

$$\left(1 + \frac{r-1}{2} M_9^2\right)^{\frac{r}{r-1}} = \pi_m \gamma_r^{\frac{r}{r-1}}$$

then

$$1 + \frac{r-1}{2} M_9^2 = \pi_m^{\frac{r-1}{r}} \gamma_r \quad (2)$$

$$M_9 = \sqrt{\frac{2}{r-1} \left(\pi_m^{\frac{r-1}{r}} \gamma_r - 1 \right)}$$

and (from ① + ②)

$$\frac{T_9}{T_0} = \gamma_m \gamma_2 \left(\pi_m^{\frac{r-1}{r}} \gamma_r \right)^{-1} = \frac{\gamma_m \gamma_2}{\gamma_r} \pi_m^{\frac{1-r}{r}}$$

Note: $\gamma_m + \pi_m$ are not connected by isentropic relation (need to use mixer equations)

then

$$\begin{aligned} \frac{u_9}{u_0} &= \sqrt{\frac{T_9}{T_0}} \frac{M_9}{M_0} = \left[\frac{\gamma_m \gamma_2}{\gamma_r} \pi_m^{\frac{1-r}{r}} \right]^{\frac{1}{2}} \frac{1}{M_0} \left[\frac{2}{r-1} \left(\pi_m^{\frac{r-1}{r}} \gamma_r - 1 \right) \right]^{\frac{1}{2}} \\ &= \frac{1}{M_0} \left[\frac{2}{r-1} \gamma_m \gamma_2 \left(1 - \frac{\pi_m^{1-r}}{\gamma_r} \right) \right]^{\frac{1}{2}} \end{aligned}$$

$$\frac{F}{\dot{m} u_0} = (1 + \beta) \frac{1}{M_0} \left[\frac{2 \gamma_m \gamma_2}{r-1} \left(1 - \frac{\pi_m^{1-r}}{\gamma_r} \right) \right]^{\frac{1}{2}} - 1$$