AerE 411, Homework 3 (13 points)

Figures are required for those HW solutions which have figures
To receive credit for computer solutions, you must include the source code

Note: this document is subject to change by the instructor during the semester.

Warning: you are probably better off converting all units to SI and solving the problem in metric. In the past there has been quite a bit of confusion about how to interpret Oates’ equations in English units.

Note: To receive credit for a numerically solved problem, you must include a listing of your source code and all of your plots must be computer generated.

1. (1 point) Show that when the afterburner is turned off, the ideal cycle analysis for the turbojet with an afterburner reduces to the equations for a turbojet.

2. (1 point each) Oates Chapter 5 (Cycle Analysis): 5.1, 5.2, 5.5, 5.6

3. (1 point) Oates Chapter 3 (Rockets): 3.8

The following two problems make use of the NASA Glenn Research Center EngineSim demonstration program, which is located at http://www.lerc.nasa.gov/WWW/K–12/airplane/ngnsim.html, or better yet use the one located at http://www.lerc.nasa.gov/WWW/K–12/airplane/engsim.html (also see the course webpage). Use the Firefox browser in the labs when running these programs, problems have been experienced when using other browsers.

4. (1 point) By selecting each of the engine components in the upper left figure, write a one paragraph discussion that qualitatively explains the trade-offs that must be considered when using the different materials listed for each part of the engine. In particular:
   a) Is it always better to use the material that can tolerate the highest temperature?
   b) Should all the engine components be built from the same material?

5. (1 point) Using the “Design Mode” select a turbojet engine design that can produce 20,000 lb of thrust at 30,000 feet altitude and a flight Mach number of 0.9 (Hint: use both the “Free Stream” and “Size” menus). While doing this, set the “Output Display” to “Graphs” and look at both pressure and temperature throughout the engine (Note: the “Zoom” and “Scale” slider bars can be used to adjust the view). When designing this engine, adjust the compressor pressure ratio manually to maximize thrust at the design point (Hint: after you have set the design Mach number and altitude, click “compressor” on the upper left engine diagram and adjust the compressor pressure ratio while observing the effect on thrust). Once you have selected a design, use the “Tunnel Test Mode” to examine the operation of your engine as a function of throttle setting, altitude and Mach number. In particular, describe what happens to thrust and fuel consumption as each of these parameters is varied.
6. (2 points) Consider a ramjet on an aircraft that is cruising with a velocity of 900 m/s at an altitude of 10,000 meters (air density is 0.41351 kg/m$^3$, air static temperature is 223.26 degrees Kelvin). The heating value of the fuel is $4.4 \times 10^7$ J/kg. The engine has a nozzle throat area of 0.5 m$^2$. The region downstream of the burner can operate up to a stagnation temperature of 1,500 degrees Kelvin.

Using an ideal cycle analysis:

a) What are the Mach number and maximum $\tau_\lambda$ for this engine?
b) What is the thrust at the design point (i.e. maximum allowable throttle setting)?
c) What is the rate of fuel consumption at the design point?
d) What is the thrust at half throttle (i.e. half the design point fuel flow rate)?

You may assume that the heating value of the fuel is $4.4 \times 10^7$ J/kg. The thermodynamic constants for air are: $R = 287$ Nm/kgK, $\gamma = 1.4$, and $c_p = 1004.9$ J/kgK.

Hints: problem (b) requires the calculation of mass flux, problem (d) requires a new calculation of both mass flux and $\tau_\lambda$.

Answers: a) $M_0 = 3.0$ and $\tau_\lambda = 6.72$, b) $F = 250,792$ N, c) $m_f = 10.15$ kg/s, d) $F = 141,986$ N.

7. (1 point) A dual combustion chamber turbojet is constructed as shown below. The two combustion chambers are b1 and b2 and the two sets of turbine stages are t1 and t2. Notice the changes in station numbering!

Both of the turbines and the compressor are connected to the same spool (i.e. shaft). Construct the equation for the power balance for this engine using the normal labeling and assumptions of ideal cycle analysis, i.e. show that:

$$\tau_c = 1 + \frac{\tau_{h1}}{\tau_t}(1 - \tau_{t1}) + \frac{\tau_{h2}}{\tau_t}(1 - \tau_{t2})$$
8. **(1 point)** Consider an ideal cycle analysis of a turbojet, where the total temperature is limited at the end of the compressor (i.e. station 3). The maximum allowable total temperature at station 3 is less than the maximum allowable total temperature at station 4 due to the fact that station 3 has the highest pressure in the engine, and cannot be passively cooled.

![Diagram of turbojet cycle analysis]

You are given the total temperature ratio limits

\[ \tau_{\lambda c} = \frac{T_{t3}}{T_0} \quad \tau_{\lambda} = \frac{T_{t4}}{T_0} \]

The design point of the engine is taken so that station 3 is maintained at the maximum allowable total temperature, but the engine is run at partial throttle so that the turbine inlet is below maximum allowable total temperature. Starting from first principles (you may assume the one-dimensional form of the thrust equation), show that the specific thrust of this engine for an ideal cycle analysis is given by

\[
\frac{F}{m} = a_0 \left\{ \left[ \frac{2 \tau_b \tau_{\lambda c}}{\gamma - 1} \left( \tau_{i} \tilde{\tau}_c \tilde{\tau}_r - 1 \right) \right]^{1/2} - M_0 \right\}
\]

9. **(1 point)** A ramjet is built with a turbine downstream of the main burner, which is used to power auxiliary equipment within the aircraft. The rate of work which is extracted from the turbine is \( W \) (and \( W \) must be positive when the shaft can provide work), i.e.

![Diagram of ramjet cycle analysis]

a) Find \( W \) as a function of the \( \pi \)'s and \( \tau \)'s of the engine.

b) Using the equations for the turbojet ideal cycle analysis, show that the specific thrust of this engine is given by

\[
\frac{F}{m} = a_0 \left\{ \left[ \frac{2 \tau_b \tau_{\lambda}}{\gamma - 1} \left( \tau_{i} \tilde{\tau}_c \tilde{\tau}_r - 1 \right) \right]^{1/2} - M_0 \right\}
\]