Nonlinearities, harmonics, and trends in dynamic applications of Terfenol-D

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ABSTRACT

We are working to understand Terfenol-D's behavior in dynamic applications. Experimental results demonstrate that the amplitudes of harmonics present in Terfenol-D transducer output displacement spectra increase with increasing drive-current amplitude. An empirical mathematical model which predicts harmonics is presented. The model approximates both the nonlinear strain versus magnetic flux density behavior (ε vs B), and the magnetic hysteresis (B vs H) of the material. Trends are shown for displacement versus drive-current amplitude as a function of frequency. Generally, displacements increase with increasing drive-current amplitude, and decrease with increasing drive frequency (when operated away from mechanical resonance). Single frequency drive-current results are compared with the trends predicted by the mathematical model.

OBJECTIVES

1) Develop a mathematical model which predicts transducer output displacement as a function of input current.
2) Compare model predictions with experimental measurements.
3) Investigate displacement trends as a function of drive-current amplitude and frequency.

INTRODUCTION

Efforts are under way to understand Terfenol-D transducer behavior when used in dynamic applications. Terfenol-D is a magnetostrictive material composed of terbium and dysprosium, alloyed with iron. The transducer in this study was of original design. It utilized a rod of Terfenol-D as the motion source (rod dimensions: 6.35 mm diameter, 50.8 mm long). The transducer is shown schematically in Fig. 1 (a).

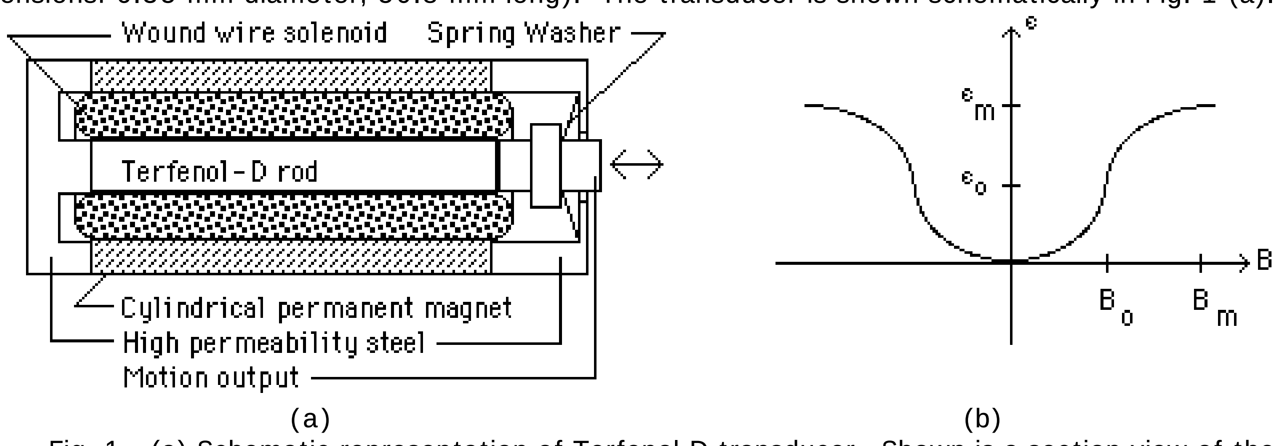

Fig. 1. (a) Schematic representation of Terfenol-D transducer. Shown is a section view of the cylindrical transducer. (b) Sketch of strain, ε, versus magnetic flux density, B, for Terfenol-D (after [1]) showing the offset flux density, B_o, and the resulting strain, ε_o.

As shown in Fig. 1 (a), the Terfenol rod is provided a constant magnetic field by the cylindrical permanent magnet. This field is added to, or subtracted from, by the field resulting from a current in the wound wire solenoid. This system was employed to obtain bidirectional motion at the output, i.e., a sinusoidal current in yielded an approximate sinusoidal motion out. This concept is further illustrated by Fig. 1 (b). The permanent magnet provides the magnetic flux density B_o shown in the figure. In response to this flux the Terfenol strains to $\varepsilon = \varepsilon_o \approx 0.5\ \varepsilon_m$, where ε_m represents the maximum extensional strain reasonably obtained from the Terfenol-D. A current in the coil is then used to vary the flux density within the material to somewhere between its maximum, B_m, and zero. Note, a negative B would again result in a positive strain. Note also the lack of hysteresis in the (nonlinear) strain-magnetic flux density relationship (Fig. 1 (b)). A sketch of B versus H, the magnetic field strength, is shown in Fig. 3 (a). The hysteresis present in output-displacement versus input-current is due primarily to this magnetic hysteresis.[1] (Actually, there is measurable hysteresis between ε and B. It seems to be a strong, and rather strange, function of prestress level.[4] Efforts to quantify this hysteresis are planned.)

Experimental measurements were performed using a Tektronix 2630 Fourier Analyzer, MTI 1000 Fotonic Sensor with an MTI 3810 probe, and a Zenith Z-386SX Data System. For all measurements reported the Tektronix was AC coupled (-3 dB @ 2.8 Hz). Measurements are typically reported in "volts." Sensitivities were: 3.25×10^4 volts/meter and 0.460 volts/ampere.

From previous investigations [2, 3] of Terfenol-D it is thought that the magnetic permeability of the material is a function of **at least** the compressive prestress, σ_o, and root-mean-square drive-current amplitude. In the present investigation, σ_o was provided by the spring washer shown in Fig. 1 (a) and maintained as a constant ($\sigma_o \approx 12$ MPa).

In this communication there is considerable discussion of the phase angle between transducer output displacement and input current. Of course, there are two sources of phase one should consider in dynamic applications of Terfenol-D transducers. The first is due to magnetic hysteresis within the material. This phase is thought to be independent of frequency.[5] The second source of phase between displacement and current is dynamic in nature, i.e., the effects of physical stiffness, damping, and mass. Thus, those experimental measurements which were designed to investigate the effects of transducer drive-current amplitude on the behavior of its magnetic circuit, were taken at frequencies well below the transducer's first physical resonance (10 Hz versus 10 000 Hz).

MATHEMATICAL MODEL

A mathematical model was sought for displacement as a function of input electric current. Toward that end, a relationship between strain and magnetic flux density was estimated. Then, magnetic flux density as a function of magnetic field strength (or equivalently, input-current) was estimated. Finally, the composite function of the second in the first was formed. For this model it was assumed that the drive-current varied sinusoidally in time, i.e., $I(t) = I\cos(\omega t)$.

