

1. Factor each polynomial completely.

(a)  $x^5 - 9x^3 - 8x^2 + 72$ .

$$x^5 - 9x^3 - 8x^2 + 72 = x^3(x^2 - 9) - 8(x^2 - 9) = (x^3 - 8)(x^2 - 9) = (x - 2)(x^2 + 2x + 4)(x - 3)(x + 3).$$

(b)  $42x^2 + 91x - 35$ .

$42x^2 + 91x - 35 = 7(6x^2 + 13x - 5)$ . Consider  $6x^2 + 13x - 5$ .  $ac = -30$ ,  $15 + (-2) = 13$ , and  $15 \cdot (-2) = -30$ . Thus  $6x^2 + 13x - 5 = 6x^2 + 15x - 2x - 5 = 3x(2x + 5) - 1(2x + 5) = (3x - 1)(2x + 5)$ . Therefore,  $42x^2 + 91x - 35 = 7(3x - 1)(2x + 5)$ .

2. Perform the indicated operations. Leave your answer in factored form.

(a)  $\frac{\frac{x^3+3x^2-4x-12}{3x^2-147}}{\frac{7x^2+35x+42}{x^2-2x-35}}$ .

$$\begin{aligned} \frac{\frac{x^3+3x^2-4x-12}{3x^2-147}}{\frac{7x^2+35x+42}{x^2-2x-35}} &= \frac{x^3 + 3x^2 - 4x - 12}{3x^2 - 147} \cdot \frac{x^2 - 2x - 35}{7x^2 + 35x + 42} \\ &= \frac{x^2(x+3) - 4(x+3)}{3(x^2-49)} \cdot \frac{(x+5)(x-7)}{7(x^2+5x+6)} = \frac{(x^2-4)(x+3)}{3(x-7)(x+7)} \cdot \frac{(x+5)(x-7)}{7(x+2)(x+3)} \\ &= \frac{(x-2)(x+2)(x+3)}{3(x-7)(x+7)} \cdot \frac{(x+5)(x-7)}{7(x+2)(x+3)} = \frac{(x-2)(x+2)(x+3)(x+5)(x-7)}{21(x-7)(x+7)(x+2)(x+3)} \\ &= \frac{(x+2)(x+5)}{21(x+7)}. \end{aligned}$$

(b)  $\frac{2x+1}{6x^2+7x-3} - \frac{3x-2}{9x^2-1}$ .

Consider  $6x^2 + 7x - 3$ .  $ac = -18$ ,  $9 + (-2) = 7$ , and  $9 \cdot (-2) = -18$ . Thus,  $6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3 = 3x(2x + 3) - 1(2x + 3) = (3x - 1)(2x + 3)$ . Therefore,  $6x^2 + 7x - 3 = (3x - 1)(2x + 3)$ . In addition,  $9x^2 - 1 = (3x - 1)(3x + 1)$ . So we have

$$\begin{aligned} \frac{2x+1}{(3x-1)(2x+3)} - \frac{3x-2}{(3x-1)(3x+1)} &= \frac{(2x+1)(3x+1)}{(3x-1)(2x+3)(3x+1)} - \frac{(3x-2)(2x+3)}{(3x-1)(3x+1)(2x+3)} \\ &= \frac{6x^2+5x+1}{(3x-1)(3x+1)(2x+3)} - \frac{6x^2+5x-6}{(3x-1)(3x+1)(2x+3)} \\ &= \frac{6x^2+5x+1-6x^2-5x+6}{(3x-1)(3x+1)(2x+3)} = \frac{7}{(3x-1)(3x+1)(2x+3)}. \end{aligned}$$

3. Solve the equation  $3x^2 - 5 = 7x$  by completing the square.

$$3x^2 - 5 = 7x \Leftrightarrow 3x^2 - 5 - 7x = 0 \Leftrightarrow 3x^2 - 7x = 5 \Leftrightarrow x^2 - \frac{7}{3}x = \frac{5}{3}.$$

So,  $m = -\frac{7}{3}$ ,  $\frac{m}{2} = -\frac{7}{6}$ , and  $\left(\frac{m}{2}\right)^2 = \frac{49}{36}$ . Hence, we have

$$x^2 - \frac{7}{3}x + \frac{49}{36} = \frac{5}{3} + \frac{49}{36} \Leftrightarrow \left(x - \frac{7}{6}\right)^2 = \frac{60}{36} + \frac{49}{36} \Leftrightarrow \left(x - \frac{7}{6}\right)^2 = \frac{109}{36}.$$

By the square root method

$$x - \frac{7}{6} = \pm \sqrt{\frac{109}{36}} \Leftrightarrow x - \frac{7}{6} = \pm \frac{\sqrt{109}}{6} \Leftrightarrow x = \frac{7 \pm \sqrt{109}}{6}.$$

The solution set is  $\left\{\frac{7 + \sqrt{109}}{6}, \frac{7 - \sqrt{109}}{6}\right\}$ .

