BLADE ELEMENT THEORY

Assumptions:

- The blade is composed of aerodynamically independent, narrow strips or elements.
- A differential blade element of chord C and width dr, located at a rauis r from the rotor axis is considered as an airfoil section.

Theory:

The airscrew is advancing at a speed of V and the velocity at the disc is given by $V_d = V_0(1 + a) = V_0 + v$.

The slipstream behind an airscrew rotates in the same sense as the blades about the airscrew axis (z-axis).

The angular velocity of the airscrew blades are Ω and the angular velocity of the flow in the plane of the blades is $a_1\Omega$ where a_1 is a constant for the element considered.

This element has a linear velocity in the plane of the rotationb of Ωr and the flow is itself rotating in the same plane and sense with an angular velocity $a_1\Omega$.

Thus the relative linear velocity of the element relative to the air in this plane is $\Omega r(1 - a_1) or r(\Omega - \omega)$.

θ is the geometric helix angle of the element measured between the zero lift-line of the element and the rotor disc.

α is the angle between the relative velocity V_R and the chord.

ϕ is the angle between the resultant velcoity V_R and the plane if rotation. Then from the geometry one can deduce:

$$\theta = \phi - \alpha\phi = arctan\tfrac{V_d}{\Omega-\omega} = arctan\tfrac{V_0(1+a)}{\Omega(1-a_1)}$$

or

$$\alpha = \theta - arctan\frac{V_0 + v}{\Omega - \omega} \tag{1}$$

STATIONARY BLADE ELEMENT WITH AIR FLOWING PAST IT

The elemental lift expressed by the blade element is:

$$\partial L = \frac{1}{2}\rho V_R^2 C_l c dr \tag{2}$$

Where $V_R^2 = (V + \omega)^2 + (r * (\Omega - \omega))^2$

The elemental drag is found to be:

$$\partial D = \frac{1}{2}\rho V_R^2 C_d c dr \tag{3}$$

Where C_l and C_d are 2-D aerodynamic characteristics of the blade section. From the force diagram

$$\partial T = \partial L \cos\phi - \partial D \sin\phi \tag{4}$$

If B is the number of blades the dT $= b\partial T$.
i.e

$$dT = bc\frac{1}{2}\rho V_R^2 (C_l \cos\phi - Cd \sin\phi) dr \tag{5}$$

Also from the force diagram: $\frac{\partial Q}{r} = \partial L \sin\phi + \partial D \cos\phi$

where ∂Q is the torque required to rotate the element about the axis of rotation.

Following the procedure used for dT one can show that the torque required to rotate B blades is given by:

$$dQ = Bc(\frac{1}{2}\rho V_R^2)(C_l \sin\phi + C_d \cos\phi) r dr \tag{6}$$

From dT and dQ one can obtain the thrust (T), torque (Q), and power required (P) using the equations given below:

$$T = \int_0^R dT \tag{7}$$

$$Q = \int_0^R dQ \tag{8}$$

$$P = Q\Omega \tag{9}$$

$\frac{dT}{dr}$ and $\frac{dQ}{dr}$ are known as the thrust grading and the torque grading respectivly.

Thrust dT from momentuem principle: $dT = \dot{m}\delta V$ = (area of annulus $*$ velocity than the annulus $*$ density) δv

or

$dT = (2\pi r dr * V(1+a)\rho)(V_e - V_0)$ $dT = (2\pi r dr * V(1+a)\rho)(v_0(1+2a) - V_0)$

$$= (2\pi r\rho V_0^2(1+a)(2a)dr \tag{10}$$

Equating 5 and 10

$bc(\frac{1}{2}\rho V_R^2)(C_l \cos\phi - C_d \sin\phi)dr = 2\pi r\rho V_0^2(1+a)(2a)dr$

upon rearrangment:

$$\frac{bc}{4\pi r}V_R^2(C_l\cos\phi - Cd\sin\phi) = V_0^2(1+a)(2a) \tag{11}$$

but from the velocity diagram:

$$V_R = V_0(1+a)1/\sin\phi \tag{12}$$

substitute (12) in (11) and rearrange to get:

$$\frac{bc}{8\pi r}1/\sin^2\phi(C_l\cos\phi - C_d\sin\phi) = \frac{a}{1+a} \tag{13}$$

Relative angular velocity of the flow far upstream is Ω

Relative angular velocity of the flow at the disc is $\Omega - a_1\Omega$ or $\Omega - \omega$

Relative angluar velocity of the flow far down stream is $\Omega - 2a_1\Omega$ or $\Omega - 2\omega$.

Elemental tourque in the annulus dQ is equal to the anglues momentum change per unit time in the annulus $= \dot{m}(\delta V_T)r$

$dQ = (2\pi r dr\rho V_o(1+a)(2a_1\Omega r)(r)$

$$dQ = 4\pi r^3\rho V_0(1+a)\Omega a_1 dr \tag{14}$$

Equating 6 and 14

$bcr\frac{1}{2}\rho V_R^2)(C_l\sin\phi + C_d\cos\phi)dr$

$$= 4\pi r^3\rho v_0(1+a)\Omega a_1 dr \tag{15}$$

From the velocity diagram:

$$V_R = \Omega r(1-a_1)\sec\phi \tag{16}$$

Substitute 12 and 16 into equation 15 and rearrange to get

$$\frac{bc}{8\pi r}(1/\sin\phi)(1/\cos\phi)(C_l\sin\phi + C_d\cos\phi = \frac{a_1}{1-a_1} \tag{17}$$

<u>Summary</u>

- Assume a and a_1
- Calculate $V_0(1+a)$ and $\Omega r(1-a_1)$ the velocity in the axial and tangential edirection at the plane of rotation.
- Calcualte α and ϕ where $\phi = \arctan\frac{V_0+v}{(\Omega - omega)r}$ $\alpha = \theta - \phi$
- calculate C_l and C_d from 2-D sectional charateristics of the airfoil used.
- calclutate new a and a_1 using : $\frac{a}{a+1} = \frac{bc}{8\pi r}1/\sin^2\phi(C_l cos\phi - C_d\sin\phi)$
  $\frac{a_1}{1-a_1} = \frac{bc}{8pir}1/\cos\phi\sec\phi(C_l\sin\phi + C_d\cos)$
- if a and a_1 are convervged go to step seven otherwise go to step 2 with new values of a and a_1. (For numerical convergance take an arithmetic mean of old a and a)
- calculate T, Q, P and C_T, C_Q, and C_P