

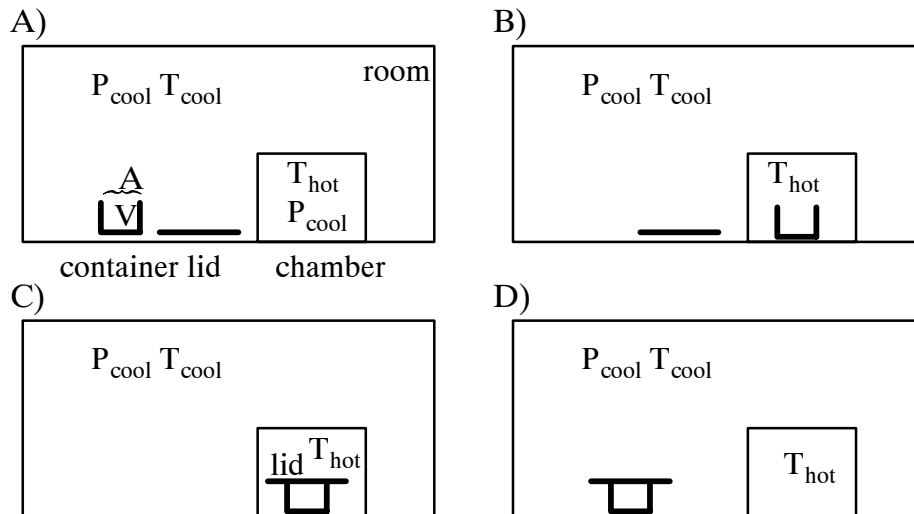
AerE 411, Homework 1 (5 points)

Figures are required for those HW solutions which have figures
To receive credit for computer solutions, you must include the source code

Note: this document is subject to change by the instructor during the semester.

1. (1 point each) Oates. Chapter 2: 2.1, 2.6

2. (1 point) An open container with volume V and cross-sectional area A in equilibrium with its surroundings is taken from a cool room at ambient temperature T_{cool} and pressure P_{cool} (figure (A) below) and is placed inside a hot chamber at temperature T_{hot} which is in equilibrium with the surrounding cool room (figure (B) below), in the sense that the pressure within the hot chamber is P_{cool} . After the system has come to equilibrium a lid is placed on top of the container so that an airtight seal forms between the container and the lid (figure (C) below). The container is then brought back into the room and allowed to cool (figure (D) below).

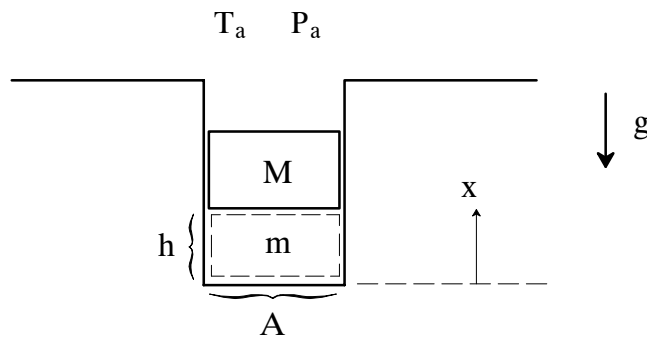


After the air within the container in figure (D) has come to thermal equilibrium with the cool room, i.e. the air temperature within the container is T_{cool} :

a) how much force (if any) is required to lift the lid from the container and break the seal. You may neglect the weight of the lid.

b) what is the force required to lift the lid if the temperature in the hot chamber is the same as the temperature in the cool room, and is your solution for part (a) consistent with this observation?

3. (1 point) Consider a simple model of combustion inside a cavity. A cylinder of mass M can freely move up and down inside a cavity with cross sectional area A . The connection between the cylinder wall and the cavity wall is air tight. Below the cylinder lies an air cavity which contains an air mass m . Initially, the entire system is in thermal and mechanical equilibrium at the ambient temperature T_a . The air within the cavity is then suddenly heated to a temperature T_{hot} . During this initial heating the cylinder does not move.



a) Show that the pressure in the air cavity immediately following the initial heating is

$$P = \left(P_a + \frac{Mg}{A} \right) \frac{T_{\text{hot}}}{T_a}$$

b) Assuming that there is no friction between the cylinder and the cavity wall and that the air within the cavity is insulated, show that the cylinder position, x , satisfies the following differential equation (where h is the initial position of the bottom of the cylinder):

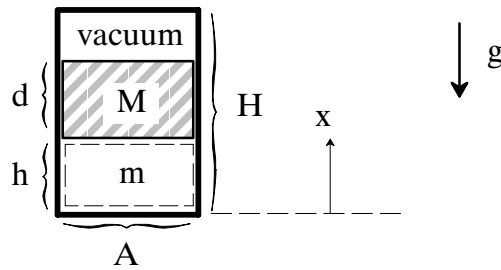
$$\frac{T_{\text{hot}}}{T_a} \left(P_a + \frac{Mg}{A} \right) \left(\frac{h}{x} \right)^\gamma A - P_a A - Mg = M \frac{d^2x}{dt^2}$$

c) Show that the final equilibrium position of the cylinder is

$$x = \left(\frac{T_{\text{hot}}}{T_a} \right)^{1/\gamma} h$$

d) Is this process reversible (yes or no)? Explain why.

4. (1 point) A piston of mass M can freely move up and down inside a closed and perfectly insulated cavity with cross sectional area A . Below the piston lies a volume of air with mass m . The connection between the piston wall and the cavity wall is air tight. The volume above the piston is a perfect vacuum. The height of the piston is d , it's cross-sectional area is A , and the total height of the cavity is H . Initially, the system is in equilibrium, with the piston lying a height h above the lower surface of the cavity. Heat is then gradually added to the air within the cavity.



a) When the temperature within the cavity satisfies $T < Mg(H - d)/(mR)$, show that the air density and temperature are:

$$\rho = \frac{m}{xA} \quad T = \frac{Mg}{mR}x$$

b) After reaching the temperature $T = Mg(H - d)/(mR)$, an extra amount of heat, Q , is added to the air. Show that the final air pressure is:

$$P = \frac{Mg}{A} + \frac{RQ}{c_v A(H - d)}$$