4. Find all real solutions to the following equations.

(a)  $\left|-\frac{1}{7}x + \frac{3}{5}\right| = \frac{5}{6}.$

Either  $-\frac{1}{7}x + \frac{3}{5} = \frac{5}{6}$  or  $-\frac{1}{7}x + \frac{3}{5} = -\frac{5}{6}.$

$$\Leftrightarrow -\frac{1}{7}x = \frac{5}{6} - \frac{3}{5} \text{ or } -\frac{1}{7}x = -\frac{5}{6} - \frac{3}{5}$$

$$\Leftrightarrow -\frac{1}{7}x = \frac{25}{30} - \frac{18}{30} \text{ or } -\frac{1}{7}x = -\frac{25}{30} - \frac{18}{30}$$

$$\Leftrightarrow -\frac{1}{7}x = \frac{7}{30} \text{ or } -\frac{1}{7}x = -\frac{43}{30}$$

$$\Leftrightarrow x = -\frac{49}{30} \text{ or } x = \frac{301}{30}. \text{ Solution set: } \left\{-\frac{49}{30}, \frac{301}{30}\right\}.$$

(b)  $\left|5x^2 - 26x + \frac{5}{2}\right| = \frac{5}{2}.$

Either  $5x^2 - 26x + \frac{5}{2} = \frac{5}{2}$  or  $5x^2 - 26x + \frac{5}{2} = -\frac{5}{2}.$

$$\Leftrightarrow 5x^2 - 26x = 0 \text{ or } 5x^2 - 26x + 5 = 0$$

$$\Leftrightarrow x(5x - 26) = 0 \text{ or } 5x^2 - 26x + 5 = 0.$$

By the zero-product property, either  $x = 0$ ,  $5x = 26$ , or  $5x^2 - 26x + 5 = 0$ .  $5x = 26$ , implies that

$x = \frac{26}{5}$ . For  $5x^2 - 26x + 5 = 0$ , use the quadratic formula.

$$x = \frac{26 \pm \sqrt{676 - 4(5)(5)}}{2(5)} = \frac{26 \pm \sqrt{676 - 100}}{10} = \frac{26 \pm \sqrt{576}}{10} = \frac{26 \pm 24}{10}. \text{ Thus } x = \frac{50}{10} \text{ or } x = \frac{2}{10},$$

which is the same as  $x = 5$  or  $x = \frac{1}{5}$ . Putting everything together, we get  $x = 0$ ,  $x = \frac{26}{5}$ ,  $x = 5$ , or

$$x = \frac{1}{5}. \text{ Solution set: } \left\{0, \frac{1}{5}, 5, \frac{26}{5}\right\}.$$

5. Express the following complex numbers in the standard form  $a + bi$ .

$$\begin{aligned}
 \text{(a)} \quad & \frac{7 - 3i}{11 - 12i} \cdot \frac{7 - 3i}{7 - 3i} = \frac{(7 - 3i)(11 + 12i)}{(11 - 12i)(11 + 12i)} = \frac{77 + 84i - 33i - 36i^2}{11^2 + 12^2} = \frac{77 + 51i - 36(-1)}{121 + 144} \\
 & = \frac{77 + 51i + 36}{265} = \frac{113 + 51i}{265} = \frac{113}{265} + \frac{51}{265}i.
 \end{aligned}$$

$$\text{(b)} \quad (5 - 6i)^4.$$

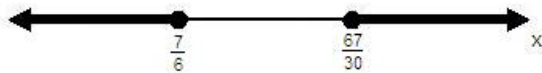
First, find  $(5 - 6i)^2$ .  $(5 - 6i)^2 = (5 - 6i)(5 - 6i) = 25 - 60i + 36i^2 = 25 - 60i - 36 = -11 - 60i$ .  
 Therefore,  $(5 - 6i)^4 = (5 - 6i)^2(5 - 6i)^2 = (-11 - 60i)(-11 - 60i) = 121 + 1320i + 3600i^2$   
 $= 121 + 1320i - 3600 = -3479 + 1320i$ .

6. Solve each of the inequalities, and graph their solution set.

$$\begin{aligned}
 \text{(a)} \quad & \left| \frac{3}{5}x - \frac{4}{3} \right| < \frac{1}{6}. \\
 & \left| \frac{3}{5}x - \frac{4}{3} \right| < \frac{1}{6} \Leftrightarrow -\frac{1}{6} < \frac{3}{5}x - \frac{4}{3} < \frac{1}{6} \\
 & \Leftrightarrow -\frac{1}{6} + \frac{4}{3} < \frac{3}{5}x < \frac{1}{6} + \frac{4}{3} \Leftrightarrow -\frac{1}{6} + \frac{8}{6} < \frac{3}{5}x < \frac{1}{6} + \frac{8}{6} \\
 & \Leftrightarrow \frac{7}{6} < \frac{3}{5}x < \frac{9}{6} \Leftrightarrow \frac{7}{6} < \frac{3}{5}x < \frac{3}{2} \\
 & \Leftrightarrow \frac{35}{6} < 3x < \frac{15}{2} \Leftrightarrow \frac{35}{18} < 3x < \frac{5}{2}. \\
 & \text{Interval notation: } \left( \frac{35}{18}, \frac{5}{2} \right).
 \end{aligned}$$



