One-Dimensional Analytical Constant Parameter Linear Electromagnetic-Magnetomechanical Models of a Cylindrical Magnetostrictive (Terfenol-D) Transducer

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ABSTRACT: A method is presented for including motional and eddy current effects when analytically modeling electrical impedance functions of cylindrical magnetostrictive transducers. To approximate eddy current effects, one-dimensional analytical constant parameter linear electromagnetic models of a cylindrical magnetostrictive transducer are developed. Maxwell's equations are solved for the magnetic field strength as a function of radial position. Closed form expressions for magnetic flux as a function of radial position are then derived, from which transducer electrical impedance functions are formulated. Two different physical models of the transducer are considered. The first model results in the classic eddy current solution for a rod in a wound wire solenoid. The second physical model includes the effects of a conducting external cylindrical housing. Motional effects are incorporated into the electromagnetic models via the magnetomechanical model, which *is* a frequency and load dependent, complex valued expression for the "dynamic magnetic permeability" of the magnetostrictive material within the transducer. The functional form for this dynamic magnetic permeability is derived by comparing the transduction equations for the magnetostrictive material with those for the transducer containing the material. Electrical impedance functions for both physical models are compared with an experimental measurement for a particular magnetostrictive transducer (using Terfenol-D) in its low signal linear range of operation. The second physical model was found to offer the better simulation of the experimental measurement of the transducer's electrical impedance, with errors in magnitude or phase of less than ±5% for excitation frequencies between 100 and 10 000 Hz (the as-run mechanical resonant frequency was 8800 Hz). Experience has shown that *measured* material parameters typically yield *simulations* of electrical impedance functions within ±10%. *Predictions* of electrical impedance

functions for Terfenol-D transducers, calculated based on *estimated or published* Terfenol-D parameters, can be in error by well over 40%. Thus, the accuracy of model results seem to be controlled primarily by the accuracy with which magnetostrictive material parameters are known.

INTRODUCTION

The problem at hand is to determine what one must know in order to formulate a reasonable prediction of a magnetostrictive transducer's *dynamic* behavior. This work represents a portion of that goal.

In this development, the transducer will be considered to be a single input, single output system. The system input is electric current; the output is displacement, velocity, acceleration, or force. All of these are vector quantities. Most of these transducers have cylindrical geometries which suggests the use of r, θ, z coordinates. The transducers usually consist of a rod of magnetostrictive material, surrounded by a cylindrical wound wire electric solenoid, which in turn is surrounded by a cylindrical housing (made from, for example, cast Alnico V, a conductive permanent magnet material). To allow for increased magnitudes of bidirectional motion, most Terfenol-D transducers are compressively prestressed and magnetically biased. This prestressed and biased point would then be the transducer's quiescent state. Typically, the origin of the coordinate system will be considered as attached to the "fixed" end of the magnetostrictive rod. The opposite end of the rod will be the one that moves or provides a force. A positive output displacement, velocity, acceleration, and/or force would be directed along the axis of the transducer, in the positive z direction. A positive current would be that which causes a positive magnetic flux which in turn results in a positive displacement (the magnetostrictive rod gets longer) when the current is near constant (DC), and the load is such that the rod can strain (the transducer is not blocked).

The theory of transducer operation is roughly as follows: 1) An electric current in the solenoid begets a magnetic field along its axis. 2) A magnetic field permeating the magnetostrictive material tends to cause the magnetic domains within the material to rotate (in an effort to align with the applied field). 3) Rotation of the magnetic domains causes the magnetostrictive rod to change length (strain).

Recall "Faraday's law," a time varying magnetic flux within a circuit induces a voltage in that circuit. Further, according to Lenz, this induced voltage will be such that the resulting current flow will contribute a flux that opposes the change in the imposed flux. In the case of a wound wire solenoid, a current flowing in the positive θ direction results in a magnetic flux density vector in the positive z direction. Varying this density with time gives an electric field directed in the negative θ direction. If the solenoid surrounds a conducting rod, for example, a rod of Terfenol-D, any annular ring within the material forms an electric circuit and will carry an induced current due to the induced electric field. These induced currents are known as "eddy currents." Their presence tends to decrease the applied field's penetration of the material, i.e., inner portions of the rod are magnetically "shielded." Also, since the eddy currents flow in a material with some resistance, there is a heating/ohmic loss associated with their existence.

Consider, initially, an air solenoid. The complex valued electrical impedance of a long, tightly wound air solenoid is typically approximated as $Z = V/I = R + j\omega L$, where Z is the complex valued impedance, V is the voltage across while I is the current through the solenoid, R is the solenoid's DC resistance, $j = \sqrt{(-1)}$, ω is the circular drive frequency, and L is the self inductance of the solenoid. Inductance, L, is defined as the total magnetic flux linkage divided by the impressed current: $L \equiv N\Phi_m/I$ where N is the number of turns of the solenoid. In the case of an air solenoid, L can also be approximated as the product of the magnetic permeability of free space, the square of the turns per unit length of the solenoid ($n = N/l_s$), and the volume enclosed by the solenoid, i.e., $L \approx \mu_o n^2 Vol$. Note that this representation for the inductance of an air solenoid is a real valued constant. If one experimentally measured Z (over the audible frequency range, say, 0 to 40000π rad./sec) they should find that its real component was basically constant with frequency and that its imaginary component increased linearly with frequency (with a slope of L). The real part of Z represents the energy dissipation mechanism, i.e., the ohmic losses, while the imaginary part represents the energy storage mechanism (energy is stored in the magnetic field down the bore of the solenoid).

Matters become more complicated with a conductor present. In particular, if one is driving a piece of Terfenol-D with an electric solenoid and measures the input voltage and current of the solenoid, the ohmic losses due to eddy currents within the Terfenol-D will appear as an increase in the solenoid resistance with increasing drive frequency, i.e., Real(Z) increases with frequency. Eddy currents result in a reduction in the magnetic field penetration (reduction in flux, thus flux linkage) which will appear as a coil inductance that decreases with increasing frequency: Imag(Z)/ω generally decreases with frequency. Following the lead above, the electrical impedance of a wound wire solenoid with Terfenol-D (or any conductor) as the core material will be modelled in this communication as: $Z = R_{DC} + j\omega L_T$ where R_{DC} is the real valued DC resistance of the coil while the *complex valued* inductance is defined as $L_T \equiv a + jb = N\Phi_m / I$. Thus, $Z = \{(R_{DC} - \omega b) + j\omega a\}$ where both a and b are real numbers that may vary with frequency, $a > 0$, $b \leq 0$. The quantity $\{-\omega b\}$ represents the ohmic losses due to eddy currents within the conducting core; thus, it should increase with frequency. Likewise, {a} should decrease with frequency. No longer can the inductance be calculated as a real valued constant, dependent primarily on the geometry of the coil.

Eddy currents affect what would be experimentally measured as a material's magnetic permeability. However, in this development it is assumed that the magnetostrictive material (Terfenol-D) has a magnetic permeability independent of eddy effects but a function of load and excitation frequency, $\mu_T(\text{load},\omega)$. (See Magnetomechanical Model section.) Magnetic flux within the transducer is formulated to include the effects of eddy currents, $\Phi_m(\text{eddy},\mu_T)$. Finally, the electrical impedance of the transducer is formulated incorporating the frequency and load dependence of both magnetic permeability and eddy currents.

The approach detailed below allows for the use of a complex valued magnetic permeability. Traditionally, a complex valued permeability is used to improve the linear flux density from applied field relationship, $B \approx \mu H$, to approximate magnetic hysteresis in the material, i.e., $B \approx (\mu e^{-j\varphi}) H$. (Stoll, 1974) This approximation was not used in this communication since the model simulations presented correspond to an experimental electrical impedance function obtained during low signal, linear operation of the transducer (hysteretic effects are assumed negligible). Transducer operation at larger signal amplitudes might be simulated more accurately by incorporating this elliptical approximation. Nonetheless, it will be shown later in this communication that a complex valued permeability (of a different form) is needed even during low signal operation in order to include magnetomechanical effects.

ELECTROMAGNETIC MODELS

Begin with Maxwell's equations and neglect the displacement current terms (assumed negligible in a conductor).

$$\mathrm{curl}(\underline{E}) = \quad x\underline{E} = -\frac{\partial \underline{B}}{\partial t} \tag{1}$$

$$\mathrm{curl}(\underline{H}) = \underline{J} \tag{2}$$

$$\mathrm{div}\underline{B} = \quad \underline{B} = 0 \tag{3}$$

where: $\underline{E}$ is the electric field vector, volts/meter,

$\underline{B}$ is the magnetic flux density vector, Tesla,

$\underline{H}$ is the magnetic field strength vector, amp-turns/meter, and

$\underline{J}$ is the current density, amps/meter^2.

Equation (1) is "Faraday's law," Equation (2) is "Ampere's law," and Equation (3) is "Gauss' law of magnetism." Equation (4) is "Ohm's law" and Equation (5) is the standard linear magnetic model.

$$\underline{J} = \sigma_e \underline{E} \tag{4}$$

$$\underline{B} = \mu \underline{H} \tag{5}$$

where: σ_e is the electrical conductivity of the material, (ohm meter)$^{-1}$, and

μ is the magnetic permeability of the substance, Tesla meter/amp-turn.

