Math 140-P2 Solutions to Homework #1:

Sec. A.1 #12: $-2x + xy = -2(-2) + (-2)(3) = 4 + (-6) = -2$.
Sec. A.1 #20: None. This is because $x^2 \geq 0$ no matter what x is, and so $x^2 + 9 \geq 9$ for all x. Hence, $x^2 + 9$ can never be zero.
Sec. A.1 #22: (b) and (d) because $x^2 - 1 = 0$ when $x = 1$ and when $x = -1$.
Sec. A.1 #28: There is a problem if $x - 6 = 0$, thus the fraction is undefined when $x = 6$. So the domain is $\{x | x \neq 6\}$.
Sec. A.1 #34: $<$.
Sec. A.1 #46: $x \geq 2$.
Sec. A.1 #54: $|x| - |y| = |3| - |-2| = 3 - 2 = 1$.
Sec. A.1 #56: $\dfrac{|y|}{y} = \dfrac{|-2|}{-2} = \dfrac{2}{-2} = \dfrac{-2}{2} = -1$.
Sec. A.1 #66: $d(D, B) = |(-1) - 1| = |-2| = 2$.
Sec. A.1 #68: $-4^2 = -4 \cdot 4 = -16$.
Sec. A.1 #70: $-4^{-2} = -(4^{-2}) = -\left(\dfrac{1}{4^2}\right) = -\left(\dfrac{1}{16}\right) = -\dfrac{1}{16}$.
Sec. A.1 #78: $\sqrt{(-3)^2} = |-3| = 3$.
Sec. A.1 #81: $(x^2y^{-1})^2 = (x^2)^2 \cdot (y^{-1})^2 = x^4 \cdot y^{-2} = x^4 \cdot \left(\dfrac{1}{y^2}\right) = \dfrac{x^4}{y^2}$.
Sec. A.1 #84: $\dfrac{x^{-2}y}{xy^2} = \left(\dfrac{x^{-2}}{x}\right) \cdot \left(\dfrac{y}{y^2}\right) = \left(x^{-2-1}\right) \cdot \left(y^{1-2}\right) = x^{-3} \cdot y^{-1}$
$= \left(\dfrac{1}{x^3}\right) \cdot \left(\dfrac{1}{y}\right) = \dfrac{1}{x^3y}$.
Another way: $\dfrac{x^{-2}y}{xy^2} = \dfrac{y}{x \cdot x^2 \cdot y^2} = \dfrac{y}{x^3y^2} = \dfrac{y}{x^3y \cdot y} = \dfrac{1}{x^3y}$.
Sec. A.1 #86: $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \dfrac{4x^{-2}y^{-1}z^{-1}}{8x^4y} = \dfrac{4}{8} \cdot \left(\dfrac{x^{-2}}{x^4}\right) \cdot \left(\dfrac{y^{-1}}{y}\right) \cdot z^{-1}$
$= \dfrac{1}{2} \cdot \left(x^{-2-4}\right) \cdot \left(y^{-1-1}\right) \cdot z^{-1} = \dfrac{1}{2} \cdot \left(x^{-6}\right) \cdot \left(y^{-2}\right) \cdot z^{-1}$
$= \dfrac{1}{2} \cdot \left(\dfrac{1}{x^6}\right) \cdot \left(\dfrac{1}{y^2}\right) \cdot \left(\dfrac{1}{z}\right) = \dfrac{1}{2x^6y^2z}$.
Another way: $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \dfrac{4x^{-2}y^{-1}z^{-1}}{8x^4y} = \dfrac{4}{8x^2yzx^4y} = \dfrac{4}{8x^6y^2z} = \dfrac{1}{2x^6y^2z}$.
Sec. A.1 #88: $\left(\dfrac{5x^{-2}}{6y^{-2}}\right)^{-3} = \dfrac{5^{-3}x^6}{6^{-3}y^6} = \dfrac{\frac{1}{5^3}x^6}{\frac{1}{6^3}y^6} = \dfrac{\frac{1}{125}x^6}{\frac{1}{216}y^6} = \dfrac{216x^6}{125y^6}$.
Another way: $\left(\dfrac{5x^{-2}}{6y^{-2}}\right)^{-3} = \left(\dfrac{6y^{-2}}{5x^{-2}}\right)^3 = \left(\dfrac{6x^2}{5y^2}\right)^3 = \dfrac{6^3x^6}{5^3y^6} = \dfrac{216x^6}{125y^6}$.

