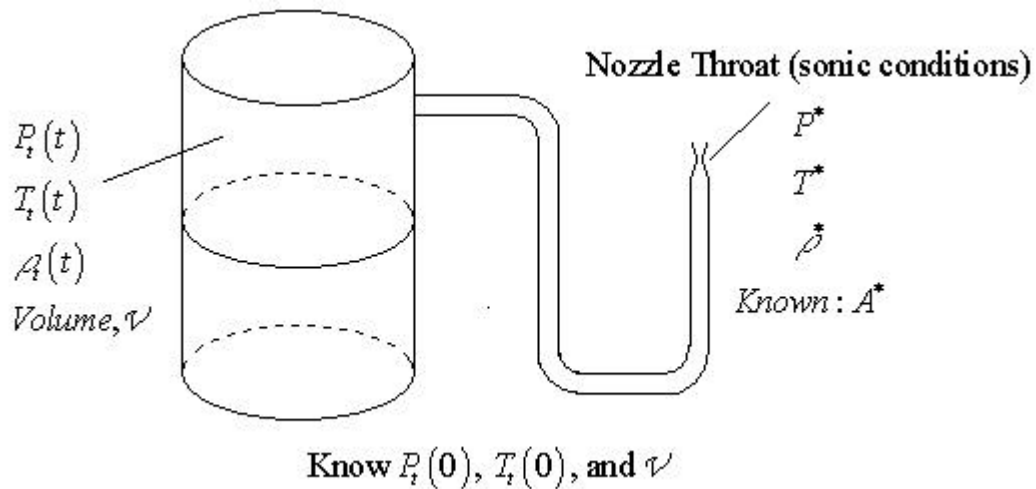


# 1. Blowdown of a Pressurized Tank

This experiment consists of starting with a tank of known initial conditions (i.e. pressure, temperature etc.) and exiting the gas through a choked nozzle. The objective is to test the theory developed which predicts the time required for the tank to be evacuated (blowdown). One of the critical issues to be resolved from this test is to characterize the expansion process in some manner. That is, we will make a couple of assumptions in the following derivation regarding the temperature in the tank. We can use experimental data to determine the validity of our assumptions.

Consider first a tank system consisting of a tank of pressurized air, a nozzle of known shape, and piping connecting the nozzle to the tank.



We will start with the unsteady mass conservation equation written in integral form:

$$\frac{\partial}{\partial t} \iiint_V \rho dV + \iint_S \rho \mathbf{V} \cdot \hat{\mathbf{n}} dS = 0 \quad (1.1)$$

The surfaces making up the boundary of the control volume in this case are the walls of the tank and piping plus the area of the nozzle throat, i.e.:

$$S = S_{\text{tank}} + S_{\text{pipe}} + S_{\text{throat}} \quad (1.2)$$

Note that no air is flowing across either the tank or piping walls, therefore

$$\begin{aligned} \mathbf{V} \cdot \hat{\mathbf{n}}_{\text{tank}} &= 0 \\ \mathbf{V} \cdot \hat{\mathbf{n}}_{\text{pipe}} &= 0 \end{aligned} \quad (1.3)$$

At the nozzle throat we will assume that the air flow is one-dimensional and perpendicular to the nozzle throat cross-section. The velocity vector is then parallel to the throat area unit vector

$$\mathbf{V} \cdot \hat{\mathbf{n}}_{throat} = \mathbf{V} \cdot \hat{\mathbf{n}}_{tank} = V^* \quad (1.4)$$

The second term in (1.1) is now non-zero at the throat only and evaluated there it becomes

$$\iint_S \mathbf{r} \mathbf{V} \cdot \hat{\mathbf{n}} dS = \mathbf{r}^* V^* A^* \quad (1.5)$$

Turning our attention now to the first term in (1.1), we know that the control volume is not changing with time, so that the partial derivative can then be brought inside the integral

$$\frac{\partial}{\partial t} \iiint_V \mathbf{r}_t dV = \iiint_V \frac{\partial \mathbf{r}_t}{\partial t} dV \quad (1.6)$$

We will also assume that the density in the tank is uniform over the entire tank volume. That is, at any given instant in time, the density does not vary from one point in the tank to another. Using this, (1.6) can be evaluated as

$$\iiint_V \frac{\partial \mathbf{r}_t}{\partial t} dV = \frac{\partial \mathbf{r}_t}{\partial t} \iiint_V dV = V \frac{\partial \mathbf{r}_t}{\partial t} \quad (1.7)$$