Cylindrical coordinates, r, θ, z, and the following assumptions will be employed in the model development:

i) no axial (z) variations (neglects end effects in transducer)

ii) displacement currents are negligible

iii) a linear magnetic model, Equation (5)

iv) no θ dependence

v) μ and σ_e are constant in space and time

vi) $\underline{H}(r,\omega,t) = H(r,\omega)\, e^{j\omega t}\, \underline{e}_z$, i.e., using separation of variables

($\underline{e}_z$ is the unit vector in the z direction)

Rationale of assumptions:

i) Neglecting axial flux variations is a simplifying assumption that reduces the problem by one dimension. In addition, the transducer tested used steel end-caps which actually extended into the solenoid on each end (approximately 5% of the solenoid's length on each end). The relatively high permeability of these steel parts was thought to offer a reduction in axial flux variations when compared to a solenoid with a core consisting of a longitudinally constant magnetic permeability.

ii) Within the conductors present in the transducer, this is a standard assumption. (Stoll, 1974) It is also applicable for those regions of the transducer where air is the medium.

iii) A linear magnetic model is a place to begin. Again, the results of this model will be compared with experimental evidence gathered when the transducer was operating in the linear region (that where linear systems analysis provides very good predictions of transducer behavior). Drive current amplitudes will be "small" implying magnetic operation within a stable minor hysteresis loop approximated well by a straight line.

iv) No angular dependence seems reasonable due to the physical symmetry of the transducer. This assumption is consistent with the ideas of a homogeneous continuum, i.e., local variations in material behavior are considered "small" (whether they are or not).

v) Magnetic permeabilities and electrical conductivities are assumed to be independent of time and position. For steady state operation (constant temperature is of particular importance), there is no known reason to assume a time dependence for these properties. Independence of position is a simplifying assumption. Looking ahead to the solution, a radial variation in applied field strength will result from field induced eddy currents occurring within the conducting rod. However, a relationship for permeability as a function of applied field amplitude (especially for Terfenol-D) is not known. As a result, it is assumed that the variation of the permeability of Terfenol-D with applied field amplitude, thus position, is negligible.

vi) Separation of variables is the standard technique for solving partial differential equations when assuming a sinusoidal time variation of the quantities. In the present case, the input current varies sinusoidally (at low amplitudes) thus it is assumed that other quantities will do likewise. The field strength discussed is that which varies in response to the time varying input current. The total field strength would be $H_{total} = H_{DC\ Bias} + H(r,\omega,t)$. Note that because of assumptions i and iv, the only component of $\underline{H}$ is in the axial ($\underline{e}_z$) direction.

Assumptions i and iv reduce the field strength, $\underline{H}$, to varying only with one spatial variable, r. In the following discussion, the time dependence cancels and the radial and frequency dependence is assumed. The object of this endeavor is to obtain a solution for $\underline{H}$ as a function of radial position, frequency, and time.

Equation (2) becomes:

$$-\frac{\partial H}{\partial r}\,\underline{e}_\theta = \underline{J} \underset{(4)}{=} \sigma_e \underline{E} = \sigma_e E \underline{e}_\theta \quad \text{or} \quad E = -\frac{1}{\sigma_e}\frac{\partial H}{\partial r} \tag{6}$$

where: $\underline{e}_\theta$ is the unit vector in the θ direction,

an equation number below an equals sign indicates the equation employed in writing the equality, and

E is the magnitude of the vector $\underline{E}$, which in the one-dimensional case represents no loss of information, that is, it is now known that the only component of $\underline{E}$ exists in the θ direction.

Equation (1) evolves as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rE\right)\underline{e}_z = -\frac{\partial \underline{B}}{\partial t} = -\frac{\partial B}{\partial t}\,\underline{e}_z \underset{(5)}{=} -\mu\frac{\partial H}{\partial t}\,\underline{e}_z = -j\omega\mu H\underline{e}_z \tag{7}$$

where the last step arises because the partial with respect to time of $\underline{H}$ is $j\omega\underline{H}$ (via assumption vi). Dropping the unit vector and using Equation (6) for E in Equation (7) yields a second order partial differential equation for H.

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} - j\omega\mu\sigma_e H = 0 \tag{8}$$

One now typically defines k_x (which is related to the inverse of the classic "skin depth") as:

$$k_x \equiv \sqrt{j\omega\mu\sigma_e} \tag{9}$$

where "x" will change to identify the material in question. Specifically, k_T will be calculated using the properties of Terfenol-D and k_{pm} calculations will use permanent magnet material properties. Using Equation (9), Equation (8) becomes

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} - k_x^2 H = 0. \tag{10}$$

Equation (10) is known as a modified Bessel equation of order zero with a general solution of the form:

$$H(r,\omega) = c_1 I_0(k_x r) + c_2 K_0(k_x r) \tag{11}$$

where: $I_0(k_x r)$ is known as the *modified Bessel function of the first kind of order zero*, i.e., $I_o(k_x r) = (-j)^o J_o(jk_x r) = J_o(jk_x r)$, and

$K_o(k_x r)$ is known as the *modified Bessel function of the second kind of order zero*.

(Definitions of this function vary between authors.)

Regardless, as $k_x r$ goes to zero, I_o approaches 1 and K_o approaches + infinity.

Equation (11) is the solution sought for the time varying applied field in a cylindrical conductor. How (much more?) complicated does the model need to be? To answer this question, physical models of the transducer were assumed, analytical expressions were derived for magnetic flux as a function of radial position, then electrical impedance functions were calculated employing analytical expressions derived from standard definitions of inductance and magnetic flux linkage. These electrical impedance functions were then compared with an experimentally measured function.

Figure 1 is a section view of the magnetostrictive transducer in this study. Shown is the view one would see if they were looking down the z axis of the transducer with the end-caps removed. Also shown in the figure is the nomenclature used for various radii.

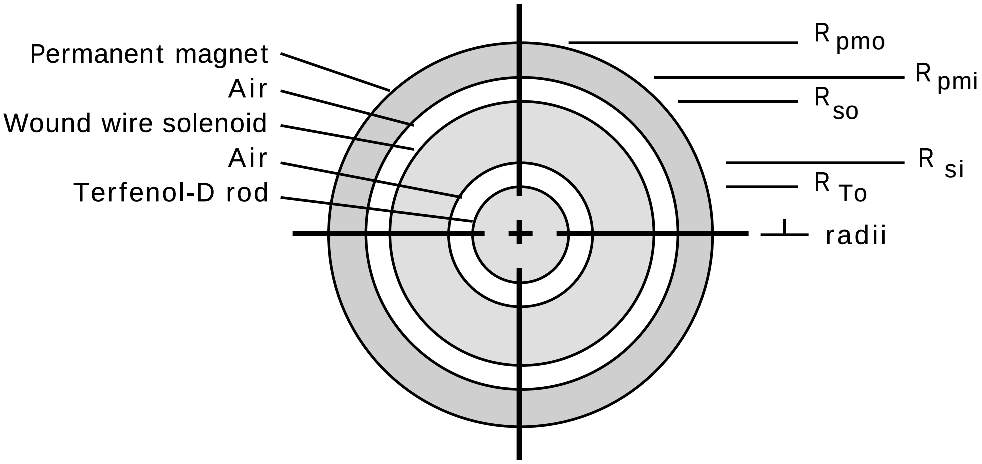

Figure 1. Schematic representation of end-view of the cylindrical magnetostrictive transducer under study (end caps removed). Subscripts on the radii read like: To = Terfenol, outer; si = solenoid, inner; so = solenoid, outer; pmi = permanent magnet, inner; and pmo = permanent magnet, outer.

Transducer Electromagnetic Model 1

The first transducer model is developed focusing on the solenoid-conductor interaction by neglecting the effects of the (also conductive) external permanent magnet. Under this assumption the following boundary conditions are applicable to Equation (11):

i) $H(0,\omega)$ = finite, which implies that $c_2 = 0$; and

ii) $H(R_{To},\omega) = nI$, which implies that $c_1 = nI/I_o(k_T R_{To})$.

Boundary condition ii) implies that H must be a continuous function of r and assumes that $H_{solenoid} = nI$. Thus, for this transducer model, Equation (11) becomes:

$$H(r,\omega) = nI\left\{\frac{I_o(k_T r)}{I_o(k_T R_{To})}\right\} \tag{12}$$

This is the classic solution for a conducting rod in a wound wire solenoid (Kahn and Spal, 1981) though it is traditionally written in terms of Kelvin functions when applied to a Terfenol-D rod. The Bessel function format will be retained in this development. Assuming each layer of the solenoid has the same flux linkage and that the rod fills the solenoid, the analytical expression for the electrical inductance is:

$$L(\omega) \approx \mu_T n^2 (\pi R_{To}^2 l_s)\left\{\frac{2}{k_T R_{To}}\,\frac{I_1(k_T R_{To})}{I_o(k_T R_{To})}\right\} \tag{13}$$

where the term in the parentheses is recognized as the volume of the solenoid and the term in the braces represents the effects of eddy currents on the standard inductance of a wound wire solenoid. Because the arguments of the Bessel functions, $k_T R_{To}$, are complex, the inductance as predicted by the model is a complex valued function (recall the previous discussion of $L_T = a + jb$). Note that frequency enters the inductance via k_T, recall Equation (9), and, as will be shown later in Equation (47), via the dynamic magnetic permeability of the magnetostrictive material, μ_T (which also appears in k_T calculations).