Sec. A.3 #18: $(x-a)^2 - x^2 = (x-a)(x-a) - x^2 = x^2 - ax - ax + a^2 - x^2$
$= x^2 - x^2 - 2ax + a^2 = -2ax + a^2$.
Sec. A.3 #22:
$(x^2+2x+1)(x^2-3x+4) = x^4 - 3x^3 + 4x^2 + 2x^3 - 6x^2 + 8x + x^2 - 3x + 4$
$= x^4 - 3x^3 + 2x^3 + 4x^2 - 6x^2 + x^2 + 8x - 3x + 4 = x^4 - x^3 - x^2 + 5x + 4$.
Sec. A.3 #30: $x^2 + 5x + 4 = (x+1)(x+4)$.
Sec. A.3 #35: $15+2x-x^2 = -x^2+2x+15 = -(x^2-2x-15) = -(x+3)(x-5)$.
Sec. A.3 #40: $3y^3 - 18y^2 - 48y = 3y(y^2 - 6y - 16) = 3y(y+2)(y-8)$.
Sec. A.3 #44: $4x^2+3x-1$. $a \cdot c = -4$. Integer factors of -4: $-4, -2, -1, 1, 2, 4$. Note that $-1+4 = 3$ and $(-1)\cdot 4 = -4$. Then, $4x^2 + 3x - 1 = 4x^2 - 1x + 4x - 1$
$= x(4x-1) + 1(4x-1) = (x+1)(4x-1)$.
Sec. A.3 #48: $x^6 + 2x^3 + 1$. Make the substitution $u = x^3$. Then $x^6 + 2x^3 + 1$ becomes $u^2 + 2u + 1 = (u+1)(u+1) = (u+1)^2$. Now, plug x^3 back in for u and get $x^6 + 2x^3 + 1 = (x^3+1)^2$. But, by the sum of cubes formula, $x^3 + 1 = (x+1)(x^2 - x + 1)$. So, $x^6 + 2x^3 + 1 = \left[(x+1)(x^2-x+1)\right]^2$
$= (x+1)^2(x^2-x+1)^2$.
Sec. A.3 #50: $x^8 - x^5 = x^5(x^3-1) = x^5(x-1)(x^2+x+1)$ by the difference of cubes formula.
Sec. A.3 #52: $5 + 11x - 16x^2 = -16x^2 + 11x + 5$. $a \cdot c = (-16)(5) = -80$. The integer multiples of 80 are as follows:
$-80, -40, -20, -16, -10, -8, -5, -4, -2, -1, 1, 2, 4, 5, 8, 10, 16, 20, 40, 80$.
Notice that $16 + (-5) = 11$ and $16(-5) = -80$. Thus $-16x^2 + 11x + 5$
$= -16x^2 - 5x + 16x + 5 = -x(16x+5) + 1(16x+5) = (-x+1)(16x+5)$
$= -(x-1)(16x+5)$.
Sec. A.3 #60: $(x-1)^2 - 2(x-1) = (x-1)\left[(x-1)-2\right]$
$= (x-1)\left[x-1-2\right] = (x-1)(x-3)$.
Sec. A.3 #63: $x^3 + 2x^2 - x - 2 = x^2(x+2) - 1(x+2) = (x^2-1)(x+2)$
$= (x-1)(x+1)(x-2)$.
Sec. A.3 #68: $\dfrac{9x^2-25}{2x-2} \cdot \dfrac{1-x^2}{6x-10} = \dfrac{(3x)^2-5^2}{2(x-1)} \cdot \dfrac{-(x^2-1)}{2(3x-5)}$
$= \dfrac{(3x-5)(3x+5)}{2(x-1)} \cdot \dfrac{-(x-1)(x+1)}{2(3x-5)} = -\dfrac{(3x-5)(3x+5)(x-1)(x+1)}{4(x-1)(3x-5)}$
$= -\dfrac{(3x+5)(x+1)}{4}$.
Sec. A.3 #72: $\dfrac{x}{x-3} - \dfrac{x+1}{x^2+5x-24} = \dfrac{x}{x-3} - \dfrac{x+1}{(x-3)(x+8)}$. The least common multiple of $(x-3)$ and $(x-3)(x+8)$ is $(x-3)(x+8)$. Thus,
$\dfrac{x}{x-3} - \dfrac{x+1}{(x-3)(x+8)} = \dfrac{x}{x-3} \cdot \dfrac{(x+8)}{(x+8)} - \dfrac{x+1}{(x-3)(x+8)}$
$= \dfrac{x^2+8x}{(x-3)(x+8)} - \dfrac{(x+1)}{(x-3)(x+8)} = \dfrac{x^2+8x-x-1}{(x-3)(x+8)} = \dfrac{x^2+7x-1}{(x-3)(x+8)}$.

