Allen – Math 160 A Quiz #7 Name: Key

1. Use implicit differentiation to determine $\frac{dy}{dx}$ if $xe^y + x^2 = y^2$

$$\frac{d}{dx}[xe^y] + \frac{d}{dx}[x^2] = \frac{d}{dx}[y^2] \Rightarrow x\frac{d}{dx}[e^y] + 1e^y + 2x = 2y\frac{dy}{dx}$$

+2

$$\Rightarrow xe^y\frac{dy}{dx} + e^y + 2x = 2y\frac{dy}{dx} \Rightarrow e^y + 2x = 2y\frac{dy}{dx} - xe^y\frac{dy}{dx}$$

+1 +1

6 $$\Rightarrow e^y + 2x = \frac{dy}{dx}(2y - xe^y) \Rightarrow \boxed{\frac{dy}{dx} = \frac{e^y+2x}{2y-xe^y}}$$ +1

+1

2. Determine an equation for the line tangent to the graph of $x \ln y = 2x^3 - 2y$ at the point $(1, 1)$.

$$\frac{d}{dx}[x\ln y] = \frac{d}{dx}[2x^3] - \frac{d}{dx}[2y] \Rightarrow x\frac{d}{dx}[\ln y] + 1\cdot\ln y = 6x^2 - 2\frac{dy}{dx}$$

+2

$$\Rightarrow x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \ln y = 6x^2 - 2\,dy/dx.$$ +1

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$$x=1,\ y=1 \Rightarrow (1)\cdot\frac{1}{(1)}\frac{dy}{dx} + \ln(1) = 6(1)^2 - 2\,dy/dx \Rightarrow \frac{dy}{dx} + 0 = 6 - 2\,dy/dx$$

$$\Rightarrow 3\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = 2;\quad m = 2,\quad \boxed{y-1 = 2(x-1)} \Rightarrow y-1 = 2x-2 \Rightarrow \boxed{y = 2x-1}$$

+3 +2

3. The area of a square with sides x inches is increasing at a rate of 10 square inches per minute. Find the rate at which the length of a side is increasing when the sides are 3 inches. Recall the formula for the area of a square is $A = x^2$.

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Know: $\frac{dA}{dt} = 10$, $x = 3$

Need: $\frac{dx}{dt}$

$$\frac{d}{dt}[A] = \frac{d}{dt}[x^2] \Rightarrow \frac{dA}{dt} = 2x\frac{dx}{dt}$$

+2 +1

$$\Rightarrow 10 = 2(3)\frac{dx}{dt} \Rightarrow 10 = 6\frac{dx}{dt} \Rightarrow \boxed{\frac{dx}{dt} = \frac{10}{6} = \frac{5}{3}}$$

+1 +1 +1

The length of a side is increasing at a rate of $5/3 \approx 1.667$ inches per minute