Transducer Electromagnetic Model 2

The second transducer model will be formulated to include the effects of a conducting external cylindrical housing, such as the permanent magnet shown in Figure 1. Hence, this model will allow for the necessity that $H(R_{To},\omega) \neq nI$ due to the external conductor. Maxwell's equations will be applied to each region of material within the transducer (recall Figure 1). The magnetic field strength, H, will be assumed to be continuous with r, $0 \le r \le R_{pmo}$, though none of its derivatives need be continuous at the material interfaces. Recall, there will be different parameters, k_T and k_{pm}, applicable to the Terfenol-D rod and external cylindrical permanent magnet, respectively.

Boundary conditions (BC) for this problem are:

i) $H(0, \omega)$ = finite,

ii) $H(R_{pmo}, \omega) = 0$, and

iii) $H(r, \omega)$ is continuous at material interfaces (there are four interfaces).

For the following derivations, the frequency dependence of H and subsequent quantities is assumed.

Region One: $0 \le r \le R_{To}$ (material in region is Terfenol-D)

The governing differential equation is, as in the first transducer model, Equation (10), with solution Equation (11). Boundary condition i) implies that c_2 is again zero. Evaluating at $r = R_{To}$ gives c_1, resulting in:

$$H(r) = H(R_{To}) \frac{I_o(k_T r)}{I_o(k_T R_{To})} \qquad 0 \le r \le R_{To}. \tag{14}$$

Keep in mind, however, that as yet neither the functional form nor the values of $H(R_{To},\omega)$ are known.

Region Two: $R_{To} < r \le R_{si}$ (material in region is air)

In air, the current density, $\underline{J}$, is zero. Equation (2) then becomes curl($\underline{H}$) = 0, which implies

$$H(r) \underset{(\text{BC iii})}{=} H(R_{To}) \qquad R_{To} < r \le R_{si}. \tag{15}$$

Region Three: $R_{si} < r \le R_{so}$ (material in region is copper wire)

The behavior of the applied field within the solenoid will be *approximated* by assuming the current density as $\underline{J} \approx I/A_{cu}\, \underline{e}_\theta$, where I is the input current and A_{cu} is the cross sectional area of a strand of the copper wire. With this approximation, Equation (2) evolves as:

$$\text{curl}(\underline{H}) = \frac{I}{A_{cu}} \underline{e}_\theta \qquad \text{or} \qquad -\frac{\partial H}{\partial r} = \frac{I}{A_{cu}} \qquad\qquad H(r) = -\frac{I}{A_{cu}} r - c_3.$$

Evaluating at $r = R_{si}$, where H is known via Equation (15), gives an expression for c_3. Using that, one can write

$$H(r) = -\frac{I}{A_{cu}} r - \left(-H(R_{To}) - \frac{I}{A_{cu}} R_{si} \right) = H(R_{To}) - \frac{I}{A_{cu}} \left(r - R_{si} \right). \tag{16}$$

It may not be immediately apparent, but it can be shown that $A_{cu} \approx (R_{so} - R_{si})/n$. As a convenient result, Equation (16) can be written as:

$$H(r) = H(R_{To}) - nI \frac{\left(r - R_{si} \right)}{\left(R_{so} - R_{si} \right)} \qquad R_{si} < r \leq R_{so}. \tag{17}$$

Note that this analysis resulted in a linear relationship for the field strength as a function of radius through the wound copper wires of the solenoid.

Region Four: $R_{so} < r < R_{pmi}$ (material in region is air)

Like in region two, H(r) is constant. The constant is obtained by invoking BC iii) at $r = R_{so}$ in Equation (17). Therefore:

$$H(r) = H(R_{To}) - nI \qquad R_{so} < r < R_{pmi}. \tag{18}$$

Region Five: $R_{pmi} \leq r \leq R_{pmo}$ (material in region is cast Alnico V, a conducting permanent magnet)

The governing differential equation is again Equation (10) with a general solution of the form:

$$H(r) = c_4 I_o(k_{pm} r) + c_5 K_o(k_{pm} r), \tag{19}$$

where k_{pm} is simply Equation (9) using the properties of the permanent magnetic material. BC ii) gives c_5 in terms of c_4 as:

$$c_5 = -c_4 \frac{I_o(k_{pm} R_{pmo})}{K_o(k_{pm} R_{pmo})}. \tag{20}$$

Continuity of H(r) at $r = R_{pmi}$ yields c_4 in terms of nI and $H(R_{To})$. In particular:

$$H(R_{pmi}) \underset{(18)}{=} H(R_{To}) - nI \underset{(19\&20)}{=} c_4 \left\{ I_o(k_{pm} R_{pmi}) - \frac{I_o(k_{pm} R_{pmo})}{K_o(k_{pm} R_{pmo})} K_o(k_{pm} R_{pmi}) \right\}.$$

Solving for c_4 gives:

$$c_4 = \frac{\left(H(R_{To}) - nI \right)}{\left\{ I_o(k_{pm} R_{pmi}) - \frac{I_o(k_{pm} R_{pmo})}{K_o(k_{pm} R_{pmo})} K_o(k_{pm} R_{pmi}) \right\}}. \tag{21}$$

Substituting Equations (20) and (21) into (19) results in the expression shown in Equation (22) for the alternating component of the magnetic field strength within the permanent magnet.

$$H(r) = \frac{\left(H(R_{To}) - nI\right)}{\left\{ I_o(k_{pm}R_{pmi}) - \frac{I_o(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} K_o(k_{pm}R_{pmi}) \right\}} \left[I_o(k_{pm}r) - \frac{I_o(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} K_o(k_{pm}r) \right]$$

$$R_{pmi} \le r \le R_{pmo} \qquad (22)$$

Expressions have been derived for $H(r,\omega)$ throughout the transducer in terms of nI, the drive field strength, and $H(R_{To},\omega)$, the yet unknown field strength at the surface of the Terfenol-D rod. To determine $H(R_{To},\omega)$ another equation is needed (again, the frequency dependence will be assumed). Following the lead of Stoll (1974), apply Faraday's induction relation at the inner radius of the cylindrical permanent magnet, i.e.,

$$\left|V\right|_{R_{pmi}} = \left[-\frac{d\Phi_m}{dt}\right]_{R_{pmi}} \qquad (23)$$

where the left-hand side is calculated as:

$$\left|V\right|_{R_{pmi}} \equiv 2\pi R_{pmi}\left|E\right|_{R_{pmi}} \underset{(6)}{=} 2\pi R_{pmi}\left[-\frac{1}{\sigma_{epm}}\frac{\partial H}{\partial r}\right]_{R_{pmi}}$$

$$= \frac{-2\pi R_{pmi}k_{pm}}{\sigma_{epm}} \frac{\left[I_1\left(k_{pm}R_{pmi}\right) + \frac{I_o(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} K_1\left(k_{pm}R_{pmi}\right) \right]}{\left\{ I_o(k_{pm}R_{pmi}) - \frac{I_o(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} K_o(k_{pm}R_{pmi}) \right\}} \left(H(R_{To}) - nI\right)$$

(defining c_x)

$$\equiv -c_x\left(H(R_{To}) - nI\right). \qquad (24)$$

The right-hand side of Equation (23) is calculated by taking the time derivative of the total magnetic flux occurring within the four regions described by $0 \le r < R_{pmi}$. The time derivative is $j\omega$ times the flux, and the total flux enclosed by the cylindrical housing can be calculated as

$$\Phi_m = \int_0^{R_{pmi}} \mu H(\rho) 2\pi\rho d\rho = \Phi_{m1} + \Phi_{m2} + \Phi_{m3} + \Phi_{m4} \qquad (25)$$

where each of the fluxes come from integration of the appropriate kernel with the corresponding limits.

