a member.

From Figure-3, the force equilibrium is

$$P_e = -P_b = P(j)$$

The elongation of the member is $(d_e - d_b)$, and strain is

$$\epsilon(j) = \frac{d_e - d_b}{L(j)} \tag{4}$$

The stress is

$$\sigma(j) = E(j)\epsilon(j) = \frac{E(j)}{L(j)}(d_e - d_b) \tag{5}$$

where $E(j)$ is the Young's Modulus of the material of member j. Another expression for stress is

$$\sigma(j) = \frac{P(j)}{A(j)} = \frac{P_e}{A(j)} \tag{6}$$

where $A(j)$ is the cross-sectional area of member j. Combining Eqns.(5,6), we find

$$P_e = -P_b = \frac{E(j)A(j)}{L(j)}(d_e - d_b) = k(j)(d_e - d_b)$$

In matrix notation

$$\left\{ \begin{array}{c} P_b \\ P_e \end{array} \right\} = \left[\begin{array}{cc} k(j) & -k(j) \\ -k(j) & k(j) \end{array} \right] \left\{ \begin{array}{c} d_b \\ d_e \end{array} \right\} \tag{7}$$

Stiffness Matrix

Element Stiffness

Equation(7) is now re-written with the matrices and vectors padded up with some zeros.

$$\begin{Bmatrix} P_b \\ 0 \\ P_e \\ 0 \end{Bmatrix} = \begin{bmatrix} k(j) & 0 & -k(j) & 0 \\ 0 & 0 & 0 & 0 \\ -k(j) & 0 & k(j) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_b \\ 0 \\ d_e \\ 0 \end{Bmatrix} \tag{8}$$

By combining Eqns(1,2,8) we get,

$$[T]\begin{Bmatrix} F_{2b-1} \\ F_{2b} \\ F_{2e-1} \\ F_{2e} \end{Bmatrix} = \begin{bmatrix} k(j) & 0 & -k(j) & 0 \\ 0 & 0 & 0 & 0 \\ -k(j) & 0 & k(j) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} [T]\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix}$$

or

$$\begin{Bmatrix} F_{2b-1} \\ F_{2b} \\ F_{2e-1} \\ F_{2e} \end{Bmatrix} = [T]^{-1} \begin{bmatrix} k(j) & 0 & -k(j) & 0 \\ 0 & 0 & 0 & 0 \\ -k(j) & 0 & k(j) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} [T]\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix} = [K(j)]\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix} \tag{9}$$

The matrix $[K(j)]$ in Eqn.(9) is known as the stiffness matrix for member j. It can be easily shown that the inverse of the $[T]$ matrix of Eqn.(3) is equal to its transpose $[T]^T$. By substituting the elements of $[T]$ and $[T]^{-1} = [T]^T$, we find

$$[K(j)] = k(j)\begin{bmatrix} C(j)^2 & C(j)S(j) & -C(j)^2 & -C(j)S(j) \\ C(j)S(j) & S(j)^2 & -C(j)S(j) & -S(j)^2 \\ -C(j)^2 & -C(j)S(j) & C(j)^2 & C(j)S(j) \\ -C(j)S(j) & -S(j)^2 & C(j)S(j) & S(j)^2 \end{bmatrix} \tag{10}$$

Recalling that $k(j) = E(j)A(j)/L(j)$ and $C(j) = \cos\theta_j$, $S(j) = \sin\theta_j$, we can construct the stiffness matrix for each of the members in the truss. The stiffness matrix for the entire truss is an assembly of the stiffness matrices of individual members. However, the ”element stiffness matrix” and the ”entire truss stiffness matrix” are of different size. For ease in assembling the truss stiffness matrix, the element stiffness matrix is made larger by padding it up with zeros so that its size becomes equal to the size of the truss stiffness matrix. This padded up element stiffness matrix is called the augmented stiffness matrix.