Sec. A.3 #76: $\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^3-x^2}=\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^2(x-1)}$. The least common multiple of $(x-1)^2$, x, and $x^2(x-1)$ is $x^2(x-1)^2$. Thus,
$\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^2(x-1)}=\dfrac{x}{(x-1)^2}\cdot\dfrac{x^2}{x^2}+\dfrac{2}{x}\cdot\dfrac{x(x-1)^2}{x(x-1)^2}-\dfrac{x+1}{x^2(x-1)}\cdot\dfrac{x}{x}$
$=\dfrac{x^3}{x^2(x-1)^2}+\dfrac{2x(x-1)^2}{x^2(x-1)^2}-\dfrac{(x+1)(x-1)}{x^2(x-1)^2}$
$=\dfrac{x^3}{x^2(x-1)^2}+\dfrac{2x(x^2-2x+1)}{x^2(x-1)^2}-\dfrac{x^2-1}{x^2(x-1)^2}$
$=\dfrac{x^3}{x^2(x-1)^2}+\dfrac{2x^3-4x^2+2x}{x^2(x-1)^2}-\dfrac{(x^2-1)}{x^2(x-1)^2}$
$=\dfrac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2}=\dfrac{3x^3-5x^2+2x+1}{x^2(x-1)^2}$.

Sec. A.3 #86: $3x^2(3x+4)^2+x^3\cdot 2(3x+4)\cdot 3=3x^2(3x+4)^2+6x^3(3x+4)$
$=(3x+4)\left[3x^2(3x+4)+6x^3\right]=(3x+4)\cdot 3x^2\left[(3x+4)+2x\right]$
$=3x^2(3x+4)[3x+4+2x]=3x^2(3x+4)(5x+4)$.

Sec. A.3 #90: $\dfrac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}=\dfrac{20x+5-(20x-8)}{(5x-2)^2}$
$=\dfrac{20x+5-20x+8}{(5x-2)^2}=\dfrac{13}{(5x-2)^2}$.

Sec. A.3 #96: $\dfrac{(x^2+9)\cdot 2-(2x-5)\cdot 2x}{(x^2+9)^2}=\dfrac{2x^2+18-(4x^2-10x)}{(x^2+9)^2}$
$=\dfrac{2x^2+18-4x^2+10x}{(x^2+9)^2}=\dfrac{-2x^2+10x+18}{(x^2+9)^2}=\dfrac{-2(x^2-5x-9)}{(x^2+9)^2}$.