1. Factor each polynomial completely.

   (a) $x^4 - 8x^2 + 16$.
   $x^4 - 8x^2 + 16 = (x^2-4)(x^2-4) = (x^2-4)^2 = [(x-2)(x+2)]^2 = (x-2)^2(x+2)^2$.

   (b) $7x^3 + 56$.
   $7x^3 + 56 = 7(x^3+8) = 7(x+2)(x^2-2x+4)$.

2. Perform the indicated operations. Leave your answer in factored form.

   (a) $\dfrac{3}{x^2+3x+2} + \dfrac{3}{x^2+5x+6}$.
   $$\frac{3}{x^2+3x+2} + \frac{3}{x^2+5x+6} = \frac{3}{(x+1)(x+2)} + \frac{3}{(x+2)(x+3)}$$
   $$= \frac{3(x+3)}{(x+1)(x+2)(x+3)} + \frac{3(x+1)}{(x+2)(x+3)(x+1)} = \frac{3x+9}{(x+1)(x+2)(x+3)} + \frac{3x+3}{(x+1)(x+2)(x+3)}$$
   $$= \frac{6x+12}{(x+1)(x+2)(x+3)} = \frac{6(x+2)}{(x+1)(x+2)(x+3)} = \frac{6}{(x+1)(x+3)}.$$

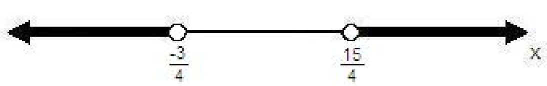
   (b) $\dfrac{2x^2-8}{x^2-49} \cdot \dfrac{x^2-4x-21}{x^2+4x-12}$.
   $$\frac{2x^2-8}{x^2-49} \cdot \frac{x^2-4x-21}{x^2+4x-12} = \frac{2(x^2-4)}{x^2-49} \cdot \frac{x^2-4x-21}{x^2+4x-12}$$
   $$\frac{2(x-2)(x+2)}{(x-7)(x+7)} \cdot \frac{(x-7)(x+3)}{(x-2)(x+6)} = \frac{2(x-2)(x+2)(x-7)(x+3)}{(x-7)(x+7)(x-2)(x+6)} = \frac{2(x+2)(x+3)}{(x+6)(x+7)}.$$

3. Solve the equation $2x^2 - 8x + 4 = 0$ by completing the square.
$2x^2 - 8x + 4 = 0 \Leftrightarrow 2x^2 - 8x = -4 \Leftrightarrow x^2 - 4x = -2$.
So, $m = -4$, $\dfrac{m}{2} = -2$, and $\left(\dfrac{m}{2}\right)^2 = 4$. Hence, we have
$x^2 - 4x + 4 = -2 + 4 \Leftrightarrow (x-2)^2 = 2$. By the square root method
$x - 2 = \pm\sqrt{2} \Leftrightarrow x = 2 \pm \sqrt{2}$.
The solution set is $\left\{2 - \sqrt{2}, 2 + \sqrt{2}\right\}$.

4. Solve each of the inequalities, and graph their solution set.

(a) $\dfrac{3}{2} \le -2x + \dfrac{5}{2} \le \dfrac{9}{2}$.
$\dfrac{3}{2} \le -2x + \dfrac{5}{2} \le \dfrac{9}{2} \Leftrightarrow \dfrac{3}{2} - \dfrac{5}{2} \le -2x \le \dfrac{9}{2} - \dfrac{5}{2}$
$\Leftrightarrow -1 \le -2x \le 2 \Leftrightarrow \dfrac{1}{2} \ge x \ge -1 \Leftrightarrow -1 \le x \le \dfrac{1}{2}$.
Interval notation: $\left[-1, \dfrac{1}{2}\right]$.

(b) $\left|\dfrac{4}{3}x - 2\right| > 3$.
$\left|\dfrac{4}{3}x - 2\right| > 3 \Leftrightarrow \dfrac{4}{3}x - 2 > 3$ or $\dfrac{4}{3}x - 2 < -3$
$\Leftrightarrow \dfrac{4}{3}x > 5$ or $\dfrac{4}{3}x < -1 \Leftrightarrow 4x > 15$ or $4x < -3$
$\Leftrightarrow x > \dfrac{15}{4}$ or $x < \dfrac{3}{4}$.
Interval notation: $\left(-\infty, \dfrac{3}{4}\right) \cup \left(\dfrac{15}{4}, \infty\right)$.



5. Find all real solutions to the following equations.

(a) $|3x-5| = 17$.
Either $3x-5=17$ or $3x-5=-17 \Leftrightarrow 3x=22$ or $3x=-12$
$\Leftrightarrow x = \dfrac{22}{3}$ or $x=-4$. Solution set: $\left\{-4, \dfrac{22}{3}\right\}$.