Augmented Element Stiffness

The element stiffness matrix is a $[4,4]$ matrix. Whereas, a truss with *npin* number of pins will be a $[2*npin, 2*npin]$ matrix. Let us examine the element stiffness in detail and determine how this can be easily augmented with zeros.

$$\begin{Bmatrix} F_{2b-1} \\ F_{2b} \\ F_{2e-1} \\ F_{2e} \end{Bmatrix} = k(j) \begin{bmatrix} C(j)^2 & C(j)S(j) & -C(j)^2 & -C(j)S(j) \\ C(j)S(j) & S(j)^2 & -C(j)S(j) & -S(j)^2 \\ -C(j)^2 & -C(j)S(j) & C(j)^2 & C(j)S(j) \\ -C(j)S(j) & -S(j)^2 & C(j)S(j) & S(j)^2 \end{bmatrix} \begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix} \tag{11}$$

Begin by setting all the elements of the augmented matrix to zero. From Eqn.(11) we find that the $(1,1)$ element $k(j)C(j)^2$ connects F_{2b-1} with U_{2b-1}. Thus the quantity $k(j)C(j)^2$ will sit at the location $(2b-1, 2b-1)$ of the augmented matrix. Continuing in this manner we find:

- Element $(2b-1, 2b) = k(j)C(j)S(j)$
- Element $(2b-1, 2e-1) = -k(j)C(j)^2$
- Element $(2b-1, 2e) = -k(j)C(j)S(j)$
- Element $(2b, 2b-1) = k(j)C(j)S(j)$
- and so on.

Truss Stiffness Matrix

The total truss stiffness matrix is the sum of all the augmented element stiffness matrices.

$$[K] = \sum_{j=1}^{nele} [K(j)^{augmented}]$$

and

$$[K]\{U\} = \{F\} \tag{12}$$

where $\{U\}$ is a vector consisting of the displacements of the pins and $\{F\}$ is a vector consisting of the forces at the pins.

Boundary Conditions

Before we solve Eqn.(12) for the nodal forces and displacements, we must specify the boundary conditions. The boundary conditions tell the program how the truss is supported and how the truss is loaded. In the following paragraphs we consider four possible situations. Recall that at pin #k, the subscript $(2k-1)$ corresponds to the x –direction and subscript $2k$ corresponds to the y –direction.

Loaded Pins:

For the loaded pin #k shown in Figure-4, the boundary conditions are

$$F_{2k-1} = 300\cos 30°, \quad F_{2k} = 300\sin 30°, \quad U_{2k-1} = unknown, \quad U_{2k} = unknown$$

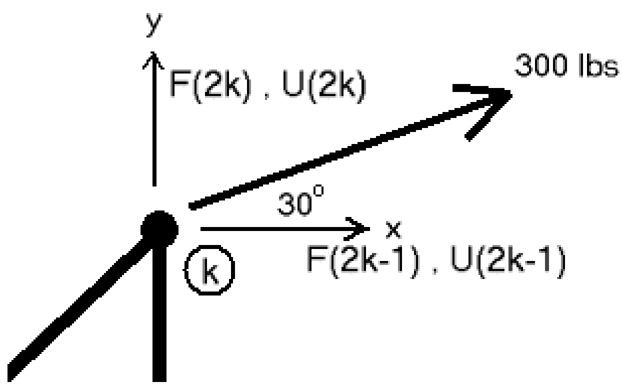

Figure-4: Loaded Pin

Unloaded Pins:

For load free pin #*k*, the boundary conditions are

$$F_{2k-1} = 0, \quad F_{2k} = 0, \quad U_{2k-1} = unknown, \quad U_{2k} = unknown$$

Fixed Support:

When pin #*k* is a fixed support as shown in Figure-5, the boundary conditions are

$$F_{2k-1} = unknown, \quad F_{2k} = unknown, \quad U_{2k-1} = 0, \quad U_{2k} = 0$$

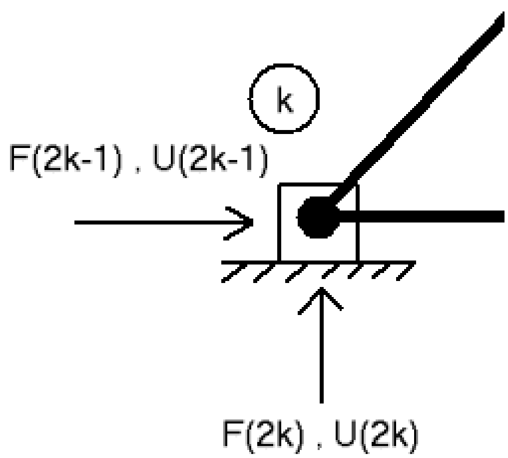

Figure-5: Fixed support.