<u>Region One</u>: $0 \le r \le R_{To}$ (material in region is Terfenol-D) $H(\rho)$ via Equation (14)

$$\Phi_{m1} = \int_0^{R_{To}} \mu_T H(\rho) 2\pi\rho d\rho = \mu_T \pi R_{To}^2 \left\{ \frac{2}{k_T R_{To}} \frac{I_1(k_T R_{To})}{I_o(k_T R_{To})} \right\} H(R_{To}) \equiv c_{10} H(R_{To}) \qquad (26)$$

<u>Region Two</u>: $R_{To} < r \le R_{si}$ (material in region is air) $H(\rho)$ via Equation (15)

$$\Phi_{m2} = \int_{R_{To}}^{R_{si}} \mu_o H(R_{To}) 2\pi\rho d\rho = \mu_o\pi\left(R_{si}^2 - R_{To}^2\right)H(R_{To}) \equiv c_{20}H(R_{To}) \qquad (27)$$

<u>Region Three</u>: $R_{si} < r \le R_{so}$ (material in region is copper wire) $H(\rho)$ via Equation (17)

Two forms of the flux in this region will be useful. Φ_{mi}, the flux enclosed between R_{si} and r_i, where r_i is the radius of the ith layer of the wound wire solenoid, will be useful in impedance calculations which include the effects of flux occurring within the windings of the coil. (The inductance given by Equation (13) ignored the flux in the air gap and the windings.) Φ_{m3} will be Φ_{mi} evaluated at $r_i = R_{so}$.

$$\Phi_{mi} = \int_{R_{si}}^{r_i} \mu_o \left\{ H(R_{To}) - nI\frac{\left(\rho - R_{si}\right)}{\left(R_{so} - R_{si}\right)} \right\} 2\pi\rho d\rho$$

$$= \mu_o\pi\left(r_i^2 - R_{si}^2\right)\left\{ H(R_{To}) + \frac{R_{si}}{R_{so} - R_{si}} nI \right\} - \frac{2\mu_o\pi}{3}\frac{r_i^3 - R_{si}^3}{R_{so} - R_{si}} nI \qquad (28)$$

using $r_i = R_{so}$ and simplifying yields Φ_{m3} as:

$$\Phi_{m3} = \mu_o\pi\left(R_{so}^2 - R_{si}^2\right)H(R_{To}) - \frac{\pi\mu_o}{3}\left\{2R_{so}^2 - R_{so}R_{si} - R_{si}^2\right\}nI$$

$$\equiv c_{30}H(R_{To}) - c_{31}nI \qquad (29)$$

<u>Region Four</u>: $R_{so} < r < R_{pmi}$ (material in region is air) $H(\rho)$ via Equation (18)

$$\Phi_{m4} = \int_{R_{so}}^{R_{pmi}} \mu_o\left(H(R_{To}) - nI\right) 2\pi\rho d\rho = \mu_o\pi\left(R_{pmi}^2 - R_{so}^2\right)\left(H(R_{To}) - nI\right)$$

$$\equiv c_{40}\left(H(R_{To}) - nI\right) \qquad (30)$$

Equation (25) can now be written in terms of the constants defined in Equations (26), (27), (29), and (30):

$$\Phi_m = \left(c_{10} + c_{20} + c_{30} + c_{40}\right)H(R_{To}) - \left(c_{31} + c_{40}\right)nI$$

with a time derivative given as:

$$\frac{\partial\Phi_m}{\partial t} = j\omega\Phi_m = j\omega\left(c_{10} + c_{20} + c_{30} + c_{40}\right)H(R_{To}) - j\omega\left(c_{31} + c_{40}\right)nI \qquad (31)$$

Equation (23) can now be rewritten using Equations (24) and (31):

$$c_x\left(H(R_{To}) - nI\right) = j\omega\left(c_{10} + c_{20} + c_{30} + c_{40}\right)H(R_{To}) - j\omega\left(c_{31} + c_{40}\right)nI$$

solving for the only unknown, $H(R_{To},\omega)$, yields:

$$H(R_{To}, \omega) = \frac{c_x - j\omega\left(c_{31} + c_{40}\right)}{c_x - j\omega\left(c_{10} + c_{20} + c_{30} + c_{40}\right)} nI \qquad (32)$$

One can now calculate $H(R_{To},\omega)$ as a function of the parameters of the problem. $H(r,\omega)$ can be estimated anywhere in the transducer via Equations (14), (15), (17), (18), or (22). The complex valued electrical inductance, $L(\omega)$, can be estimated by summing the flux linkage per ampere of each layer of the solenoid's windings. Assuming the solenoid has N turns and p layers (each consisting of N/p turns per layer), the inductance can be estimated using Equations (26), (27), and (28) as:

$$L(\omega) = \frac{N\Phi_m}{I} = \frac{N\left(\Phi_{m1} + \Phi_{m2}\right) + \frac{N}{p}\sum_{i=1}^{p}\Phi_{mi}}{I} . \qquad (33)$$

Then the impedance can be calculated as $Z = R_{DC} + j\omega L(\omega)$. All inductance calculations presented in this communication used the expression in Equation (33). The simple formulation for $L(\omega)$ of Equation (13) was not used for either model.

Analytical electromagnetic models have been developed for two physical models of the magnetostrictive transducer by solving Maxwell's equations in cylindrical coordinates. Including magnetomechanical effects is the subject of the next section.

MAGNETOMECHANICAL MODEL

The linear transduction equations for the magnetostrictive material and the transducer containing the magnetostrictive material will be compared (ignoring eddy current effects) in this section of the communication. The two sets of transduction equations will be compared with an eye towards deriving an expression for the electrical impedance of the loaded magnetostrictive transducer. Bearing in mind the following observations, this expression will imply the functional form of the magnetic permeability of the magnetostrictive material within the transducer.

From an engineering viewpoint, it is observed that:

1) The transducer is basically an R-L circuit.

2) In its simplest form, the electrical impedance of an R-L circuit is: $Z = R + j\omega L \approx R + j\omega\mu n^2 \mathrm{Vol}$.

3) R, n, and the volume of the coil (Vol) are all real valued constants; hence, the "strange" variations in magnitude and phase of measured electrical impedance functions of these transducers *must* enter the formulation through variations of the magnetic permeability of the core material (μ) with frequency and load. (The variations in impedance are especially pronounced at frequencies near mechanical resonance of the loaded transducer. For an example, see Figure 2.)

4) The magnetic permeability of magnetostrictive materials is already known to depend upon mechanical conditions. (Clark, 1980) When dealing with magnetostrictives, one speaks of a "blocked permeability" applicable when the magnetostrictive material is prohibited from straining, and speaks of another permeability when the material is held at a constant stress throughout the strain cycle. In this communication this concept (a mechanical strain/displacement influence on magnetic permeability) is extended to include the effects *on* the magnetic permeability *of* the frequency dependent/dynamic material strains. A significant result of this section of the communication will thus be an expression for the dynamic magnetic permeability of the magnetostrictive material within the transducer. This expression *is* the magnetomechanical model.

The low signal linear transduction equations for the magnetostrictive material (as opposed to a transducer containing the material, which is addressed below) are given as (Clark, 1980):

$$\varepsilon = \frac{\sigma}{E_y^H} + qH \tag{34a}$$

$$B = q\sigma + \mu^\sigma H \tag{34b}$$

where: ε is the mechanical strain of the material,

σ is the mechanical stress in the material (positive is tensile, negative is compressive),

$E_y{}^H$ is the "Young's modulus" for the material at constant applied field strength, H (traditionally, a superscript implies which quantity is held constant during the measurement of the material property, here, E_y is measured while H is held constant),

q is the linear coupling coefficient (aka the "d constant"), it is *defined* as the local slope of a plot of strain from applied magnetic field strength *at constant stress*,

H is the applied magnetic field strength,

B is the magnetic flux density within the material, and

μ^σ is the magnetic permeability of the material at constant stress.

It should be noted that these are scalar equations, are usually employed in the time domain, and since they are linear, they can represent either the overall values of the state variables or simply the oscillating values (recall, $H_{total} = H(r,t) + H_{DC\ Bias}$, the same can be said for the ε, σ, and B). All of the quantities are those applicable in the longitudinal direction of the rod, i.e., the strain is the longitudinal strain as a function of position and time, $\varepsilon_{33}(r, \theta, z, t, \sigma, H, B)$. The dependence on position and time is understood. The equations themselves represent the functional dependence on the other variables. Dynamic effects, mass and damping, are completely neglected (as is demonstrated below).

To solve these simultaneous equations, the coefficients, q, $E_y{}^H$, and μ^σ, along with two of the four state variables (ε, σ, B, and H) must be known. When using Terfenol-D, keep in mind that q, $E_y{}^H$, and μ^σ all vary with magnetic bias point, compressive prestress level, and drive levels. (Savage, Clark, and Powers, 1975; Clark et al., 1990; Hall, 1994) As a result, every effort should be made to obtain appropriate values for the coefficients.

According to Hunt (1982), the canonical form of the transduction equations is:

$$V = Z_e I + T_{em} v \qquad (35a)$$

$$F = T_{me} I + z_m v \qquad (35b)$$

where: V is the voltage measured over the terminals of the transducer, volts,

Z_e is the blocked electrical impedance of the transducer, ohms,

I is the electric current passing through the transducer, Amps,

T_{em} is the transduction coefficient for electrical effects from the mechanical velocity, volt sec./meter ("em" electrical from mechanical),

v is the transducer output velocity, m/s,

F is the output force of the transducer, N,

T_{me} is the transduction coefficient for mechanical effects from the electric current, N/Amp ("me" mechanical from electrical), and

z_m is the mechanical impedance of the transducer, N s/m.

