divide by P_i

$$\frac{C_v}{R}\left(\frac{P_f}{P_i}\frac{V_c}{V}+\frac{P_f}{P_i}-1\right)=-\frac{P_f}{P_i}\frac{V_c}{V}$$

$$c_p - c_v = R \qquad \gamma - 1 = R/c_v$$

rearrange above

$$\frac{P_f}{P_i}\frac{V_c}{V}+\frac{P_f}{P_i}-1=-(\gamma-1)\frac{P_f}{P_i}\frac{V_c}{V}$$

$$\left(1+\gamma-1\right)\frac{P_f}{P_i}\frac{V_c}{V}=1-\frac{P_f}{P_i}$$

$$\frac{V_c}{V}=\frac{1}{\gamma}\frac{P_i}{P_f}\left(1-\frac{P_f}{P_i}\right)$$

so back to

$$T_{cf}=\frac{\frac{P_f}{P_i}T_i\frac{V_c}{V}}{\left[1-\left(\frac{P_f}{P_i}\right)^{1/\gamma}\right]}$$

$$\Rightarrow \quad T_{cf}=T_i\frac{\frac{P_f}{P_i}\frac{1}{\gamma}\frac{P_c}{P_f}\left(1-P_f/P_i\right)}{1-\left(\frac{P_f}{P_i}\right)^{1/\gamma}}$$

$$\boxed{T_{cf}=\frac{1}{\gamma}T_i\frac{1-\frac{P_f}{P_i}}{1-\left(\frac{P_f}{P_i}\right)^{1/\gamma}}}$$

QED