The relationship shown in the first quadrant of Fig. 1 (b) was approximated by a straight line plus a sinusoid. See Fig. 2. The quantity of interest was $\varepsilon - \varepsilon_o$ as a function of flux density. This quantity is a measure of the displacement from the equilibrium position (current, $I(t) = 0$ amperes). This relationship, extended to cover the first and second quadrants, was approximated as:

$$\varepsilon - \varepsilon_o \approx \varepsilon_o\left(\frac{|B| - B_o}{B_m - B_o}\right) + \varepsilon_w \sin\left(\pi\,\frac{|B| - B_o}{B_m - B_o}\right)$$

(1a)

where: ε_w is the amplitude of a sinusoid, centered at B_o, which lends curvature to the straight line given by the first term of the equation; B_m and B_o are both assumed to be positive values; and $|B|$ is the absolute value of the total flux density. If it is further assumed that $B_m = 2B_o$, Eqn. (1a) reduces to:

$$\varepsilon - \varepsilon_o = \varepsilon_o\left(\frac{|B|}{B_o} - 1\right) - \varepsilon_W \sin\left(\pi\frac{|B|}{B_o}\right)$$

(1b)

It is interesting to note that in oscillatory use, B should not change sign (recall Fig. 1 (b)). This means that the transducer is operated in either the first and second, or third and fourth quadrants of Fig. 3 (a). (The figure is labeled as though one were operating in the first and second quadrants; however, that need not be the case.) Since the hysteresis loop is wider at one end than the other, the displacement is not symmetric about I = 0. It requires more time to traverse the wide end of the hysteresis loop than the narrow end. This is demonstrated in Fig. 4, experimental measurements of transducer output displacement versus time for two (30 Hertz) sinusoidal input

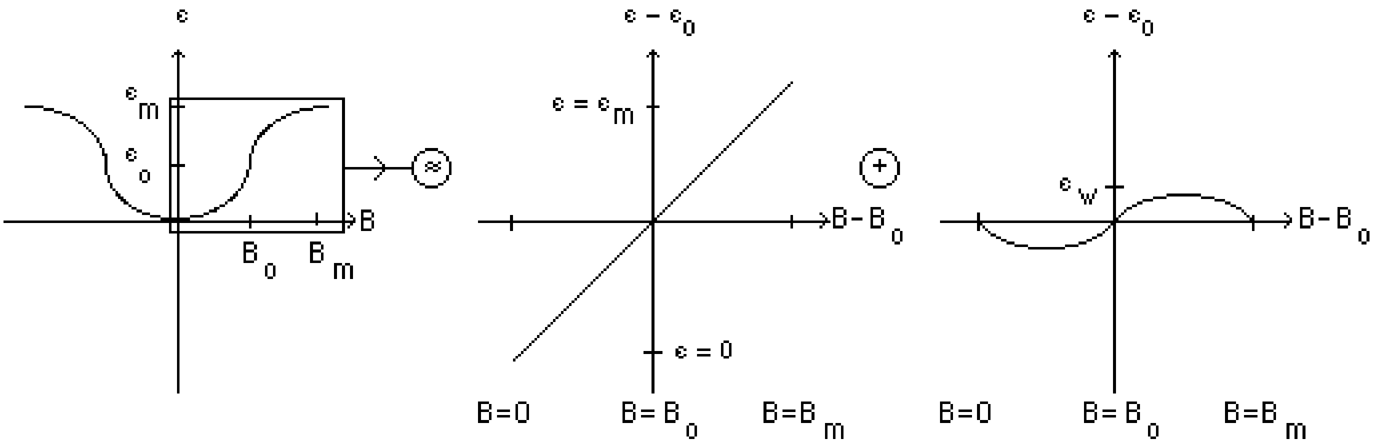

Fig. 2. Schematic of mathematical model of $ε - ε_0$ as a function of B-B_0 for the first quadrant of Fig. 1 (b).

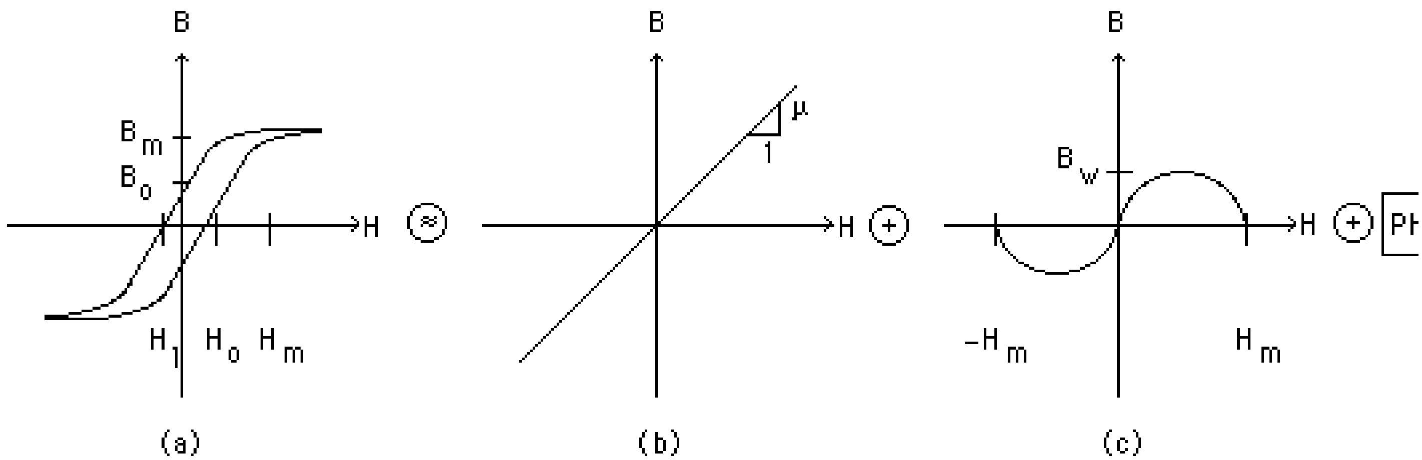

Fig. 3. (a) Sketch of B, magnetic flux density, versus H, magnetic field strength, showing magnetic hysteresis. Views (b) and (c) show symbol definitions for the approximations used in the mathematical model developed in this study. Note the addition of phase.