(b) $x(3x-7) = -1$.
$x(3x-7) = -1 \Leftrightarrow 3x^2-7x=-1 \Leftrightarrow 3x^2-7x+1=0$.
Using the quadratic formula.
$x = \dfrac{7 \pm \sqrt{49-4(3)(1)}}{2(3)} = \dfrac{7 \pm \sqrt{49-12}}{6} = \dfrac{7 \pm \sqrt{37}}{6}$. Solution set: $\left\{\dfrac{7-\sqrt{37}}{6}, \dfrac{7+\sqrt{37}}{6}\right\}$.

(c) $3x^3-12x^2-12x+48=0$.
$3x^3-12x^2-12x+48=0 \Leftrightarrow 3\left(x^3-4x^2-4x+16\right)=0 \Leftrightarrow x^3-4x^2-4x+16=0$
$\Leftrightarrow x^2(x-4)-4(x-4)=0 \Leftrightarrow (x^2-4)(x-4)=0 \Leftrightarrow (x-2)(x+2)(x-4)=0$.
By the zero-product property, either $x=2$, $x=-2$, or $x=4$.
Solution set: $\{-2, 2, 4\}$.

(d) $\left|x^2-7x+6\right| = 6$.
Either $x^2-7x+6=6$ or $x^2-7x+6=-6$.
$\Leftrightarrow x^2-7x=0$ or $x^2-7x+12=0$
$\Leftrightarrow x(x-7)=0$ or $(x-4)(x-3)=0$.
By the zero-product property, either $x=0$, $x=7$, $x=3$, or $x=4$.
Solution set: $\{0, 3, 4, 7\}$.

6. Express the following complex numbers in the standard form $a + bi$.

   (a) $\dfrac{1-i}{2+3i}$.
   $\dfrac{1-i}{2+3i} = \dfrac{(1-i)(2-3i)}{(2+3i)(2-3i)} = \dfrac{2-3i-2i+3i^2}{2^2+3^2} = \dfrac{2-5i-3}{4+9}$
   $= \dfrac{-1-5i}{13} = -\dfrac{1}{13} - \dfrac{5}{13}i.$

   (b) $(2+i)^3$.
   $(2+i)^3 = (2+i)(2+i)(2+i) = (4+2i+2i+i^2)(2+i) = (4+4i-1)(2+i) = (3+4i)(2+i)$
   $= 6+3i+8i+4i^2 = 6+11i-4 = 2+11i.$

7. Solve the following equations over the complex numbers.

   (a) $5x^2 + 2 = -3x$.
   $5x^2+2 = -3x \Leftrightarrow 5x^2+3x+2 = 0$. Using the quadratic formula,
   $x = \dfrac{-3 \pm \sqrt{9-4(5)(2)}}{2(5)} = \dfrac{-3 \pm \sqrt{9-40}}{10} = \dfrac{-3 \pm \sqrt{-31}}{10} = \dfrac{-3 \pm \sqrt{31}i}{10} = -\dfrac{3}{10} \pm \dfrac{\sqrt{31}}{10}i.$
   Solution set: $\left\{-\dfrac{3}{10} - \dfrac{\sqrt{31}}{10}i, -\dfrac{3}{10} + \dfrac{\sqrt{31}}{10}i\right\}$.

   (b) $x^4 - x^3 + 8x - 8 = 0$.
   $x^4-x^3+8x-8 = 0 \Leftrightarrow x^3(x-1)+8(x-1) = 0 \Leftrightarrow (x^3+8)(x-1) = 0 \Leftrightarrow (x+2)(x^2-2x+4)(x-1) = 0.$
   By the zero-product property, either $x = -2$, $x = 1$, or $x^2 - 2x + 4 = 0$. Use the quadratic formula on $x^2 - 2x + 4 = 0$ and get
   $x = \dfrac{2 \pm \sqrt{4-4(1)(4)}}{2(1)} = \dfrac{2 \pm \sqrt{-12}}{2} = \dfrac{2 \pm \sqrt{12}i}{2} = \dfrac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i.$
   Putting everything together, we have $x = -2$, $x = 1$, or $x = 1 \pm \sqrt{3}i$. So the solution set is $\{-2, 1, 1-\sqrt{3}i, 1+\sqrt{3}i\}$.

8. **Extra Credit** Solve $x^6 - 1 = 0$ over the complex number system. (Suggestion: Factor using the difference of squares to get started.)
   $x^6 - 1 = 0 \Leftrightarrow (x^3-1)(x^3+1) = 0 \Leftrightarrow (x-1)(x^2+x+1)(x+1)(x^2-x+1) = 0$. Using the quadratic formula on $x^2+x+1 = 0$, we get $x = \dfrac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i.$
   Using the quadratic formula on $x^2 - x + 1 = 0$, we get $x = \dfrac{1 \pm \sqrt{1-4(1)(1)}}{2(1)} = \dfrac{1 \pm \sqrt{-3}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i.$
   Solution set: $\left\{-1, 1, -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i, -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i, \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i, \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right\}$.