$\Rightarrow$

$$\frac{C_v}{R} P_f V_{cyl} + \frac{C_v}{R} P_f V_c - \frac{C_v}{R} P_i V_c = -P_f V_{cyl}$$

but:

$$C_v = \frac{R}{r-1}$$

$\Rightarrow$

$$P_f V_{cyl} + P_f V_c - P_i V_c = -(r-1) P_f V_{cyl}$$

$$(P_f - P_i) V_c = -r P_f V_{cyl}$$

need V_{cyl}/V_c

$$\frac{V_{cyl}}{V_c} = \frac{1}{r P_f}(P_i - P_f) = \frac{1}{r}\left(\frac{P_i}{P_f} - 1\right)$$

$$\frac{T_{cyl}}{T_i} = \frac{P_f}{P_i}\frac{1}{r}\left(\frac{P_i}{P_f} - 1\right)\left[1 - \left(\frac{P_f}{P_i}\right)^{1/r}\right]^{-1}$$

$$\boxed{\frac{T_{cyl}}{T_i} = \frac{1}{r}\,\frac{1 - P_f/P_i}{1 - (P_f/P_i)^{1/r}}}$$

Q.E.D.