These equations assume a single degree of freedom, steady state operation, and that time enters the problem only in the form $e^{j\omega t}$, that is, these equations are in the frequency domain. To solve the equations, all of the coefficients of v and I must be known along with two of the four state variables (V, F, I, and v). The coefficients represent "effective" parameters for the *transducer*. For the case of magnetostrictive transducers, when the coefficients are determined experimentally, they will include the effects of eddy currents within transducer components, leakage flux within the transducer, and all of the detrimental effects of the overall magnetic circuit on the performance of the magnetostrictive material within the transducer.

The dependence of transducer performance on the load can be included by noting that the force applied to a load would be $F_{load} = z_L v$, where z_L is the mechanical impedance of the load. Assuming the output velocity of the transducer is the same as the velocity of the load, the output force of the transducer would be equal and opposite to F_{load}, thus F of Equation (35b) would be given as:

$$F = -z_L v.$$

With this in mind, Equation (35b) becomes

$$0 = T_{me} I + \left(z_m + z_L\right) v. \tag{36}$$

Solving for velocity per ampere, v/I,

$$\frac{v}{I} = \frac{-T_{me}}{\left(z_m + z_L\right)} . \tag{37}$$

The electrical impedance, as would be measured experimentally, is the complex ratio of voltage from current, V/I, called Z_{ee}. Dividing Equation (35a) by current, using Equation (37) for v/I, and combining yields

$$\frac{V}{I} = Z_e - \frac{T_{em} T_{me}}{\left(z_m + z_L\right)} \equiv Z_{ee}. \tag{38}$$

As shown, the measured electrical impedance consists of two terms, the blocked electrical impedance, Z_e, (what one would measure if the transducer output velocity were held at zero, a very difficult task to perform in reality) and a term due to motional effects that is traditionally called the motional impedance, Z_{mot}, defined as:

$$Z_{mot} \equiv \frac{-T_{em} T_{me}}{\left(z_m + z_L\right)} . \tag{39}$$

The motional impedance represents a modification of the transducer's electrical impedance due to motional effects. Combining Equations (38) and (39):

$$Z_{ee} = Z_e + Z_{mot}. \tag{40}$$

It is important to note that if one can calculate Z_e and Z_{mot}, thus Z_{ee}, they are actually solving the pair of linear simultaneous equations given by Equations (35), and they are very close to being able to characterize the performance of the transducer in its linear range (this procedure will be outlined later).

To calculate Z_{ee}, it is necessary to determine the forms of the various coefficients of Equations (35), when the equations are applied to cylindrical magnetostrictive transducers. To determine these coefficients, the two sets of transduction equations, Equations (34) and (35), will be compared. Recall that the typical transducer is basically a magnetostrictive rod, of length l_r and area A, in a wound wire solenoid of N turns. For the comparison, the following approximations and simplifying assumptions are made. For the solenoid, it is assumed that: the rod fills the solenoid; the rod and solenoid are the same length ($l_r = l_s$); $H = nI = NI/l_s$ (recall, n is turns per length of the solenoid), magnetic flux is given as $\Phi_m = BA$; all of the turns of the solenoid have the same magnetic flux linkage, thus the total flux linkage is $N\Phi_m$; and $B = \mu H$. The result of these assumptions is that the electrical inductance of the solenoid is approximately: $L \approx \mu n^2 A l_s$. (The effects of eddy currents are being tacitly ignored in this part of the model development. Their effects will be included via the transducer electromagnetic models developed previously.) Further, it is also assumed that stress and strain are spatially independent, thus $F = \sigma A$ and displacement is strain times length ($u = \varepsilon l_r$). This assumption implies that the analysis is only strictly correct when operating conditions are such that the rod behaves like a linear spring. Lastly, it is assumed that quantities (B, H, ε, σ, V, I, F, and v) vary in time like $e^{j\omega t}$; ergo, velocity and displacement are related by $v = j\omega u$, and the two sets of equations can be compared.

Equations (34) can be rearranged and simplified to obtain a form similar to Equations (35). This will be done because the following variables will be related: V and B, I and H, F and σ, and v and ε. With that aim, Equations (34) become:

$$B = \mu^{\sigma}\left(1 - k^2\right)H + qE_y^H\varepsilon$$

$$\sigma = -qE_y^H H + E_y^H\varepsilon,$$

where:

$$k^2 \equiv \frac{q^2 E_y^H}{\mu^{\sigma}}$$

using variables defined above. "k" is known as the magnetomechanical coupling factor, or one of the "figures of merit" for the material. Making the assumptions that $H = nI$, that $v = j\omega u$, and that $u = l_r \varepsilon$ (which implies that $\varepsilon = v/j\omega l_r$) gives

$$B = \left[\mu^{\sigma}\left(1 - k^2\right)n\right]I + \left[\frac{qE_y^H}{j\omega l_r}\right]v$$

and

$$\sigma = \left[-qE_y^H n\right]I + \left[\frac{E_y^H}{j\omega l_r}\right]v.$$

For a solenoid, the voltage drop across its leads is equal to the DC resistance times the current, $R_{DC}I$, plus the time rate of change of the total flux linkage, i.e., $V \approx R_{DC}I + d(NAB)/dt \approx R_{DC}I + j\omega NAB$. Using B as above, $n = N/l_s$, and comparing the result with Equation (35a) gives

$$V = R_{DC}I + j\omega NAB = \left[R_{DC} + j\omega\mu^{\sigma}\left(1 - k^2\right)n^2 Al_s\right]I + \left[Nq\frac{E_y^H A}{l_r}\right]v = \left|Z_e\right|I + \left|T_{em}\right|v.$$

Therefore, the blocked electrical impedance of the transducer is given as

$$Z_e = R_{DC} + j\omega\left\{\mu^{\sigma}\left(1 - k^2\right)n^2 Al_s\right\} = R_{DC} + j\omega L_{blocked} \qquad (41)$$

where the term in the braces is recognized as an approximation of the inductance of a solenoid containing a material of permeability, $\mu = \mu^{\sigma}(1-k^2) = \mu^{\varepsilon}$ = blocked permeability of the magnetostrictive material.

The transduction coefficient, electrical due to mechanical, is

$$T_{em} = Nq\frac{E_y^H A}{l_r} = Nqk_m^H \qquad (42)$$

where $k_m{}^H$ is the mechanical stiffness of a rod of the magnetostrictive material, when operated at constant field strength. Ignoring losses, constant field strength translates to constant electric current.

The output force of the transducer, F, is equated to stress times area. Using stress as above, and comparing with Equation (35b), yields

$$F = \sigma A = \left[-qE_y^H An\right]I + \left[\frac{E_y^H A}{j\omega l_r}\right]v = \left|T_{me}\right|I + \left|z_m\right|v.$$

Therefore,

$$T_{me} = -Nq\frac{E_y^H A}{l_r} = -Nqk_m^H = -T_{em} \tag{43}$$

and

$$z_m = \frac{1}{j\omega}\,\frac{E_y^H A}{l_r} = \frac{k_m^H}{j\omega}. \tag{44}$$

Note that the transduction, mechanical due to electrical, is the negative of electrical due to mechanical. This relationship was anticipated; it is due to the inherent spatial orthogonallity of electric current and magnetic field. (Hunt, 1982) Note also that the mechanical impedance of the transducer consists of only a stiffness term. Dynamic effects, mass and damping, are tacitly ignored in Equations (34). As a result, use of Equations (34) should be restricted to frequencies well below the first axial resonance of the transducer as run with a given load.

Next, an equation for the electrical impedance of a magnetostrictive transducer, Z_{ee}, including motional effects, will be developed. This will be useful for a number of reasons, for example, it could be of use because the real component of Z_{ee} gives the input electric power per squared ampere of the transducer. (Recall, Power = $I^2\ ||Z_{ee}||\ \cos(ø) = I^2\ \mathrm{Real}\{Z_{ee}\}$.) Another example of its utility is that Z_{ee} could be used in analyzing proposed control algorithms. In the present study, Z_{ee} will be used to imply the functional form for the magnetic permeability of the magnetostrictive rod.

To formulate Z_{ee}, begin with the motional impedance, Z_{mot}, as defined in Equation (39). Using the relationships detailed above for n, l_s, l_r, k^2, k_m^H, T_{me}, and T_{em}, the motional impedance for a magnetostrictive transducer can be written as:

$$Z_{mot} = q^2 E_y^H \frac{E_y^H A / l_r}{\left(z_m + z_L\right)}\,\frac{l_s}{l_r}\,n^2 A l_s = j\omega\mu^\sigma\,\frac{q^2 E_y^H}{\mu^\sigma}\,\frac{k_m^H}{j\omega\left(z_m + z_L\right)}\,n^2 A l_s = j\omega\mu^\sigma k^2\,\frac{k_m^H}{j\omega\left(z_m + z_L\right)}\,n^2 A l_s.$$

Using Z_e, as defined in Equation (41), and Z_{mot} as above, allows Equation (40) to become:

$$Z_{ee} = R_{DC} + j\omega\left\{\mu^\sigma\left(1 - k^2\right)n^2 A l_s\right\} + j\omega\left\{\mu^\sigma k^2\,\frac{k_m^H}{j\omega\left(z_m + z_L\right)}\,n^2 A l_s\right\}$$

or

$$Z_{ee} = R_{DC} + j\omega\left\{\left[\mu^\sigma\left(1 + k^2\left(\frac{k_m^H}{j\omega\left(z_m + z_L\right)} - 1\right)\right)\right]n^2 A l_s\right\} \tag{45}$$

where the term in the braces is the electrical inductance, and the bracketed term is μ_T, the *complex valued* magnetic permeability of the magnetostrictive rod in the solenoid–*including motional effects* as determined by the mechanical impedances of the transducer *and* the load. In particular, the dynamic magnetic permeability of Terfenol-D in a wound wire solenoid, as a function of material coupling and mechanical impedances (thus frequency) is given as:

$$\mu_T = \mu^{\sigma}\left(1 + k^2\left(\frac{k_m^H}{j\omega\left(z_m + z_L\right)} - 1\right)\right) \tag{46}$$

In these transducers, displacements are enhanced or reduced due to dynamic and load effects, thus the property typically called magnetic permeability is effected. More on this topic follows.

