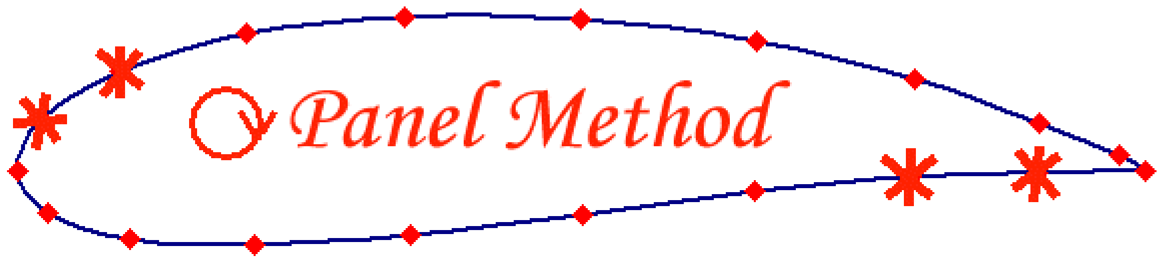

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Goal

Calculate the pressure distribution on an airfoil and the lift of an airfoil when it is placed in an inviscid parallel flow approaching the airfoil with an angle of attack.

The Method

- The contour defining the airfoil is partitioned into N number of straight panels or elements.
- A fictitious source of constant density m_j is distributed on the j-th (j = 1 , 2 , 3 , ... , N) panel.
- A fictitious point vortex of constant density is distributed on all the panels. Note that the source density changes from panel to panel, but the vortex density remains the same.
- The N number of source densities and the vortex density are computed by solving N+1 number of simultaneous equations. These equations are obtained by enforcing the no-penetration condition at one control point per panel and the Kutta condition at the trailing edge.

The Velocity Potential

The velocity potential is constructed by using the principle of superposition with contributions from the free stream, the source distribution, and the vortex distribution. This potential as seen by an observer in the fluid is written as

$$\phi = V_\infty (x\cos\alpha + y\sin\alpha) + \sum_{j=1}^{N} \int_{\text{element } j} \left(\frac{m_j}{2\pi} \ln r - \frac{\gamma}{2\pi}\theta \right) ds$$

On the j-th element, an element of length *ds* is taken. With *r* being the distance between this element and the observer, and θ being the angle this *r* makes with the x-axis; the integrand is the velocity potential due to source and vortex distribution on the element *ds*. By integrating this elemental potential over panel j and then summing over all the panels, the total potential is obtained.

Panel Parameters

The starting and the ending points of a panel are at (x_i, y_i) and (x_{i+1}, y_{i+1}), repectively. Five quantities are associated with each panel, namely, the length of the panel, the coordinate of the control point on the panel, and the orientation of the panel. These quantities are calculated as follows:

$$(\bar{x}_p, \bar{y}_p) = \left(\frac{x_p + x_{p+1}}{2}, \frac{y_p + y_{p+1}}{2} \right)$$

$$\ell_p = \sqrt{(x_{p+1} - x_p)^2 + (y_{p+1} - y_p)^2}$$

$$\sin\theta_p = \frac{y_{p+1} - y_p}{\ell_p} \quad , \quad \cos\theta_p = \frac{x_{p+1} - x_p}{\ell_p}$$

No-Penetration Condition

The no-penetration condition enforced at the control point on each panel yields the following set of equations.

$$\sum_{j=1}^{N} m_j A_{ij} + \gamma A_{i,N+1} = b_i \quad \text{for } i = 1,2,3,\ldots,N$$

where

$$A_{ij} = \frac{1}{2\pi}\sin(\theta_i - \theta_j)\ln\left(\frac{r_{i,j+1}}{r_{ij}}\right) + \frac{1}{2\pi}\beta_{ij}\cos(\theta_i - \theta_j)$$

$$A_{i,N+1} = \sum_{j=1}^{N}\left[\frac{1}{2\pi}\cos(\theta_i - \theta_j)\ln\left(\frac{r_{i,j+1}}{r_{ij}}\right) - \frac{1}{2\pi}\beta_{ij}\sin(\theta_i - \theta_j)\right]$$

and

$$b_i = V_\infty \sin(\theta_i - \alpha)$$

Kutta Condition

The Kutta Condition states that the tangential components of velocities approaching the trailing edge along the top and bottom surfaces are equal. This equality condition on velocities is imposed at the two nodes, one on the top surface of the airfoil and the other on the bottom surface, adjacent to the trailing edge.