Roller Support:

When pin #*k* is roller supported as shown in Figure-6, the boundary conditions are

$$F_{2k-1} = 0, \quad F_{2k} = unknown, \quad U_{2k-1} = unknown, \quad U_{2k} = 0$$

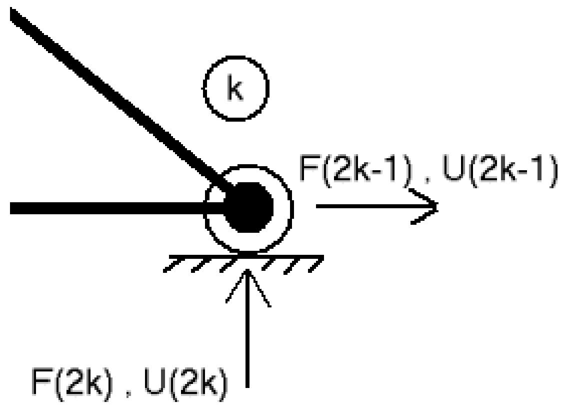

Figure-6: Roller support.

When pin #k is roller supported as shown in Figure-7, the boundary conditions are

$$F_{2k-1} = unknown, \quad F_{2k} = 0, \quad U_{2k-1} = 0, \quad U_{2k} = unknown$$

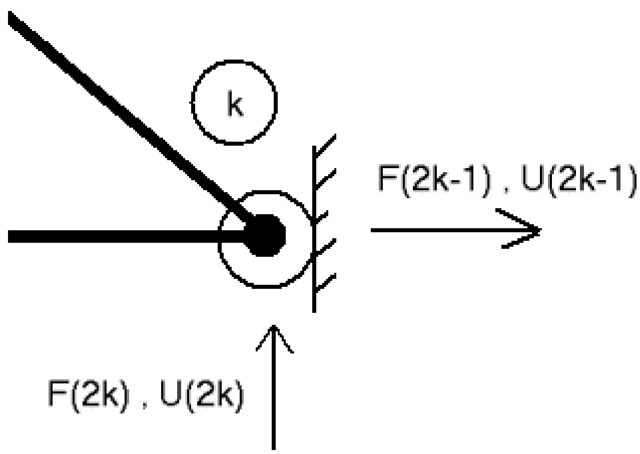

Figure-7: Roller support.

Boundary Data

For a truss with *npin* number of pins, this segment of the data file will have $2 * npin$ number of lines. Because for each pin we must specify the data for two directions, namely x and y. In general, line # $(2k-1)$ contains the boundary data for the x –direction of pin #k and line # $2k$ contains the boundary data for the y –direction of pin #k.

The format of the data in line j is $[flag(j) \quad value(j)]$. The variable $flag(j)$ is a literal string. When $flag(j) = "d"$, then the program knows that the displacement $U(j)$ is specified and $U(j) = value(j)$. When the $flag(j) = "f"$, then the program knows that the force $F(j)$ is specified and $F(j) = value(j)$.

Inserting the Boundary Conditions

It is easy to understand how the boundary conditions are inserted in Eqn.(12) if we rewrite it as follows

$$[K]\{U\} = [I]\{F\} \tag{13}$$

where $[I]$ is the identity matrix with ones on the diagonal and zeros everywhere else. This equation is transformed into a set of equations

$$[A]\{x\} = \{b\} \tag{14}$$

by inserting the boundary conditions. $\{x\}$ contains the unknown forces and displacements, and $\{b\}$ is the contribution from the boundary conditions.