Another form for the dynamic magnetic permeability of the magnetostrictive material can be obtained by employing the simple, single degree of freedom mechanical impedance functions:

$$z_m = j\omega m_m + b_m + \left(k_m^H + k_{mps}\right) / j\omega \quad \text{and} \quad z_L = j\omega m_L + b_L + k_L / j\omega$$

where ω is the frequency at which the transducer is operated. Subscript "m" refers to the transducer and "L" refers to the load. The term k_{mps} represents the linear stiffness of the transducer's prestressing mechanism. (Recall, Terfenol-D must be run in an overall state of compressive stress, thus a prestressing spring, wire, beam, etc., is usually used in parallel with the rod.) Of the subscripted quantities: m is the dynamic mass, b is the damping coefficient, and k is the linear stiffness. (Note that these impedances are based on velocities, as opposed to displacements.) Defining m, b, ω_n^2, and $2\zeta\omega_n$ as follows: $m = m_m + m_L$, $b = b_m + b_L$, $\omega_n^2 = (k_m^H + k_{mps} + k_L)/m$, and $2\zeta\omega_n = b/m$; allows Equation (46) to be written as:

$$\mu_T = \mu^{\sigma}\left(1 + k^2\left\{\left(\frac{k_m^H}{k_m^H + k_{mps} + k_L}\right) / \left(1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\,\frac{\omega}{\omega_n}\right) - 1\right\}\right) \tag{47}$$

This formulation shows the mechanical impedance function in a dimensionless form, and one more familiar to some readers.

The behavior of this function for the dynamic magnetic permeability should be checked against known trends in the classic magnetic permeability of magnetostrictive materials. For this discussion it is assumed that k_{mps} is approximately zero. If k_L goes to infinity (the transducer is blocked by the stiffness of the load), the numerator of the frequency dependent fraction goes to zero and the permeability approaches the blocked permeability of the material ($\mu_T = \mu^\sigma(1-k^2) = \mu^\varepsilon$). This trend is in agreement with theory and intuition.

Similarly, if $k_L << k_m^H$, the numerator of the frequency dependent fraction is about one, and what happens depends upon the frequency of excitation. At frequencies, $\omega << \omega_n$, corresponding to the stiffness controlled range of transducer operation, the fraction is very nearly one. Thus, the permeability is approximately μ^σ, i.e., the permeability is unaffected at low frequencies. For excitation frequencies near resonance, $\omega \approx \omega_n$, the permeability is increased due to the increased amplitude of the rod's displacement. In addition, a phase is present owing to the time lag between displacement and current at these frequencies. In the context of $B = \mu_T H$, Equation (47) is really saying that the total time varying axial B field is the sum of that due to the time varying H, and that due to the displacement, u. (This is acceptable because fields add.) Expressed in the frequency domain (and assuming $k_{mps} = k_L = 0$):

$$B = \left(B_{\text{from } H}\right) + \left\{B_{\text{from } u}\right\} = \left(\mu^\sigma\left(1 - k^2\right)H\right) + \left\{\mu^\sigma k^2\left(1 \Big/ \left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + j2\zeta\frac{\omega}{\omega_n}\right]\right)H\right\}$$

where the second term *is* the motional related field. Due to *dynamic* motional effects, the displacement, thus the displacement related B field, is delayed in time when compared to H. In addition, the displacement amplitude varies with frequency, reaching its maximum amplification at $\omega \approx \omega_n$ (assuming low damping). Thus the maximum displacement related B field will occur when $\omega \approx \omega_n$. For frequencies such that $\omega >> \omega_n$, corresponding to the mass controlled range of transducer operation, displacements and the fraction in Equation (47) go to zero. In this case, the permeability again approaches the blocked value, i.e., $\mu_T \approx \mu^\sigma(1-k^2) = \mu^\varepsilon$.

Finally, it is noteworthy that the square of the magnetomechanical coupling, k^2, multiplies the term in Equation (47) which reflects displacement related contributions to the axial flux density. Thus, increasing the coupling increases the magnetic effects of the mechanical displacements.

Most of the above trends agree with those normally expected of the magnetic permeability of a magnetostrictive material. The frequency related effects are a consequence of the applied and motional fields adding, and the motional or displacement related field being amplified/attenuated and delayed in time due to dynamic effects.

The complex valued expression for the dynamic magnetic permeability of the magnetostrictive material, Equation (47), *is* the magnetomechanical model. It is used in *all* electromagnetic model calculations that call for a permeability of the magnetostrictive material (Terfenol-D in this study), including k_T, as given by Equation (9). Using the dynamic permeability in k_T calculations improves the accuracy of transducer simulations, when compared to simulations using k_T based on a constant magnetic permeability. (Hall, 1994) It also has some interesting implications on the frequency dependence of eddy current effects.

The eddy current losses occurring within the magnetostrictive rod are related to the k_T term defined in Equation (9) and repeated here: $k_T = (j\omega\ \mu_T\ \sigma_{eT})^{1/2}$. Eddy current losses increase with any term in the expression that increases. One typically only considers increasing frequency, ω. However, with a frequency dependence for the magnetic permeability, as implied by Equation (47), eddy current losses are less straight forward. As the frequency of excitation is increased toward that of mechanical resonance, the model predicts an increase in eddy current losses owing to the increase in both frequency *and* permeability. At frequencies slightly higher than that of mechanical resonance, the model predicts a relative *decrease* in eddy current losses owing to the destructive interference of the displacement related and drive field related magnetic fluxes (the displacement is approaching -180° phase). Lower overall time rates of change of flux density translate to lower induced electric fields within the magnetostrictive rod, which mean lower eddy current losses. Thus, the model implies that, over a range of frequencies above that of mechanical resonance, one can expect relatively low eddy current losses, i.e., the transducer will run cooler. This implication warrants further experimental investigation.

Taken together, the magnetomechanical model, which includes motional effects, and the electromagnetic models, which include the effects of eddy currents, result in analytical models of electrical impedance functions for two popular cylindrical magnetostrictive transducer geometries.

RESULTS & DISCUSSION

Figure 2 shows two example calculations of Z_{ee} using the combined electromagnetic-magnetomechanical models described above, as applied to a particular Terfenol-D transducer (Hall and

Flatau, 1992) in its low signal linear range of operation. The figure also shows the experimentally measured electrical impedance function (100 averages, 400 frequency lines, Hanning window) obtained using pseudo-random, constant voltage excitation. In the figure, plot (a) shows magnitudes, (b) shows phases. Recall that Model 1 was developed neglecting the effects of the external conducting cylindrical permanent magnet, whereas, Model 2 included its effects. As is shown in the figure, both models are capable of simulating the distinctive characteristics of the coupled electromagnetic-magnetomechanical system, that is, both models exhibit large variations in magnitude and phase near mechanical resonance. The second model provides the best simulation of the transducer's measured impedance with typical errors in magnitude or phase of less than ±5% for frequencies between 100 and 10 000 Hz. The relatively large errors in phase estimates at frequencies below 100 Hz are thought to be due primarily to end effects. At these lower frequencies of excitation the steel ends, which actually extended into the wound wire solenoid on each end, result in an increased average magnetic permeability for the transducer. The models neglect this effect, thus they underestimate the inductance at the low frequencies. As the frequency of excitation increases, eddy currents within the end-caps begin to shield them magnetically, resulting in a reduction in their apparent permeability and their effects on the transducer's inductance become less appreciable.

It is interesting to note the effects of the conducting external housing as shown in Figure 2. Eddy currents occurring in the external housing result in a lower electrical impedance magnitude for the transducer. This trend is due to lower flux linkage in the solenoid. Recall, the flux in the coil is that due to the electric current in the solenoid plus the fluxes due to the eddy currents occurring in the rod and the cylindrical housing. These fluxes have different phases, thus they sum to a lower net value yielding a reduced impedance magnitude.

At first glance one might like the idea of lowering the electrical impedance magnitude of the transducer. There are at least two problems with this. First, the amplifier driving the transducer is supplying the energy for all of the currents flowing in the transducer. Each current has an associated ohmic loss, thus more currents translate to higher losses. The second problem is that for the reduced inductance case, less energy is stored in the magnetic field which means there is less energy available to transduce from magnetic to mechanical (which *is* the primary function of the transducer).

