

AerE 411, Homework 2 (8 points)

Figures are required for those HW solutions which have figures
To receive credit for computer solutions, you must include the source code

Note: this document is subject to change by the instructor during the semester.

1. (2 points) Oates. Chapter 2: 2.8. **Note:** this is the most difficult problem of the homework set.

2. (1 point) Consider the model of a combustion chamber given in class

$$(\dot{m}_{\text{air}} + \dot{m}_{\text{fuel}})h_{t2} - \dot{m}_{\text{air}}h_{t1} = \dot{m}_{\text{fuel}}h$$

The mass flow rate of air through the burner is 800 kg/s. The total temperature of the air entering the burner is 500K. The fuel has a heating value of 43,000 kJ/kg and is being burned at a fuel mass flow rate of 8 kg/s. Assuming that the combustion chamber can be modeled as a constant area heat addition, what is the total temperature and total pressure ratio across the burner (i.e. $\tau_b = T_{t2}/T_{t1}$ and $\pi_b = P_{t2}/P_{t1}$) and the downstream Mach number, as a function of upstream Mach number. Assume that the upstream Mach number is subsonic in the range 0 to 0.4.

Note: you may assume that $R = 287 \text{ Nm/kgK}$, $\gamma = 1.4$, and $c_p = 1004.9 \text{ J/kgK}$.

Hint: the problem can be solved using the quadratic formula with + signs on all the square roots, providing that you stay in the 0–0.4 upstream Mach number range.

3. (1 point) Starting from the differential form of the inviscid total energy equation

$$\frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \nabla \cdot \left[\rho \left(e + \frac{V^2}{2} \right) \vec{V} \right] = - \nabla \cdot [P\vec{V}]$$

and using the one–dimensional approximation

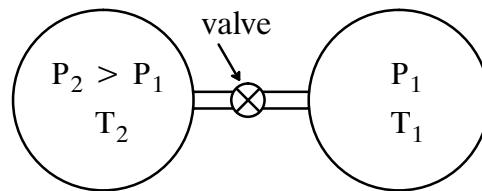
$$\vec{V} = u(x, t)\hat{\mathbf{i}} \quad \nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x}$$

show that the inviscid total energy equation may be simplified to give:

$$\rho \frac{\mathcal{D}h_t}{\mathcal{D}t} = \frac{\partial P}{\partial t}$$

Hints: write the equation in scalar form, expand the left hand side using a chain rule, use the mass conservation equation to eliminate terms, write the equation in terms of total enthalpy, then use the mass conservation equation to eliminate terms.

4. (2 points) Two well insulated containers of equal volume are separated by a small pipe and a valve. Initially, the containers have the same pressure and temperature, and each contain the same mass of air. With the valve closed, air is slowly pumped into the left container, increasing the pressure (and there is an accompanying change in temperature). During this pressure increase, all processes are adiabatic. Just before the connecting valve is opened, the pressure in the left container is greater than the pressure in the right container and the temperatures are unequal (all starting temperatures shown below are known from measurements). Assume throughout that all process changes occur slowly, the containers are insulated and no heat transfer occurs back through the pipe.



a) After the valve is opened and the two containers have come to equilibrium, show that the final temperatures within the containers are equal, i.e.

$$T_{2f} = T_{1f}$$

Hints:

- a) start from pressure equilibrium
- b) use perfect gas relations
- c) no conservation laws are required for part (a)

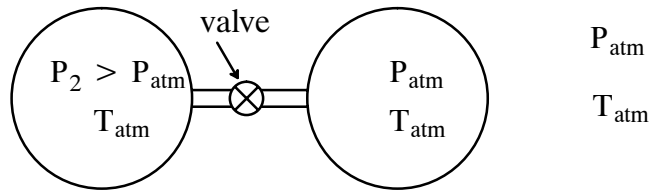
b) Show that the final temperature is given by:

$$T_{2f} = T_1 \left[\frac{1}{2} \left(1 + \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} \right) \right]^{\gamma-1}$$

Hints:

- a) you may always use perfect gas relations in each of the containers
- b) a mass balance is required (you also need to account for extra mass in the left container)
- c) you must account for the initial pressurization, which is adiabatic
- d) no other conservation law is required
- e) the containers are in equilibrium when the pressures are equal (i.e. when $P_{2f} = P_{1f}$)

5. (2 points) Consider two containers of equal size that are initially at atmospheric pressure and temperature. The tanks are sealed and the line between them is closed. The tank on the left is very slowly pumped to a pressure that is larger than that of the tank on the right (i.e. $P_2 > P_{\text{atm}}$). That is, the left tank is pumped up to a pressure P_2 slowly enough that it always stays in thermal equilibrium with its surroundings. After the compression is complete and the left tank has reached a known pressure P_2 , the valve in the line connecting the tanks is opened and air is allowed to rapidly flow from the left tank to the right tank. During this transfer of air no heat transfer occurs between any parts of the system. Assume that the pipe between the containers is never choked.



a) After the tanks have come to equilibrium, show that the final density in the left tank is:

$$\rho = \frac{P_{\text{atm}}}{RT_{\text{atm}}} \frac{1 + \frac{P_2}{P_{\text{atm}}}}{1 + \left(\frac{P_{\text{atm}}}{P_2}\right)^{1-\frac{1}{\gamma}}}$$

b) After the tanks have come to equilibrium, show that the final density in the right tank is:

$$\rho = \frac{P_{\text{atm}}}{RT_{\text{atm}}} \frac{1 + \frac{P_2}{P_{\text{atm}}} \left(\frac{P_{\text{atm}}}{P_2}\right)^{1-\frac{1}{\gamma}}}{1 + \left(\frac{P_{\text{atm}}}{P_2}\right)^{1-\frac{1}{\gamma}}}$$