The steps in transforming Eqn.(13) into Eqn.(14) are:

- Set $[A] = [K]$ and $\{b\} = \{0\}$.
- If $flag(j) = "d"$ then $U_j = value(j)$. Multiply the $j-th$ column of $[A]$ by $value(j)$ and subtract the resulting column from $\{b\}$. This is how the known U_j is pushed to the right-hand-side of Eqn.(14). When U_j gets pushed to the right-hand-side, the $j-th$ column of $[A]$ is set to zero. Now we need to bring the unknown F_j to the left-hand-side. This is done by setting the element $A(j,j) = -1$, because F_j has a coefficient of unity on the right-hand-side of Eqn.(13).
- If $flag(j) = "f"$ then $F_j = value(j)$. In this situation, F_j should remain in the right-hand-side and thus, is simply added to the $j-th$ row of $\{b\}$.

Solution

In Eqn.(14), the matrix $[A]$ is of order $[2*npin, 2*npin]$ where $npin$ is the number of pins in the truss. Vectors $\{x\}$ and $\{b\}$ are of length $\{2*npin\}$. $\{x\}$ contains the unknown quantities - either pin displacement or pin forces. $\{b\}$ contains the contribution from the boundary conditions.

The Eqn.(14) can be solved by utilizing any solver for a system of simultaneous equations. After the elements of $\{x\}$ are determined, the complete arrays for force and displacement are constructed as follows:

- If $flag(j) = "d"$ then $U(j) = value(j)$ and $F(j) = x(j)$
- If $flag(j) = "f"$ then $F(j) = value(j)$ and $U(j) = x(j)$

Stress and Strain Calculation

From Eqn.(4), the expression for strain in the member j is

$$\epsilon(j) = \frac{1}{L(j)}(d_e - d_b)$$

This equation is written in matrix form as

$$\epsilon(j) = \frac{1}{L(j)}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} d_b \\ 0 \\ d_e \\ 0 \end{bmatrix} \tag{15}$$

Combining Eqn.(15) and Eqn.(1), we write

$$\epsilon(j) = \frac{1}{L(j)}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}[T(j)]\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix}$$

By substituting $[T(j)]$ from Eqn.(3)

$$\epsilon(j) = \frac{1}{L(j)}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} C(j) & S(j) & 0 & 0 \\ -S(j) & C(j) & 0 & 0 \\ 0 & 0 & C(j) & S(j) \\ 0 & 0 & -S(j) & C(j) \end{bmatrix}\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix}$$

or

$$\epsilon(j) = \frac{1}{L(j)}\begin{bmatrix} -C(j) & -S(j) & C(j) & S(j) \end{bmatrix}\begin{Bmatrix} U_{2b-1} \\ U_{2b} \\ U_{2e-1} \\ U_{2e} \end{Bmatrix}$$

or

$$\epsilon(j) = \frac{1}{L(j)}[C(j)(U_{2e-1} - U_{2b-1}) + S(j)(U_{2e} - U_{2b})] \tag{16}$$

where b and e are the beginning and ending pins of member j. The stress in member j can now be calculated from

$$\sigma(j) = E(j)\epsilon(j) \tag{17}$$

The loading $P(j)$ on member j can be calculated from

$$P(j) = A(j)\sigma(j) \tag{18}$$

where $A(j)$ is the cross-sectional area of member j.