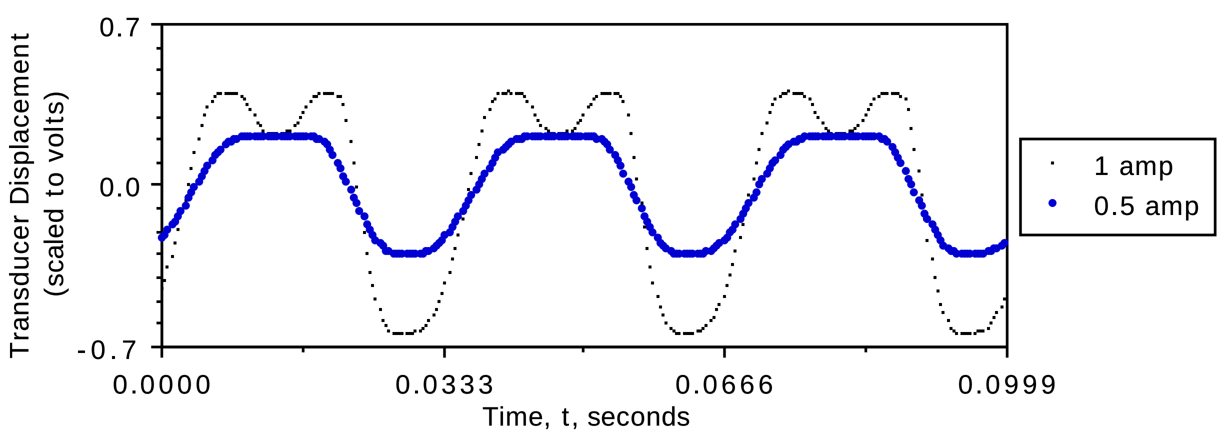

Fig. 4. Experimental transducer output displacement versus time for two different 30 Hz sinusoidal current inputs. Note difference in widths of positive and negative displacement peaks of 0.5

amp trace, and the change in direction of output travel appearing as double positive peaks in the 1 amp trace.
currents. Note that the peaks are wider for the positive displacements than for the negatives (see 0.5 amp trace). The 1.0 amp trace shows the effects of current amplitudes high enough to cause the magnetic flux density to change sign, the output "unexpectedly" reversed its direction of travel. (In these traces a positive displacement corresponds to the Terfenol-D rod becoming shorter. The transducer is operating in quadrants three and four of Fig. 3 (a).)

Referring to Fig. 3, an approximation of flux density as a function of the applied field strength was formulated as:

$$B \approx \mu H_o + \mu n I \cos(\omega t - ø) + B_w \sin\left(\pi \frac{H}{H_m}\right)$$

(2a)

where: μ = magnetic permeability of Terfenol-D, ≠ a constant,
$H_o = (H_m + H_l)/2$, see Fig. 3 ($H_l < 0$, $H_m > 0$),
n is the turns per unit length of the wound wire solenoid,
I is the amplitude of the input current (I>0),
ω is the circular frequency,
t is time,
ø is the phase angle between B and H, (ø determines the width of the hysteresis loop of Fig. 3 (a)),
B_w is the amplitude of the sinusoid used to provide curvature to the hysteresis loop,
$H = H_o + nI \cos(\omega t)$; however, relative to B use: $H = H_o + nI \cos(\omega t - ø)$,
$H_m = H_o + nI_m$ (H_m is always greater than zero), and
I_m is the current amplitude required to oscillate between H_m and H_l, see Fig. 3 (a).

This formulation resulted in reasonable approximations of displacement versus current at "large" drive-current amplitudes. What are meant by "large" amplitudes are those approaching the amplitudes required to trace out the major hysteresis loop (aka, magnetization curve), as depicted in Fig. 3 (a). (For the transducer in this study, current amplitudes of around 3/4 amperes or greater, at 10 Hz, fit the description of "large.") If, on the other hand, one is operating/oscillating in a stabilized minor hysteresis loop, one which resides entirely within the major hysteresis loop, the assumption that every minor loop looks the same gives:

$$B = \mu H_o + \mu n I \left\{ \cos(\omega t - ø) + \frac{k}{\pi} \sin\big(\pi \cos(\omega t - ø)\big) \right\}$$

(2b)

where: μH_o represents the "DC" magnetic flux density as provided by the permanent magnet, $B_o = \mu |H_o|$
(the quantity μH_o is assumed to be independent of the drive-current amplitude),
$\mu nI\{ \ldots \}$ is the oscillating, current induced, contribution to the flux density, and
k is a parameter which controls the slope at either end of the minor hysteresis loop.
(Note: k=1 would give a zero slope to a B vs. cos(ωt-ø) plot at cos(ωt-ø) = ± 1.)

This model resulted in improved agreement with experimental data for current amplitudes of 0.5 to 0.1 amps (at 10 Hz).

The assumption that minor hysteresis loops are adequately represented as ellipses, i.e., ignoring the nonlinearities of the hysteresis loops, can be modelled by setting k = 0 in Eqn. (2b). This approach parallels that taken by [5]. For the transducer used in this investigation, current amplitudes below approximately 0.1 amps (at 10 Hz) were adequately modelled via this assumption. Note that for "small" current amplitudes, thus "small" variations in B, linearization of Eqn. (1b) might be justified. Assuming that, the above elliptic assumption yields a model which is consistent with classical linear systems analysis. ([6] used a linear systems approach to predict transducer output acceleration, or force, as a function of drive current over a range of operating conditions.)

The fourth and most simple model would be one where hysteresis is ignored entirely, i.e., k = 0 and ø = 0 in Eqn. (2b). This approximation is mentioned only for completeness. Experimental displacements for current amplitudes low enough for possible use of this model were too small to measure with the measurement system employed.