$$\sum_{j=1}^{N} m_j A_{N+1,j} \quad + \quad \gamma A_{N+1,N+1} \quad = \quad b_{N+1}$$

where

$$A_{N+1,j} = \sum_{k=1,N}\left[\frac{1}{2\pi}\beta_{kj}\sin(\theta_k - \theta_j) - \frac{1}{2\pi}\cos(\theta_k - \theta_j)\ln\left(\frac{r_{k,j+1}}{r_{kj}}\right)\right]$$

$$A_{N+1,N+1} = \sum_{k=1,N}\sum_{j=1}^{N}\left[\frac{1}{2\pi}\beta_{kj}\cos(\theta_k - \theta_j) + \frac{1}{2\pi}\sin(\theta_k - \theta_j)\ln\left(\frac{r_{k,j+1}}{r_{kj}}\right)\right]$$

and

$$b_{N+1} = -V_\infty \cos(\theta_1 - \alpha) - V_\infty \cos(\theta_N - \alpha)$$

r_{ij} and β_{ij} Calculation

$$r_{ij} = \sqrt{(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_j)^2}$$

$$\beta_{ij} = \text{(D)ATAN2}\left[\begin{matrix}(\bar{y}_i - y_{j+1})(\bar{x}_i - x_j) - (\bar{x}_i - x_{j+1})(\bar{y}_i - y_j), \\ (\bar{y}_i - y_{j+1})(\bar{y}_i - y_j) + (\bar{x}_i - x_{j+1})(\bar{x}_i - x_j)\end{matrix}\right] \text{ when } i \neq j$$

$$= \pi \text{ when } i = j$$

Pressure Calculation

The tangential component of velocity at each control point can be calculated from the equation

$$V_{ti} = V_\infty \cos(\theta_i - \alpha) + \sum_{j=1}^{N} \frac{m_j}{2\pi}\left[\beta_{ij} \sin(\theta_i - \theta_j) - \cos(\theta_i - \theta_j) \ln\left(\frac{r_{i,j+1}}{r_{ij}}\right)\right]$$

$$+ \frac{\gamma}{2\pi} \sum_{j=1}^{N} \left[\sin(\theta_i - \theta_j) \ln\left(\frac{r_{i,j+1}}{r_{ij}}\right) + \beta_{ij} \cos(\theta_i - \theta_j)\right]$$

The pressure at each control point can be calculated by utilizing the Bernoulli Equation

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p_i + \frac{1}{2}\rho V_{ti}^2$$

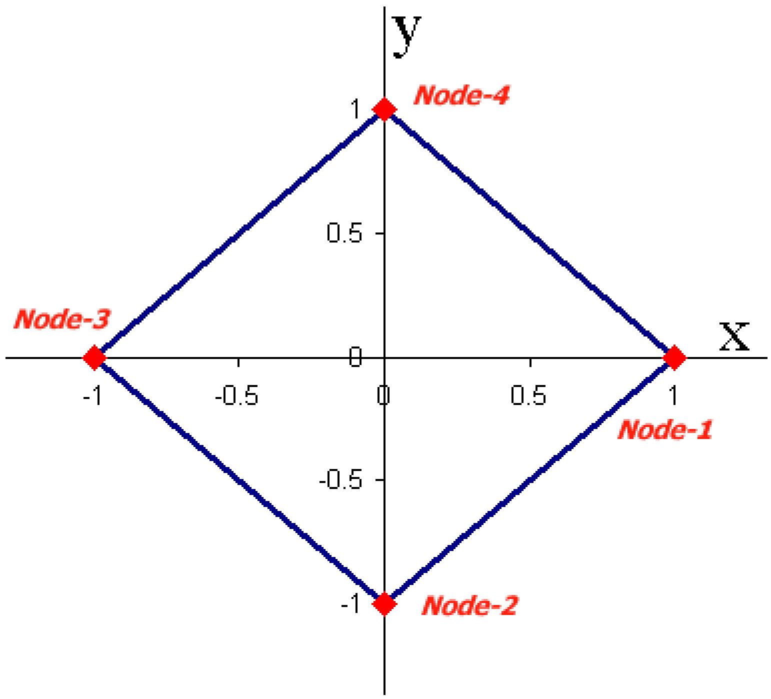
Lift Coefficient

The lift coefficient is calculated from the equation

$$C_L = \frac{2\gamma}{V_\infty} \sum_{j=1}^{N} \ell_j$$

Test Case (Freestream velocity=10, Attack angle=0, Density=1)

For testing purpose, create a 4-panel grid for a circle of unit radius.



The results for the A_{ij} and b_i for the 4-noded circle problem are:

A_{ij} Matrix

0.5	0.1281	0.1476	0.1281
0.1281	0.5	0.1281	0.1476
0.1476	0.1281	0.5	0.1281
0.1281	0.1476	0.1281	0.5

b_i Vector

-7.071	7.071	7.071	-7.071

The test results for source strengths m_i for the 4-noded circle problem are:

m_i Vector

-20.06	20.06	20.06	-20.06

The tests results for u_i, v_i, and p_i for the 4-noded circle problem are:

p_i, u_i, v_i

Node	p_i	u_i	v_i
1	-50	10	10
2	-50	10	-10
3	-50	10	10
4	-50	10	-10

Panel Project

Study case parameters:

- NACA 2412 airfoil
- Freestream velocity = 300 ft/sec, Density = 0.00238 slug/cu-ft

Study case plots:

1. ***p*** vs. ***xbar*** (x-coordinate of the control points) for the ***top*** and ***bottom*** surfaces of the airfoil for one attack-angle.
2. Lift coefficient versus attack-angle plot.