Examples

Example-1

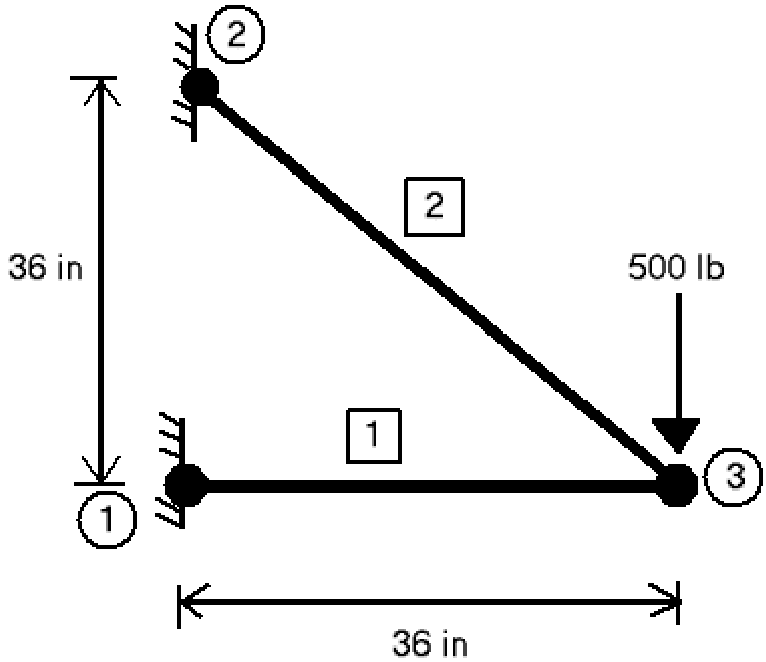

Figure-8: Truss for Example-1.

Data File

- nele= 2
- Area and Young's modulus of member #1: 8 1.9E6
- Area and Young's Modulus of member #2: 8 1.9E6
- npin = 3
- Coordinates of pin #1: 0 0
- Coordinates of pin #2: 0 36
- Coordinates of pin #3: 36 0
- IJK of member #1: 1 3 (starting pin =1, ending pin =3)
- IJK of member #2: 2 3 (starting pin =2, ending pin = 3)
- x-displacement given at pin #1: *d* 0
- y-displacement give at pin #1: *d* 0
- x-displacement given at pin #2: *d* 0
- y-displacement give at pin #2: *d* 0
- x-force given at pin #3: *f* 0
- y-force give at pin #3: *f* -500

From this data, we find

$$L(1) = 36, L(2) = 50.9117, C(1) = 1.0, S(1) = 0, C(2) = 0.707107, S(2) = -0.707107$$

Element stiffness matrix for member #1 is

$$4.22 \times 10^5 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_5 \\ F_6 \end{Bmatrix}$$

and augmented stiffness matrix for member #1 is

$$[K(1)^{augmented}] = 4.22 \times 10^5 \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

Element stiffness matrix for member #2 is

$$2.98 \times 10^5 \begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \begin{Bmatrix} U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}$$

and augmented stiffness matrix for member #2 is

$$[K(2)^{augmented}] = 2.98 \times 10^5 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 & -0.5 & 0.5 \\ 0 & 0 & -0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & -0.5 & 0.5 & 0.5 & -0.5 \\ 0 & 0 & 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} \tag{20}$$

Adding the matrices in Eqns.(19,20) we get the truss stiffness matrix and Eqn.(13) becomes

$$10^5\begin{bmatrix} 4.22 & 0 & 0 & 0 & -4.22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.49 & -1.49 & -1.49 & 1.49 \\ 0 & 0 & -1.49 & 1.49 & 1.49 & -1.49 \\ -4.22 & 0 & -1.49 & 1.49 & 5.71 & -1.49 \\ 0 & 0 & 1.49 & -1.49 & -1.49 & 1.49 \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}$$

Inserting the boundary conditions

$$U_1 = U_2 = U_3 = U_4 = 0 \quad and \quad U_5, U_6 \quad unknown$$

and

$$F_1, F_2, F_3, F_4 \quad unknown \quad and \quad F_5 = 0, \quad F_6 = -500$$

the Eqn.(14) becomes

$$10^5\begin{bmatrix} -1 & 0 & 0 & 0 & -4.22 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1.49 & 1.49 \\ 0 & 0 & 0 & -1 & 1.49 & -1.49 \\ 0 & 0 & 0 & 0 & 5.71 & -1.49 \\ 0 & 0 & 0 & 0 & -1.49 & 1.49 \end{bmatrix}\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}$$

By solving this set of equations, we find

$$F_1 = 500lb, F_2 = 0, F_3 = -500lb, F_4 = 500lb, U_5 = -1.18 \times 10^{-3}in, U_6 = -4.54 \times 10^{-3}in$$

The forces with odd subscript are along the *x*-axis. Summing these forces we get

$$F_1 + F_3 + F_5 = 500 - 500 + 0 = 0 \tag{21}$$

The forces with even subscript are along the *y*-axis. Summing these forces we get

$$F_2 + F_4 + F_6 = 0 + 500 - 500 = 0 \tag{22}$$

Eqns.(21,22) verify the equilibrium of the truss. This is a standard check for the proper functioning of the truss solver code.