The development which follows will use Eqn. (2b), assuming k and ø ≠ 0. This is the simplest model which retains the nonlinearities in the electromagnetic realm. It is the model which seems applicable for "medium" drive-current amplitudes and represents a first approximation of transducer behavior for drive conditions beyond those for which classical linear systems analysis is applicable. It is the model which will be emphasized in this communication.
The composite function for the mathematical model of displacement as a function of current was formed by using Eqn. (2b), in Eqn. (1b). Utilizing the relations displayed below Eqns. (2a) and (2b), the composite function may be written as:

$$\varepsilon - \varepsilon_o = \varepsilon_o \left\{ (a_1 - 1) - \frac{\varepsilon_w}{\varepsilon_o} \sin(\pi a_1) \right\}$$

(3a)
or

$$\varepsilon = \varepsilon_o \left\{ a_1 - \frac{\varepsilon_w}{\varepsilon_o} \sin(\pi a_1) \right\}$$

(3b)
where:

$$a_1 = \frac{|B|}{B_o} = \left| \operatorname{sig}(H_o) + b \frac{I}{I_m} \left\{ \cos(\omega t - ø) + \frac{k}{\pi} \sin\left[\pi \cos(\omega t - ø)\right] \right\} \right| ,$$

(4)

$$\operatorname{sig}(H_o) = \text{the sign of } H_o \equiv \begin{cases} +1 : H_o > 0 \\ -1 : H_o < 0 \end{cases} ,$$

$$b = \frac{\mu(I, \omega, \sigma_o, \ldots)}{\mu(I_m, \omega, \sigma_o, \ldots)} = \frac{\text{permeability as a function of current, frequency, prestress, ...}}{\text{baseline permeability associated with major hysteresis loop}} ,$$

(5)

b is the apparent "dynamic permeability ratio" ($b \geq 0$),
and output displacement = Terfenol-D rod length x Eqn. (3a).

During the derivation of Eqn. (4), it was tacitly assumed that the variability in magnetic permeability did not affect the field strength associated with the magnetic biasing, i.e., H_o. The validity of this assumption is demonstrated by examining the behavior predicted by the model. As is shown below, b goes to zero as the drive-current amplitude goes to zero. In the case of no drive current, one would expect the transducer to return to its biased position, i.e., $\varepsilon = \varepsilon_o$. Via Eqn. (3b) we anticipate that a_1 should go to one as I goes to zero. If the assumption about the variability in magnetic permeability not affecting H_o is **not** made, one finds that sig(H_o) term of Eqn. (4) must be multiplied by b. Thus, the b in Eqn. (4) would appear outside the absolute value signs, and the quiescent state of strain would be predicted as zero.

COMPARISONS OF MODEL WITH EXPERIMENT

For comparisons of the model predictions with experimental measurements, it was necessary to experimentally determine the sign of H_o and the values of I_m and ε_o. The transducer under test was found to be incorrectly magnetically biased ($H_{o\ exp} < 0$, and $| H_{o\ exp} | < | H_{o\ theory} |$). In such a case, the following assumption proved useful:

$$B_{o\ theory} = c\ \mu\ | H_{o\ exp} | = c\ \mu\ n\ I_x \tag{6}$$

where: I_x is the experimentally determined maximum current amplitude possible without causing the transducer to reverse directions (as depicted in the one amp trace of Fig 4), I_x = 0.75 amps in this experiment; and c is a yet to be determined multiplier (in this case, c > 1). Employing this relation, Eqn. (4) becomes:

$$a_1 = \frac{|B|}{B_o} = \frac{1}{c} \left| \mathrm{sig}\left(H_o\right) + b\ \frac{I}{I_x} \left\{ \cos(\omega t - ø) + \frac{k}{\pi} \sin\left|\pi \cos(\omega t - ø)\right] \right\} \right| \tag{7}$$

As a further consequence of the transducer bias condition, $c\ \varepsilon_{o\ exp} \approx \varepsilon_{o\ theory} = \varepsilon_o$ of Eqn. (3b) was used, and displacement was calculated as:

$$\text{displacement} = \text{Terfenol-D rod length x (Eqn. (3b)} - \varepsilon_{o\ exp}\ ) \tag{8}$$

The positive strain estimated during the drive-current amplitude = I_x measurements **was** $\varepsilon_{o\ exp}$.

The model parameters, c = 4/3, k = 0.3, ø = 0.37, and $\varepsilon_W/\varepsilon_o$ = 0.01 were estimated by visual comparisons of model predictions with experimental measurements which were obtained using a 10 Hz sinusoidal one-amp drive-current. At one-amp, b was assumed equal to one, i.e., it was assumed that we were approximating the major magnetic hysteresis loop as it would be measured at 10 Hz. This represents the baseline magnetic permeability of the material within the transducer in the as-run configuration. Changes in permeability with changing drive current amplitude, i.e., the dynamic permeability ratio b(I), will be noted. Parameters c, k, and $\varepsilon_W/\varepsilon_o$ remained constant for the following comparisons. Recall: c represents a variation from the ideal transducer magnetic biasing (it should remain constant); k was a minor hysteresis loop parameter (which should remain constant in view of the assumption that all minor loops looked the same); and $\varepsilon_W/\varepsilon_o$ represents the curvature of the strain vs magnetic flux density relationship of the material (which, too, should remain constant). A comparison of one model prediction with experimental measurement is shown in Fig. 5. Fig. 6 (a) shows model predictions for current amplitudes ranging from 0.12 to 0.5 amps. Compare with Fig. 6 (b), the corresponding experimental measurements.

As shown in Fig. 5, the agreement seems to be reasonable. Most noteworthy was the fact that the model was incapable of duplicating the "sharp" corners displayed in the experimental measurements. Thus it is unlikely that the model will correctly predict the amplitudes of **all** of the harmonics in the transducer's output displacement spectrum. More on this topic in the discussion of Fig. 8, below. As displayed in Figs. 6 (a) and (b), the model does yield the correct trends: the curves are nested hysteresis loops, and displacement increases with increasing drive-current amplitude. In the literature one often encounters the magnetostrictive "d constant" which is simply the slope of a plot of strain versus applied magnetic field. Fig. 6 is, of course, simply a scaled version of such a plot. A glance at Fig. 6 (b), noticing its varying slopes, informs the reader that the "d constant" is not (constant). Certainly, one is ill-advised to take the name literally.

In order to produce the model curves of Fig. 6 (a) it was necessary to vary the dynamic permeability ratio, b, along with the amplitude of the drive-current. This trend makes sense since we are oscillating in stabilized minor hysteresis loops which do not have "effective" slopes as great as that of the major hysteresis loop. It seems that one would be ill-advised to assume that the permeability was a constant value for dynamic applications of Terfenol-D which involve varying drive amplitudes. The ratio of magnetic permeabilities, b, as a function of current amplitude, I, as predicted by this model, is shown in Fig. 7. Note the small change in b from I = 1.0 to I = 0.75 amps. This trend lends credence to the assumption that b **should** equal 1.0 at I = 1.0 amps (at 10 Hz using this transducer in its present configuration ...). Recall, for this transducer, B changed sign (or equivalently, the output direction of travel reversed unexpectedly, refer to Fig. 4) when I ≈ 3/4 amps. This small change in b for currents of one to 3/4 amps implies that the one amp current was indeed approaching the major hysteresis loop of the material (as assumed in the discussion below Eqn. 8).