Substituting (1.7) and (1.5) into (1.1) we get the equation governing the blowdown of the tank:

$$V \frac{\partial \mathbf{r}_t}{\partial t} + \mathbf{r}^* V^* A^* = 0 \quad (1.8)$$

The velocity through the throat can be related to the thermodynamic properties at the throat:

$$\begin{aligned} V^* &= M^{*1} a^* \\ V^* &= \sqrt{gRT^*} \end{aligned} \quad (1.9)$$

Now we note that if the flow through the nozzle is choked, then the properties at the nozzle throat will remain constant. Moreover, these properties can be related to total conditions (i.e. conditions in the tank) by assuming an isentropic expansion in the nozzle and applying the isentropic relations at the throat. Note that assuming the tank conditions are equal to the total conditions in the tank neglects any losses in the pipes. Since the flow is choked, Mach number at the throat is unity, and the isentropic relations reduce to

$$\frac{T^*}{T_t} = \frac{2}{g+1}$$

$$\frac{r^*}{r_t} = \left( \frac{2}{g+1} \right)^{\frac{1}{g-1}} \quad (1.10)$$

Applying (1.9) and (1.10) to (1.8) gives, after some simplification, the governing differential equation in terms of tank variables only

$$\boxed{V \frac{\partial r_t}{\partial t} + \sqrt{gRA^*} \left( \frac{2}{g+1} \right)^{\frac{g-1}{2(g-1)}} T_t^{1/2} r_t = 0} \quad (1.11)$$

Equation (1.11) is a single differential equation with two unknowns,  $\rho_t$  and  $T_t$ , assuming that we know the properties of the gas, the volume of the tank, and the area of the nozzle throat. We must close the system by adding an extra equation or making an additional assumption. The approach taken here is to make two different assumptions regarding the expansion in the tank and compare results with the experiments. It is not clear what we can assume about the density in the tank, so we will make an assumption about the tank temperature and solve for the density.

### **Isothermal Expansion**

Our first assumption will be to assume that the temperature in the tank is constant (isothermal). This is most accurate for situations in which the tank wall is thin and expansion is slow. In this case, any drop in temperature inside the tank would create a heat flux from a constant temperature reservoir, say the room, and the tank temperature would warm back up to room temperature. The isothermal approximation assumes that this process is instantaneous and the temperature of the tank stays effectively constant. Since the temperature is now known, we have a first order differential equation for the tank density of the form

$$A \frac{dx}{dt} + Bx = 0 \quad (1.12)$$

where A and B are constants.

Equation (1.12) has a solution of the form

$$x = c_1 e^{(-B/A)t} \quad (1.13)$$

The constant  $c_1$  is determined from initial conditions, i.e.  $c_1 = x(0) = \rho_t(0)$ . Applying the solution (1.13) to (1.11) gives the tank density as a function of time:

$$\mathbf{r}(t) = \mathbf{r}_t(0) e^{-I t} \quad (1.14)$$

where

$$I = \frac{1}{V} \sqrt{g} R A^* \left( \frac{2}{g+1} \right)^{\frac{g+1}{2(g-1)}} T_t^{1/2} \quad (1.15)$$

In the lab it is most convenient to measure the tank gauge pressure. The solution can be rewritten in terms of gauge pressure by using the equation of state

$$\begin{aligned} P_t &= \mathbf{r}_t R T_t \\ \mathbf{r}_t &= \frac{P_t}{R T_t} = \frac{P_g + P_{atm}}{R T_t} \\ \mathbf{r}_t(0) &= \frac{P_g(0) + P_{atm}}{R T_t(0)} \end{aligned} \quad (1.16)$$

where  $P_g$  is the tank gauge pressure and  $P_{atm}$  is the atmospheric pressure. Substituting (1.16) into (1.14) gives

$$\boxed{P_g(t) = (P_g(0) + P_{atm}) e^{-I t} - P_{atm}} \quad (1.17)$$

Equation (1.17) is valid for the isothermal blowdown of a tank which exits through a choked nozzle. For all other situations this equation is an approximation. Note that our original equation, (1.1) made no assumption as to whether the flow is viscous or inviscid, however recall that (1.10) assumed an isentropic expansion through the diffuser of the nozzle and neglected losses in the pipe, so (1.17) is also subject to these conditions.