From Eqn.(16), the strain in member #1 is (starting node $b = 1$, ending node $e = 3$)

$$\epsilon(1) = \frac{1}{L(1)}[C(1)(U_5 - U_1) + S(1)(U_6 - U_2)] = -3.289 \times 10^{-5}$$

The strain in member #2 is (starting node $b = 2$, ending node $e = 3$)

$$\epsilon(2) = \frac{1}{L(2)}[C(2)(U_5 - U_3) + S(2)(U_6 - U_4)] = 4.652 \times 10^{-5}$$

From Eqn.(17), the stresses in the members are

$$\sigma(1) = E(1)\epsilon(1) = -62.5psi$$

$$\sigma(2) = E(2)\epsilon(2) = 88.39psi$$

From Eqn.(18), the forces in the members are

$$P(1) = \sigma(1)A(1) = -500lb$$

$$P(2) = \sigma(2)A(2) = 707.1lb$$

Example-2

In Figure-9, a loaded truss with pin and element numbering is shown. The members in the truss are made of steel ($E = 29 \times 10^6 psi$) and have a cross-sectional area of $1in^2$.

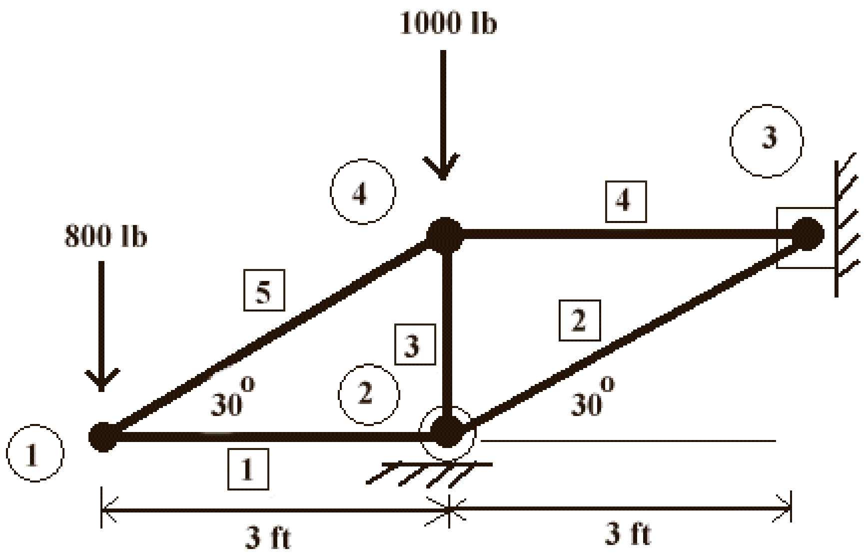

Figure-9: Truss for example-2

The data file for the solution of this truss is:

```
5
1 29d6
1 29d6
1 29d6
1 29d6
1 29d6
4
-36 0
```

```
0 0
36 20.78
0 20.78
1 2
2 3
4 2
3 4
4 1
f 0
f -800
f 0
d 0
d 0
d 0
f 0
f -1000
```

The output from the program is:

```
 Member # 1
 Strain= -0.4779D-04 Stress= -0.1386D+04 Load=
-0.1386D+04
 Member # 2
 Strain= -0.5518D-04 Stress= -0.1600D+04 Load=
-0.1600D+04
 Member # 3
 Strain= -0.6207D-04 Stress= -0.1800D+04 Load=
-0.1800D+04
 Member # 4
 Strain= 0.4779D-04 Stress= 0.1386D+04 Load= 0.1386D+04
 Member # 5
 Strain= 0.5518D-04 Stress= 0.1600D+04 Load= 0.1600D+04
*******************************************
 Pin # 1
 displacements 0.4369D-02 -0.1643D-01
 Pin # 2
 displacements 0.2648D-02 0.0000D+00
 Pin # 3
 displacements 0.0000D+00 0.0000D+00
 Pin # 4
 displacements -0.1720D-02 -0.1290D-02
```