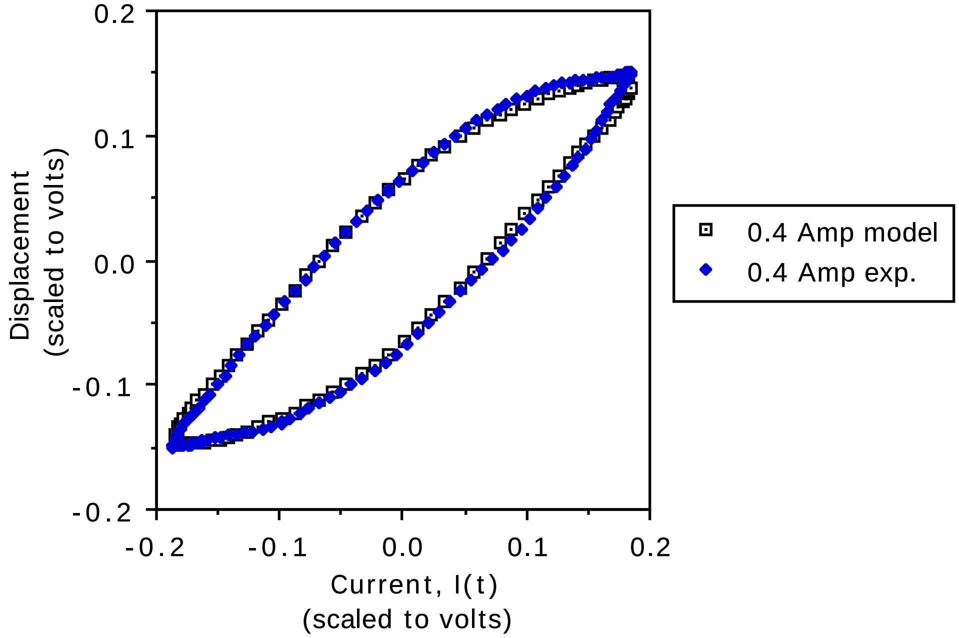

Fig. 5. Transducer output displacement versus input drive-current as predicted by model and measured experimentally. Experiment: ten time averages at I(t) = 0.4 cos(20π t) amps. Model parameters given in Table 1.

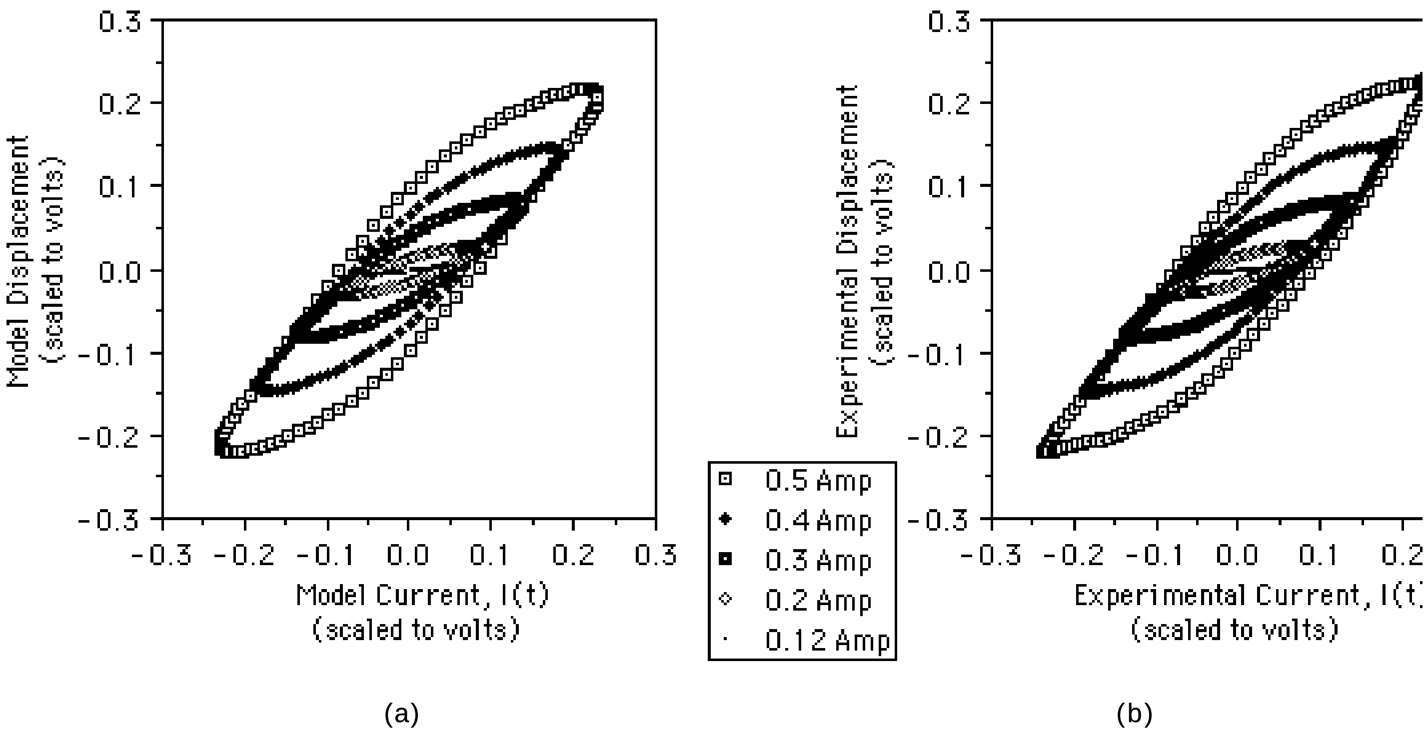

Fig. 6. Model (a) and experimental (b) transducer output displacement versus input current for five, 10 Hz, sinusoidal drive-current amplitudes. Model parameters are given in Table 1.