It may have been obvious to some from the start, and perhaps it is more apparent to others now, but the transducer *should not* include a conducting cylindrical external housing. The simple modification of slitting one side of the cylindrical housing lengthwise, i.e., making it "C" shaped when viewed down the axis, would interrupt the annular conducting path, dramatically reducing the eddy current problem. (This was not done here; however, it was done in (Hall, 1994).)

The following values were employed in both model calculations shown in Figure 2: $k_{mps} = k_L = 0$, $\mu^\sigma = 2.9\ \mu_o$, $k^2 = 0.025$ (low coupling due to low signal amplitudes), $2\zeta = 0.09$, $\omega_n = 2\pi \times 8800$ rad./s (Hall and Flatau, 1992), and $\sigma_{e\ T} = 1/6 \times 10^7$ (ohm meter)$^{-1}$ (EDGE, 1988). For Model 2, the following values were used for the cylindrical permanent magnet: $\sigma_{e\ pm} = 1/45 \times 10^8$ (ohm meter)$^{-1}$ and $\mu_{pm} = 4\ \mu_o$ (Hansen, 1993). The magnetic permeability of the permanent magnet, natural frequency of oscillation, and both electrical conductivities were obtained from the literature. The damping parameter of the transducer (ζ), and the coupling (k^2) and magnetic permeability (μ^σ) of the Terfenol-D rod were found empirically for these simulations, but that need not be the case.

As formulated, the dynamic magnetic permeability of the magnetostrictive rod, Equation (47), contains a "mixed" batch of parameters. Linear transduction theory applied to the magnetostrictive material *as run in the transducer* could supply values for: μ^σ, k^2, and $E_y{}^H$ (which implied $k_m{}^H$ in this study). (Savage, Clark, and Powers, 1975; Hall, 1994) Linear transduction theory as applied to the transducer, as loaded, could give values for: k_L, ω_n, and ζ. (Hunt, 1982; Hall, 1994) In general, one would like to know these parameters before a transducer is built. In that case, one could use the procedure presented here to calculate a *prediction* of the transducer's performance (Figure 2 shows *simulations* of transducer performance–accurate values for μ^σ, ζ, k^2, and $E_y{}^H$, were not known before the transducer was tested). The accuracy of the prediction would depend heavily upon the accuracy to which the parameters were known. Experience has shown that Terfenol-D properties vary significantly from test-to-test and rod-to-rod. Thus, one should anticipate larger errors in *predictions* than in *simulations*. The simulation using Model 2 in Figure 2 was a good one, i.e., simulated values were generally within ±5% of the experimental measurement. Average simulations run about ±10%. *Predictions* calculated using this approach can be in error by more than 40%. (Hall, 1994, p. 128)

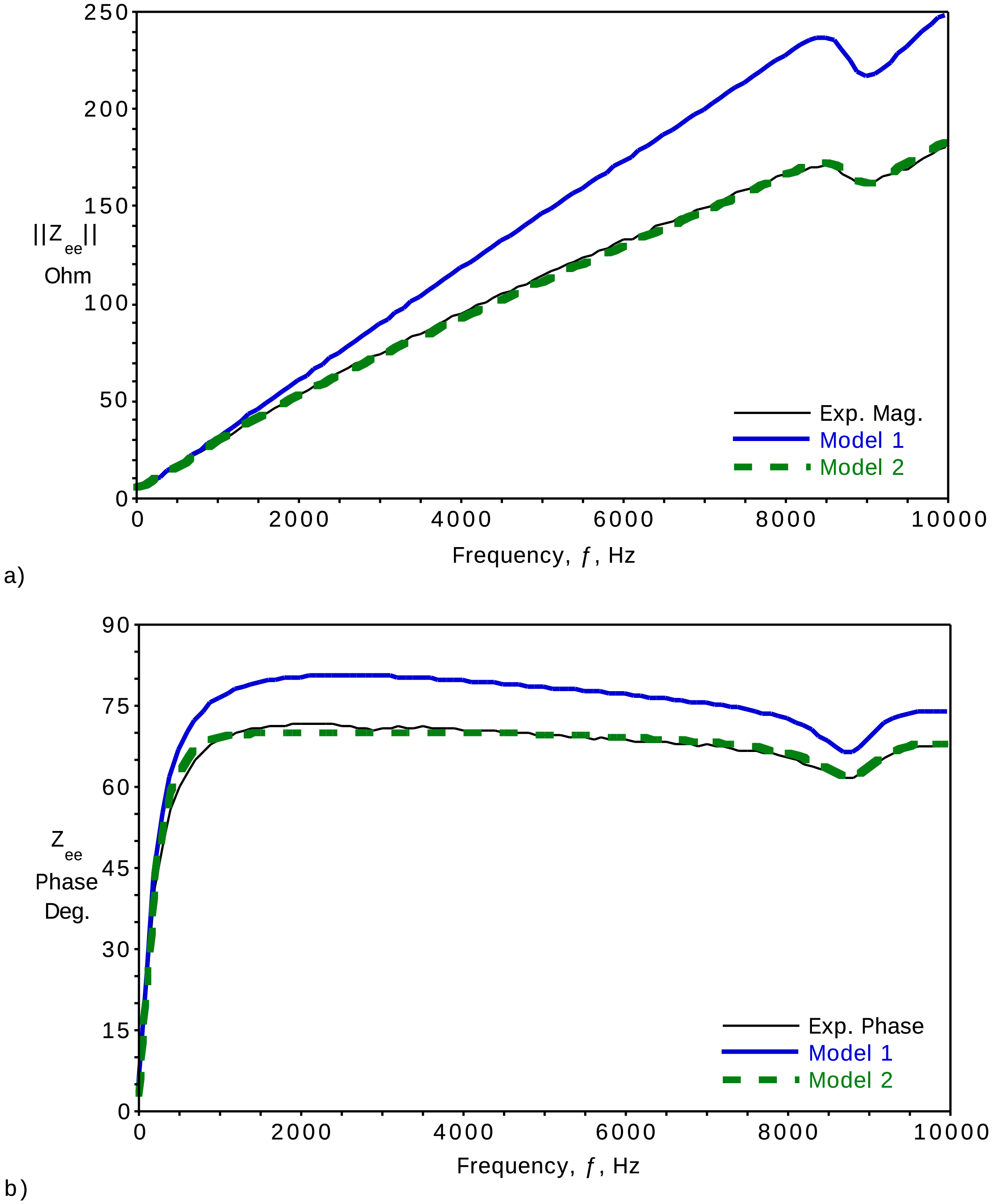

Figure 2. Comparisons of electrical impedance functions, magnitude (a) and phase (b), for a Terfenol-D transducer as simulated by the electromagnetic-magnetomechanical models and as measured experimentally (100 averages, 400 frequency lines, Hanning window).

CONCLUSIONS & RECOMMENDATIONS

Analytical electrical impedance function models for two common magnetostrictive transducer geometries are presented. The models include the motional effects of the magnetostrictive material and the effects of eddy currents occurring either in the conducting rod (Model 1), or in the rod and conducting cylindrical housing (Model 2). Transducer Electromagnetic Model 2 results in an electromechanical mathematical model that provides reasonable *simulations* of the measured electrical impedance of the cylindrical Terfenol-D transducer tested in this study. This is as it should be since the transducer under study *had* a conducting cylindrical housing and transducer Model 1 ignored its existence.

The deleterious effects of the cylindrical housing are shown in Figure 2. From the figure it is concluded that the external housing should be slit longitudinally to break-up the external cylindrical conducting path. Alternatively, one might use a housing of much lower electrical conductivity. Standard, commercially available Terfenol-D actuators include *both* a solid cylindrical conducting permanent magnet (like the one in this study) *and* a solid cylindrical external aluminum housing (which is *also* subject to induced eddy currents). It is recommended that these designs be changed.

An implicit assumption of the modeling procedure presented in this communication is that the magnetostrictive material (Terfenol-D in this study) has a magnetic permeability that is independent of eddy currents but that it is a function of load and excitation frequency. The effects of eddy currents are calculated separately. Though it is impossible to separate the effects of eddy currents from experimental estimates of magnetic permeability, the success of Model 2 indicates that they might be separable mathematically. The frequency and load effects on the magnetic permeability came from an extension of the widely held view that the material property called magnetic permeability is effected by the mechanical conditions to which the magnetostrictive material is subjected (i.e., blocked, or constant stress).

The eddy current losses occurring within the magnetostrictive rod are related to the k_T parameter defined in Equation (9), repeated here: $k_T = (j\omega\, \mu_T\, \sigma_{eT})^{1/2}$. Eddy current losses increase with any term in the expression which increases. With a frequency dependence for the magnetic permeability, as implied by Equation (47), eddy current losses should increase disproportionately near mechanical resonance and exhibit a relative decrease over a frequency range above resonance. Further research is recommended to validate this implication of the magnetomechanical model.