$$\begin{aligned}
 \text{(b)} \quad & \left| \frac{17}{16} - \frac{5}{8}x \right| \geq \frac{1}{3}. \\
 & \left| \frac{17}{16} - \frac{5}{8}x \right| \geq \frac{1}{3} \\
 & \Leftrightarrow \frac{17}{16} - \frac{5}{8}x \geq \frac{1}{3} \text{ or } \frac{17}{16} - \frac{5}{8}x \leq -\frac{1}{3} \\
 & \Leftrightarrow -\frac{5}{8}x \geq \frac{1}{3} - \frac{17}{16} \text{ or } -\frac{5}{8}x \leq -\frac{1}{3} - \frac{17}{16} \\
 & \Leftrightarrow -\frac{5}{8}x \geq \frac{16}{48} - \frac{51}{48} \text{ or } -\frac{5}{8}x \leq -\frac{16}{48} - \frac{51}{48} \\
 & \Leftrightarrow -\frac{5}{8}x \geq -\frac{35}{48} \text{ or } -\frac{5}{8}x \leq -\frac{67}{48} \\
 & \Leftrightarrow -5x \geq -\frac{35}{6} \text{ or } -5x \leq -\frac{67}{6} \Leftrightarrow x \leq \frac{7}{6} \text{ or } x \geq \frac{67}{30} \\
 & \text{Interval notation: } \left( -\infty, \frac{7}{6} \right] \cup \left[ \frac{67}{30}, \infty \right).
 \end{aligned}$$



7. Solve the following equations over the complex numbers.

(a)  $-5x^4 - 35x^3 + 5x + 35 = 0$ .

$$-5x^4 - 35x^3 + 5x + 35 = 0 \Leftrightarrow -5(x^4 + 7x^3 - x - 7) = 0 \Leftrightarrow x^4 + 7x^3 - x - 7 = 0.$$

$$x^3(x+7) - 1(x+7) = 0 \Leftrightarrow (x^3 - 1)(x+7) = 0 \Leftrightarrow (x-1)(x^2 + x + 1)(x+7) = 0.$$

By the zero-product property, either  $x = 1$ ,  $x = -7$ , or  $x^2 + x + 1 = 0$ . Use the quadratic formula on  $x^2 + x + 1 = 0$  and get

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i. \text{ So we have } x = -7, x = -1, \text{ and}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i. \text{ Solution set: } \left\{ -7, 1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right\}.$$

(b)  $x(7x - 13) = -109$ .

$$x(7x - 13) = -109 \Leftrightarrow 7x^2 - 13x = -109 \Leftrightarrow 7x^2 - 13x + 109 = 0. \text{ Using the quadratic formula,}$$

$$x = \frac{13 \pm \sqrt{169 - 4(7)(109)}}{2(7)} = \frac{13 \pm \sqrt{169 - 3052}}{14} = \frac{13 \pm \sqrt{-2883}}{14} = \frac{13 \pm \sqrt{2883}i}{14}$$

$$= \frac{13 \pm \sqrt{961}\sqrt{3}i}{14} = \frac{13 \pm 31\sqrt{3}i}{14} = \frac{13}{14} \pm \frac{31\sqrt{3}}{14}i. \text{ Solution set: } \left\{ \frac{13}{14} - \frac{31\sqrt{3}}{14}i, \frac{13}{14} + \frac{31\sqrt{3}}{14}i \right\}.$$

(c)  $x^6 + 9x^3 + 8 = 0$ .

$x^6 + 9x^3 + 8 = 0 \Leftrightarrow (x^3 + 8)(x^3 + 1) = 0 \Leftrightarrow (x+2)(x^2 - 2x + 4)(x+1)(x^2 - x + 1) = 0$ . By the zero-product property, either  $x = -2$ ,  $x = -1$ ,  $x^2 - 2x + 4 = 0$ , or  $x^2 - x + 1 = 0$ . Use the quadratic formula on  $x^2 - 2x + 4 = 0$  and get

$$x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm \sqrt{12}i}{2} = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i.$$

Use the quadratic formula on  $x^2 - x + 1 = 0$  and get

$$x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i. \text{ Putting everything together, we have}$$

$$x = -2, x = -1, x = 1 \pm \sqrt{3}i, \text{ and } x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i. \text{ So the solution set is}$$

$$\left\{ -2, -1, 1 - \sqrt{3}i, 1 + \sqrt{3}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i \right\}.$$