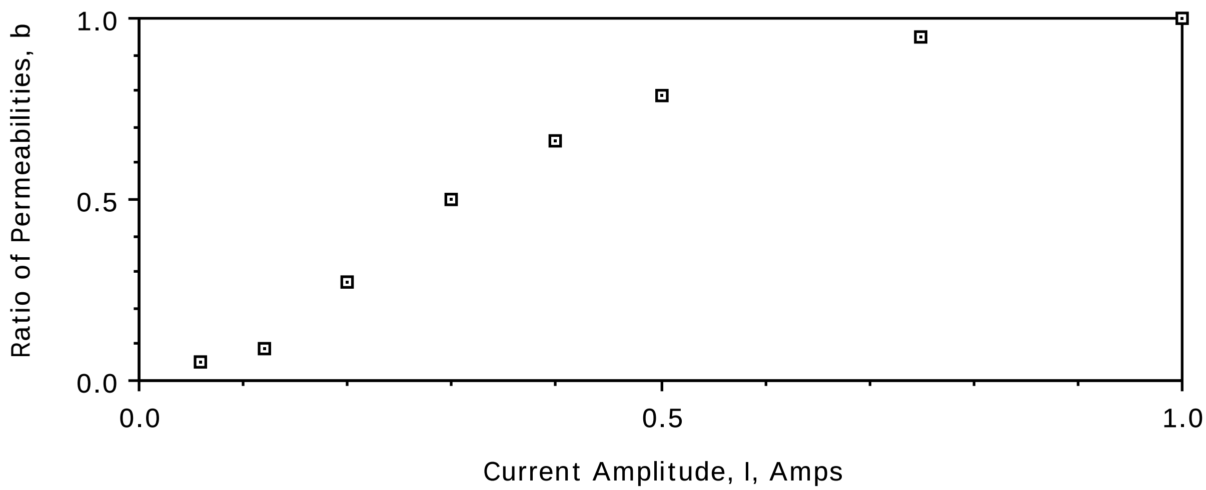

Fig. 7. Ratio of magnetic permeabilities (Eqn. (5)) versus drive-current amplitude required for reasonable agreement between mathematical model displacement predictions and experimental measurements. Values are given in Table 1.

Table 1. Model parameters which varied during simulations of Terfenol-D transducer. For these tests the transducer was driven by a 10 Hz sinusoidal input current.

I, Current (amps)	b, Ratio of Permeabilities $\mu(I)/\mu(I_m)$	ø, Phase of H relative to B (radians)
1.00	1.000	0.37
0.75	0.950	0.37
0.50	0.790	0.37
0.40	0.660	0.37
0.30	0.500	0.40
0.20	0.270	0.40
0.12	0.085	0.25
0.06	0.055	0.10

Shown in Fig. 8 are discrete autospectral density functions as predicted by the mathematical model and calculated from the corresponding experimental measurement. The fundamental frequency of excitation was 100 Hz. The model was tailored to mimic the experiment in the time domain. Then both were transferred to the frequency domain as shown below. Note that the model does a better job of predicting the amplitudes of the even harmonics than those of the odd. However, the model does predict the existence of all of the measured harmonics. Further, it predicts the trends of odd to even harmonic amplitudes displayed by the experimental spectrum, i.e., the second harmonic is larger in amplitude than the first, the fourth is larger than the third, etc. It should be noted that there was a significant difference in noise level in the two "measurements." For the experiment, the "noise floor" was at approximately -130 dB; for the model it was approximately -270 dB. There was also some pesky 60 Hz noise in the experiment, which means that there is also some contamination of the 300, 600, and 900 Hz experimental components below.

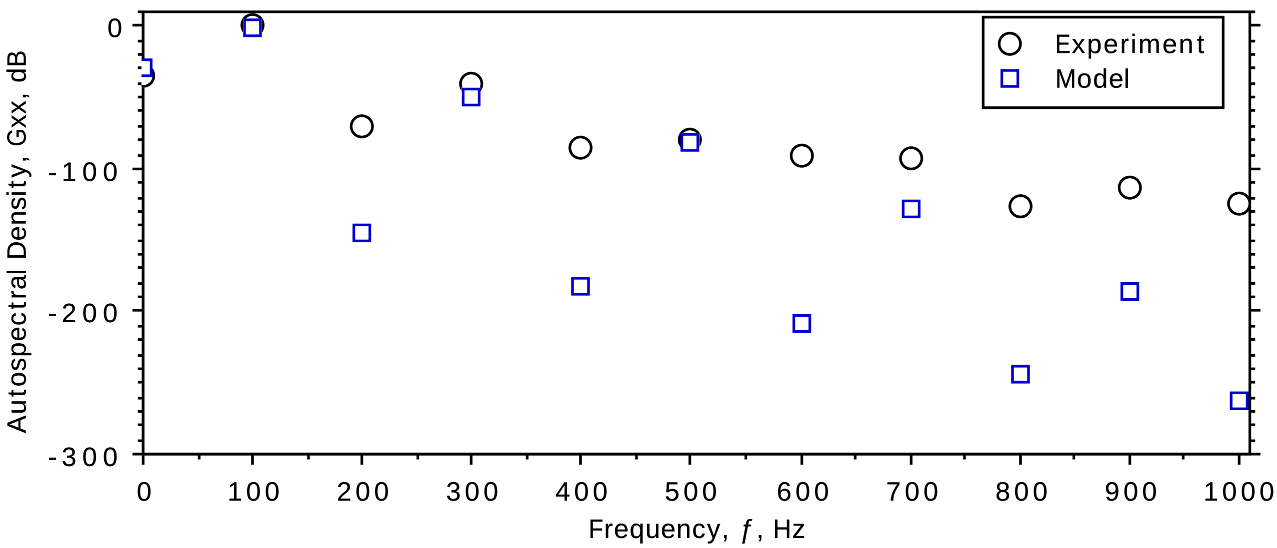

Fig. 8. Autospectral density functions of a mathematical model displacement simulation and experimental measurement. Shown are amplitudes for only the fundamental frequency, 100 Hz, and the resulting harmonics. All datums reported were approximately 10 dB or greater above the noise bases.