A method for estimating magnetostrictive transducer electrical impedance, Z_{ee}, as a function of magnetostrictive material and transducer parameters and as a function of mechanical load has been presented. Other quantities of engineering interest can also be calculated via Equations (35), assuming all of the coefficients are found. The procedure is as follows: (Hall, 1994) Z_{ee} is calculated using the dynamic magnetic permeability given by Equation (47) and the improved inductance approximation of Equation (33). (Recall that for this calculation, z_m and z_L, the mechanical impedances of the transducer and the load, respectively, must be known.) Z_e, the blocked electrical impedance of the transducer, is then calculated using the same algorithm used for the Z_{ee} calculation, only this time through using the blocked permeability everywhere a permeability of the magnetostrictive material appears. This way, both Z_{ee} and Z_e will include the effects of eddy currents. At this point, formulations for the transduction coefficients will allow the general use of Equations (35) to model transducer behavior. Recall that it was shown that $T_{em} = -T_{me}$. Equations (39) and (40) can be used to estimate T_{em}, including the deleterious effects of eddy currents (recall that Equations (42) and (43) *did not* include eddy effects). Thus, $T_{em} = \{(Z_{ee} - Z_e)(z_m + z_L)\}^{1/2}$, and Equations (35) are now at the designer's disposal for transducer modeling.

Equations (34), the low signal linear transduction equations traditionally used to describe magnetostrictive material behavior, are shown to neglect dynamic effects. Thus, they are limited to use at only very low frequencies (when compared with the axial resonant frequency of the rod as loaded). A new functional form for the complex valued dynamic magnetic permeability of the magnetostrictive material within a cylindrical transducer is presented in Equation (47). It includes the dynamic effects of the transducer as loaded.

The models of this study were compared with a transducer operating in its low signal linear range. Consequently, the effects of magnetic hysteresis were ignored. It is thought that simulations of transducer operation at larger excitation amplitudes might be improved by using the elliptical relationship, $B \approx (\mu e^{-j\varphi}) H$, discussed earlier. ($\mu^{\sigma}$ in Equation (47) would be replaced with the elliptical approximation.) It seems likely that both μ and φ are functions of at least excitation amplitude. As future research, experimental measurements should be performed to determine these functional relationships.

It is important to recall that the models in this study are linear. They ignore the harmonic frequencies that might be present in the transducer voltage, current, velocity, etc. With Terfenol-D, increasing the drive amplitude has the effect of increasing the relative magnitudes of the integer harmonics. The linear modeling procedure presented in this communication is thus limited to applications where these harmonics can be treated as noise. Assuming the harmonics can be ignored in a given application, Hall (1994) reports that this technique provides reasonable *simulations* of transducer performance (±10% errors) up to strains of about 1/3 of the magnetostrictive rod's low frequency saturation strain.

Most of the physical parameters employed in the models were obtained from the literature. This adds hope that this approach may be widely applicable. It is thought that a transducer might be designed by formulating a physical model and dynamic model of the proposed design, for use in Equation (47), estimating permeability, stiffness, and coupling of the rod at the proposed magnetic biasing and drive levels (perhaps from information provided by the manufacturer), running the electromagnetic-magnetomechanical simulation, and comparing the predictions with the design parameters. Validation of this transducer design procedure awaits further research into Terfenol-D property values–as run in a typical transducer–as functions of at least magnetic biasing, prestress level, and low signal AC drive levels.

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Running Title: 1-D Analytical Electromechanical Models

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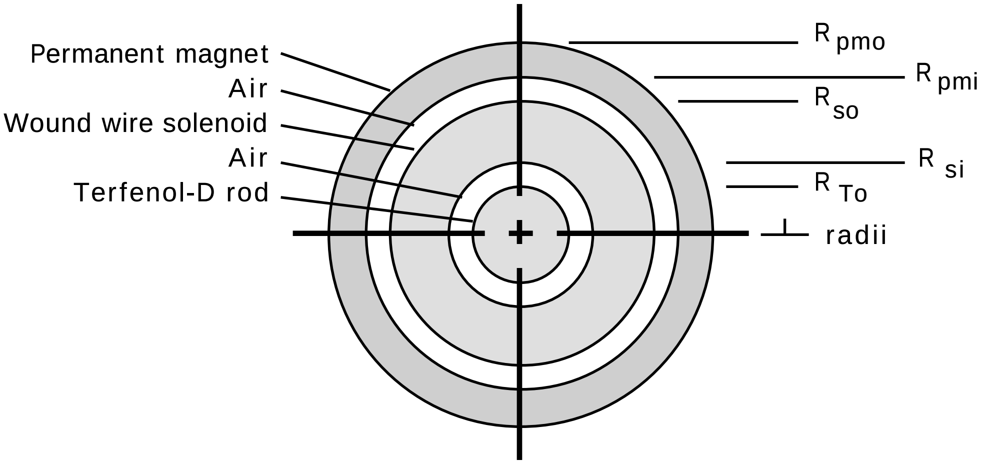

Figure 1. Schematic representation of end-view of the cylindrical magnetostrictive transducer under study (end caps removed). Subscripts on the radii read like: To = Terfenol, outer; si = solenoid, inner; so = solenoid, outer; pmi = permanent magnet, inner; and pmo = permanent magnet, outer.

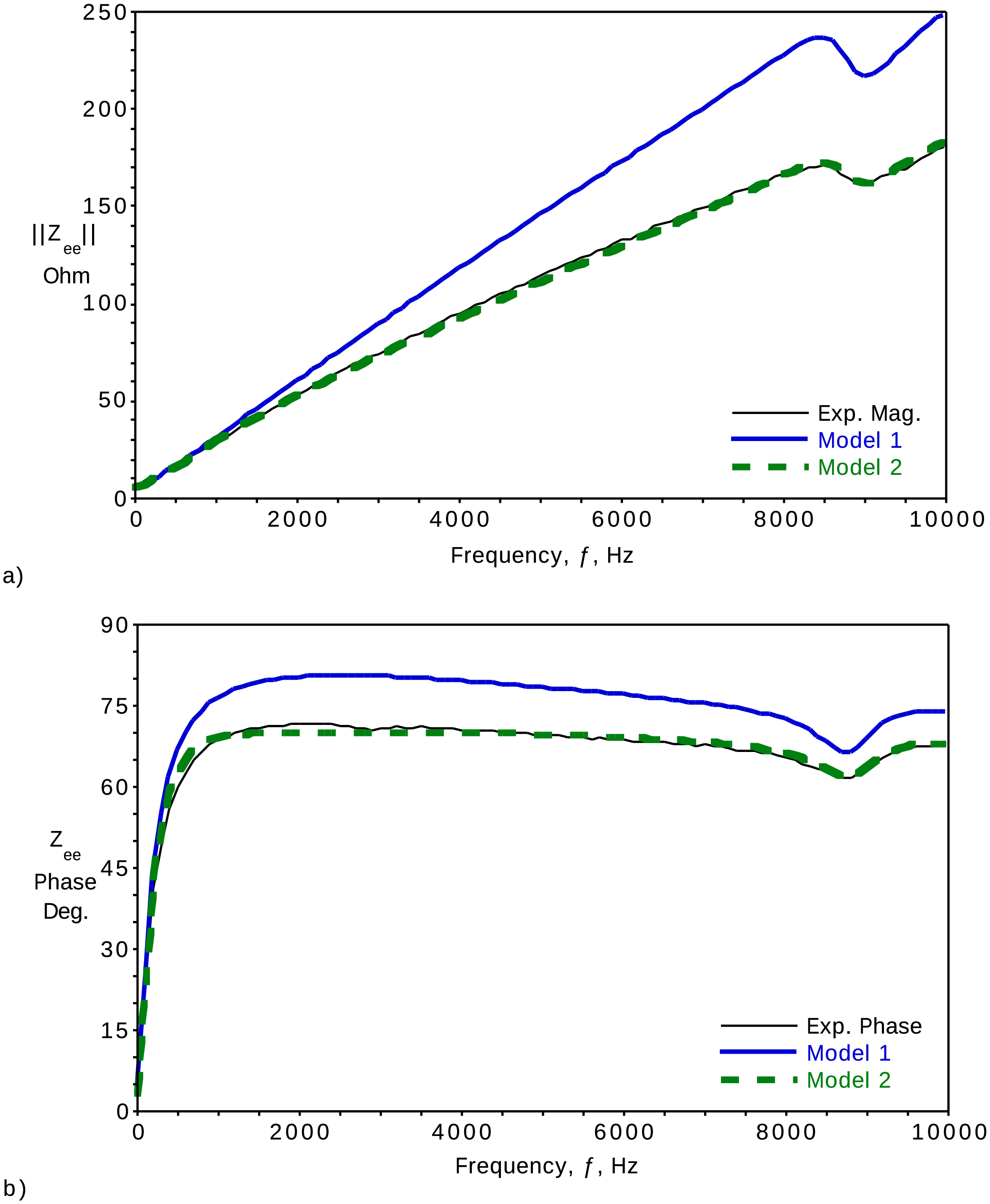

Figure 2. Comparisons of electrical impedance functions, magnitude (a) and phase (b), for a Terfenol-D transducer as simulated by the electromagnetic-magnetomechanical models and as measured experimentally (100 averages, 400 frequency lines, Hanning window).