Allen – Math 265 B1 Quiz #3 Name: Key

1. Describe geometrically the domain of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 16}$.

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Need $x^2+y^2+z^2-16 \geq 0 \leftarrow +1$

$\Rightarrow x^2+y^2+z^2 \geq 16.$ $+1$

The domain is everything on or outside the sphere centered at $(0,0,0)$ with radius 4.

+1 +1 +1

2. Show that $f(x, y) = x^3y - xy^3$ is harmonic. In other words, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

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$f_x(x,y) = 3x^2y - y^3$ $+1$ $f_y(x,y) = x^3 - 3xy^2$ $+1$

$f_{xx}(x,y) = 6xy$ $+1$ $f_{yy} = -6xy$ $+1$

Thus $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 6xy - 6xy = 0$ ✓. $+1$

3. Let

$$f(x, y) = \begin{cases} \dfrac{x^2 - 4y^2}{x - 2y}, & \text{if } x \neq 2y \\ g(x), & \text{if } x = 2y \end{cases}$$

If f is continuous in the whole plane, find a formula for $g(x)$.

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Note that

$$\frac{x^2-4y^2}{x-2y} = \frac{(x-2y)(x+2y)}{x-2y} = x+2y \quad +3$$

When $x=2y$, $y = \frac{1}{2}x$. Thus, $g(x) = x + 2(\frac{1}{2})x$ $+2$

$\Rightarrow \boxed{g(x) = 2x}$ $+1$

-2 for $g(x) = x+2y$.

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4. Show that the function defined by

$$f(x,y,z) = \begin{cases} \dfrac{xyz}{x^3+y^3+z^3}, & \text{for } (x,y,z) \neq (0,0,0) \\ 0, & \text{for } (x,y,z) = (0,0,0) \end{cases}$$

is not continuous at $(0,0,0)$.

+4

Consider the path $x=t,\ y=t,\ z=t$. Along this path,

$f(x,y,z) = \dfrac{t \cdot t \cdot t}{t^3+t^3+t^3} = \dfrac{t^3}{3t^3} = \dfrac{1}{3}$. Thus

$\lim\limits_{\substack{(x,y,z)\to(0,0,0) \\ \text{along } x=t,\,y=t,\,z=t}} f(x,y,z) = \dfrac{1}{3}$. +2 But $f(0,0,0) = 0 \neq \dfrac{1}{3}$. +1

Hence, f cannot be continuous at $(0,0,0)$.

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5. Find the slope of the tangent to the curve of intersection of the surface $2z = \sqrt{9x^2+9y^2-36}$ and the plane $y=1$ at the point $(2, 1, \frac{3}{2})$.

use f_x +3

$z = \frac{1}{2}(9x^2+9y^2-36)^{\frac{1}{2}} = f(x,y)$

$f_x(x,y) = \frac{1}{4}(9x^2+9y^2-36)^{-\frac{1}{2}}(18x) = \frac{9x}{2}(9x^2+9y^2-36)^{-1/2}$ +2

Slope of line $= f_x(2,1) = \frac{9(2)}{2}(9(2)^2+9(1)^2-36)^{-1/2}$

$= 9(36+9-36)^{-1/2} = 9(9)^{-1/2}$

$= 9 \cdot \frac{1}{\sqrt{9}} = 9 \cdot \frac{1}{3} = \boxed{3}$

+2