DISPLACEMENT TRENDS OF A TERFENOL-D TRANSDUCER

Transducer output displacement tends to increase with increasing drive-current amplitude. That trend was displayed in Fig. 6 (b). Fig. 9 shows that displacement tends to decrease with increasing frequency. Fig. 9 is a plot of displacement versus drive-current for four different frequencies of excitation. In all cases the current amplitude was 1/4 amp. The first resonant frequency of the transducer, as tested, was approximately 10 kHz. Thus the frequencies in Fig. 9 are well below resonance and below the frequency at which eddy current shielding should be significant.[1] Notice how, with increasing frequency of excitation, the hysteresis loops become progressively better approximations of ellipses.

Fig. 10 displays that the amplitudes of the harmonics present in the transducer's output displacement increase with increasing drive-current amplitude. Shown are only the amplitudes for the fundamental frequency (100 Hz) and related harmonics from autospectral density experimental displacement measurements. These amplitudes have all been normalized by that of the corresponding fundamental frequency. Note that the first harmonic increases disproportionately with increasing current amplitude. The balance of the harmonics seem to increase along with the fundamental, for these current amplitudes. This last trend again adds credence to the assumption made earlier about all the hysteresis loops looking the same.

As shown in Table 1, the relative phase angle required for model predictions to resemble experimental measurements was nearly constant ($0.2 \le I \le 0.5$ amps). At the lower current amplitudes, however, the phase needed to be reduced. Not coincidentally, at these lower current amplitudes the experimentally measured hysteresis loops became better and better approximations of ellipses. (Ellipses can be modelled as simply straight lines with a relative phase.[5]) The same trend is shown in Fig. 9, this time due to increasing frequency. In turn, as one lowers the current amplitude the ellipses become better and better approximations of straight lines. Thus, as drive amplitudes increase from zero and/or as frequencies of excitation decrease, the complexity of the model required to approximate transducer behavior increases. The model presented in Eqn. (2a) yields reasonable results for low frequency-"large" current amplitude simulations. The model of Eqn. (2b), the one emphasized in this communication, agrees well with experimental results for "medium" current amplitudes at low frequencies, and large currents at high frequencies. An elliptic

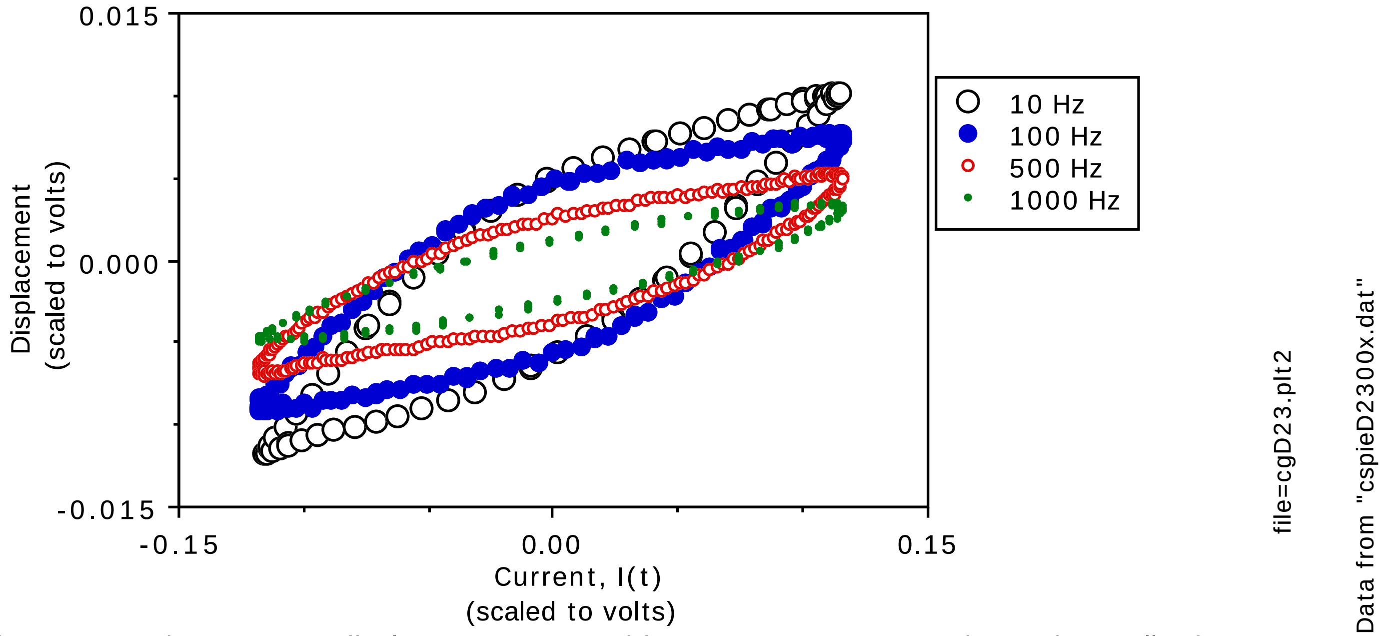

Fig. 9. Transducer output displacement versus drive-current, as measured experimentally, for I(t) = 0.25cos(2π*f* t) amps and *f* = 10, 100, 500, and 1000 Hz. Each set of data represents one cycle of oscillation with fifty time averages having been performed.

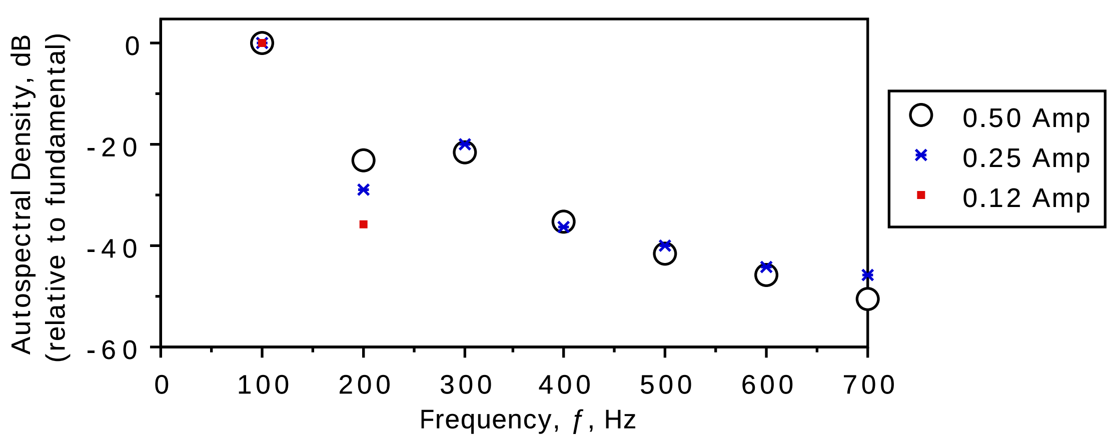

Fig. 10. Amplitudes in dB (relative to the fundamental frequency's) for different drive-current amplitudes. All tests were 100 averages of experimental displacement measurements. Shown are only the fundamental's and related harmonics' amplitudes. Only two datums for 0.12 amps were above the noise base.

