use $\rho = \frac{P}{RT}$

$$\frac{P_1}{T_1} + \frac{P_2}{T_2} = \frac{P_{1f}}{T_{1f}} + \frac{P_{2f}}{T_{2f}} = P_{2f}\left(\frac{1}{T_{1f}} + \frac{1}{T_{2f}}\right)$$

need to get P's in terms of T's

initial pressurization

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

divide above by P_1

$$\frac{P_{1f}}{P_1} = \left(\frac{T_{1f}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{1}{T_1} + \frac{1}{T_2}\frac{P_2}{P_1} = \frac{P_{1f}}{P_1}\left(\frac{1}{T_{1f}} + \frac{1}{T_{2f}}\right)$$

$$\frac{1}{T_1} + \frac{1}{T_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_{1f}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{T_{1f}} + \frac{1}{T_{2f}}\right)$$

$$T_{1f} = T_{2f}$$

$$\frac{1}{T_1} + \frac{1}{T_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_{1f}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}\frac{2}{T_{1f}}$$

mult by T_1

$$1 + \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}-1} = T_1^{1-\frac{\gamma}{\gamma-1}}\, 2\, T_{1f}^{\frac{\gamma}{\gamma-1}-1}$$

$$\frac{\gamma}{\gamma-1} - 1 = \frac{\gamma - \gamma + 1}{\gamma - 1} = \frac{1}{\gamma-1}$$