For a given nozzle and exit pressure, the total pressure corresponding to first critical ( $P_{t1st}$ ) in the nozzle can be found. This is the point at which the choked flow assumption first becomes invalid. Substituting the value for  $P_{t1st}$  into (1.17) and solving for  $t$  can then give the time to first critical and an estimation of the blowdown time of the tank.

## **Isentropic Expansion**

An isentropic expansion is effectively the opposite of an isothermal expansion. In this case, we assume that there is no heat transfer through the walls of the tank. That is the expansion is adiabatic. The theory of thermodynamics has then shown that the density and temperature can be related by

$$\frac{\rho_t(t)}{\rho_t(0)} = \left( \frac{T_t(t)}{T_t(0)} \right)^{\frac{1}{\gamma-1}}$$

*or*

(1.18)

$$T_t = T_t(0) \left( \frac{\rho_t}{\rho_t(0)} \right)^{\gamma-1}$$

We can now eliminate the tank temperature in (1.11). After some simplification, the equation becomes

$$V \frac{\partial \rho_t}{\partial t} + \sqrt{\gamma R A^*} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma-1}{2(\gamma-1)}} T_t^{1/2}(0) \rho_t^{\frac{1-\gamma}{2}}(0) \rho_t^{\frac{\gamma-1}{2}} = 0$$
(1.19)

Clearly we can determine the initial tank conditions and the only unknown left is once again the tank density. Equation (1.19) is a first order non-linear differential equation of the form

$$\frac{dx}{dt} + Dx^n = 0$$
(1.20)

for n any number. This equation has a solution of the form

$$x = \left[ D(n-1)t + c_2 \right]^{\frac{1}{1-n}}$$
(1.21)

where  $c_2$  is again determined from initial conditions. If we apply (1.19) to (1.21), we get a solution for the density in the tank as a function of time. In our case

$$n = \frac{\gamma+1}{2}$$
(1.22)

The resulting solution is

$$\mathbf{r}_t(t) = \left[ \frac{(\mathbf{g}-1)\sqrt{\mathbf{g}RA^*}}{2V} \left( \frac{2}{\mathbf{g}+1} \right)^{\frac{\mathbf{g}+1}{2(\mathbf{g}-1)}} T_t^{1/2}(0) \mathbf{r}_t^{\frac{1-\mathbf{g}}{2}}(0)t + c_2 \right]^{\frac{2}{1-\mathbf{g}}} \quad (1.23)$$

If we set  $t = 0$ , we can determine the unknown constant

$$\begin{aligned} \mathbf{r}_t(0) &= c_2^{\frac{2}{1-\mathbf{g}}} \\ c_2 &= \mathbf{r}_t^{\frac{1-\mathbf{g}}{2}}(0) \end{aligned} \quad (1.24)$$

Note that we can factor (1.24) out of (1.23). In this case, (1.23) becomes

$$\mathbf{r}_t(t) = \mathbf{r}_t(0) \left[ \frac{(\mathbf{g}-1)\sqrt{\mathbf{g}RA^*}}{2V} \left( \frac{2}{\mathbf{g}+1} \right)^{\frac{\mathbf{g}+1}{2(\mathbf{g}-1)}} T_t^{1/2}(0)t + 1 \right]^{\frac{2}{1-\mathbf{g}}} \quad (1.25)$$

We again wish to write the final solution in terms of the tank gauge pressure. The isentropic relation between pressure and density is

$$\frac{\mathbf{r}_t(t)}{\mathbf{r}_t(0)} = \left( \frac{P_t(t)}{P_t(0)} \right)^{\frac{1}{\mathbf{g}}} = \left( \frac{P_g(t) + P_{atm}}{P_g(0) + P_{atm}} \right)^{\frac{1}{\mathbf{g}}} \quad (1.26)$$

The final solution for the tank gauge pressure is

$$P_g(t) = (P_g(0) + P_{atm}) \left[ \frac{(\mathbf{g}-1)\sqrt{\mathbf{g}RA^*}}{2V} \left( \frac{2}{\mathbf{g}+1} \right)^{\frac{\mathbf{g}+1}{2(\mathbf{g}-1)}} T_t^{1/2}(0)t + 1 \right]^{\frac{2\mathbf{g}}{1-\mathbf{g}}} - P_{atm} \quad (1.27)$$

Once again, assuming the tank pressure at first critical is known, this equation can be solved for the time to first critical and the tank blowdown time.