## Curvature, Velocity, and Acceleration Formulas:

**Unit Tangent Vector:**

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\vec{v}(t)}{\|\vec{v}(t)\|}.$$

**Curvature:**
1.) If a 2-D curve is given by $y = g(x)$, then

$$K(x) = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}} = \frac{|g''(x)|}{\left[1 + (g'(x))^2\right]^{3/2}}.$$

2.) If a curve is defined by parametric equations, or by a curvilinear motion $\vec{r}(t)$, then

$$K(t) = \frac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|^3} = \frac{\|\vec{v}(t) \times \vec{a}(t)\|}{\|\vec{v}(t)\|^3}.$$

Remarks: a.) Formula (2) is useful even if the curve is 2-D. Just set the $z$-component to zero.
b.) If you know $a_N(t)$ already, then $K(t)$ is easy to compute using formula (2).
c.) The primes in (1) denote differentiation with respect to $x$. In (2) (and also in the remaining formulas on this sheet), the primes denote differentiation with respect to $t$.

**Tangential Component of Acceleration:**
1.) If you already know what $\vec{T}(t)$ is, then it's probably easier to use

$$a_T(t) = \vec{r}\,''(t) \cdot \vec{T}(t) = \vec{a}(t) \cdot \vec{T}(t).$$

2.) You can also use the formula

$$a_T(t) = \frac{d}{dt}\,\|\vec{r}\,'(t)\| = \frac{d}{dt}\,\|\vec{v}(t)\|.$$

**Normal Component of Acceleration:**
1.) If you computed $a_T(t)$ first, it's easier to use the Pythagorean identity,

$$\|\vec{a}(t)\|^2 = a_T^2(t) + a_N^2(t),$$

and then solve for $a_N(t)$.
2.) Otherwise, you can use

$$a_N(t) = \frac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|} = \frac{\|\vec{v}(t) \times \vec{a}(t)\|}{\|\vec{v}(t)\|} = \left\|\vec{a}(t) \times \vec{T}(t)\right\|.$$

Remark: If you know $K(t)$ already, then $a_N(t)$ is easy to compute using formula (2).

**Unit Normal Vector:**
1.) If you know $a_T(t)$ and $a_N(t)$ already, then it is easier to use the tangential and normal decomposition formula for acceleration,

$$\vec{a}(t) = a_T(t)\vec{T}(t) + a_N(t)\vec{N}(t),$$

then solve for $\vec{N}(t)$.
2.) Otherwise, you can use

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}.$$

**Binormal Vector:**

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t).$$