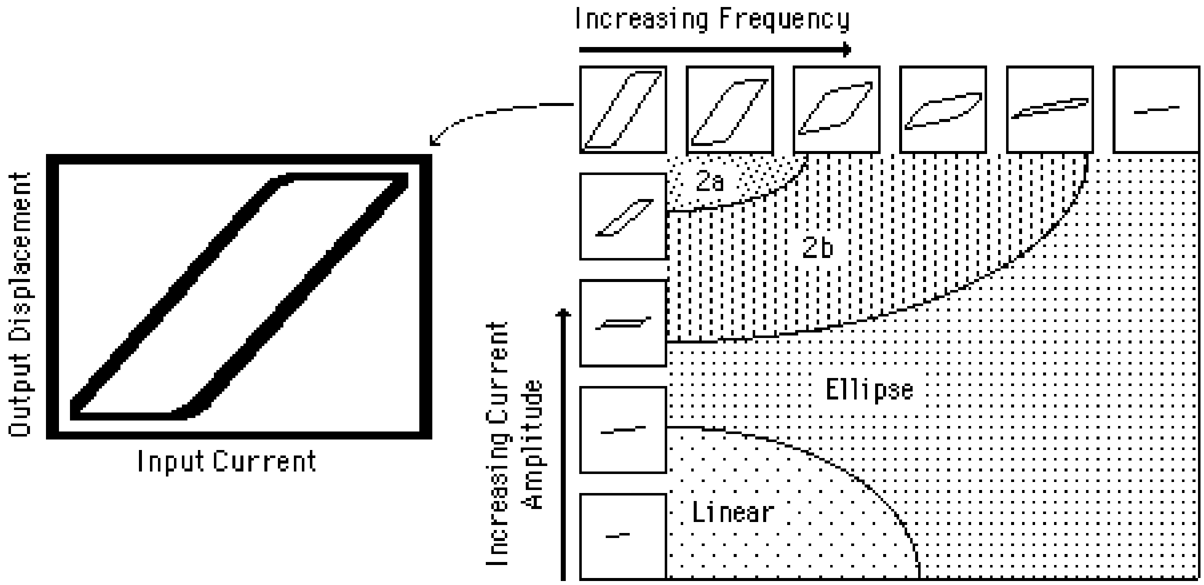

Fig. 11. Schematic representation of the evolution of displacement versus current relationships with increasing drive-current amplitude and increasing frequency of excitation. Also shown are regions where various mathematical models of displacement from current seem to be applicable. "2a" refers to the model resulting from the B-H model of Eqn. (2a). "2b" refers to the model demonstrated here resulting from the use of Eqn. (2b) ("all hysteresis loops look the same," $k \neq 0$, $ø \neq 0$). "Ellipse" is simply a linear model with a relative phase (the classic linear systems approach). "Linear" is the elliptic model with no phase.

relationship between current and displacement can be employed for "low" amplitude currents at low frequencies, to medium and large amplitudes at high frequencies. A straight line seems reasonable for very low amplitude currents at low frequencies and is usually limited very quickly by dynamic effects (which introduce a phase, i.e., we need an ellipse) owing to increasing frequencies. These trends are depicted in Fig. 11.

When operating in the region of Fig. 11 where the elliptic model applies, one can obtain standard frequency response functions, one frequency at a time or using broadband excitation, which adequately predict transducer behavior. In this region the transducer behaves linearly, in a least squares sense. You have a classic linear system. Terfenol-D transducer displacements tend to be comparable with piezoelectric transducers in this range. Coherence functions calculated for frequency response functions tend to be very good. However, as drive-current amplitudes increase, or frequencies decrease, the coherence of the measurement decreases, indicating that the transducer is beginning to produce appreciable harmonic amplitudes.[6] During this evolution, the amplitudes of the harmonics, relative to the fundamental frequency of excitation, increase disproportionately. The range where transducer displacement is no longer modelled well via linear systems analysis, i.e., "2b" of Fig. 11, corresponds to the range where displacement performance begins to show advantages over the typical piezoelectric. Thus, it would be nice to exploit this region. Use of the model introduced here may help toward that end (though it is considerably more difficult to use than standard linear systems analysis).

Though there are two sources of phase between current and displacement, that due to magnetic hysteresis and that from dynamic effects, the discussion above does not differentiate between the two. As frequencies of excitation increase, it seems less and less appropriate to speak of displacement as a function of current. It might be more appropriate to convert to a forcing function due to current and then model the dynamics separately. This conversion may be as simple as multiplying the strain, Eqn. (3b), by a modulus of elasticity. However, judging from Fig. 9 where the maximum displacement does not seem to decrease like the inverse squared frequency (as one might anticipate), it probably will not be that simple. That work is yet to be performed.

CONCLUSIONS

1. The empirical model emphasized in this communication, Eqn. (3a), represents a first approximation of a Terfenol-D transducer's behavior (displacement due to current) as conditions become such that the linear systems approach fails. It approximates the nonlinearities of the material and provides some insight into the sources of harmonics, and their behavior, as dynamic application conditions vary. The model predicts the existence of harmonics in the displacement spectra.
2. Terfenol-D transducer output displacement increases with increasing drive-current amplitude. The increase is not linear with current amplitude for those currents which result in minor hysteresis loops as approximated in Eqn. (2b). In the region where classical linear systems analysis applies, the increase seems to be linear.
3. Terfenol-D transducer output displacement decreases with increasing frequency of excitation.
4. Harmonic content of the displacement spectra increases disproportionately with increasing current amplitude over the range of operating conditions corresponding to the evolution from behavior modelled well by a linear systems approach to that modelled by Eqn. (2b). The relative amplitudes of the harmonics then tend to remain approximately constant over the range of operating conditions appropriate for the use of Eqn. (2b). The harmonics result from the nonlinearities, magnetic and magnetomechanical, of the material.

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