Stat 544 Exam 1

March 2, 2006

I have neither given nor received unauthorized assistance on this examination.

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signature date

1. Below are tables giving pmfs $f(x|\theta)$ for an observable X, for a parameter θ with possible values $1, 2,$ and 3. Then there is a table giving a pmf $g(\theta)$ for a simple discrete prior distribution for the parameter.

x	$f(x\|1)$
1	.4
2	.3
3	.2
4	.1

x	$f(x\|2)$
1	.25
2	.25
3	.25
4	.25

x	$f(x\|3)$
1	.1
2	.2
3	.3
4	.4

θ	$g(\theta)$
1	.2
2	.3
3	.5

a) Suppose that a single observation takes the value $X = 1$. Give a pmf for the posterior distribution of θ.

b) Suppose that (conditioned on θ) X_1 and X_2 are iid with marginal pmf $f(x|\theta)$. If $X_1 = 1$, find the posterior predictive distribution (give a pmf) for X_2.

c) Suppose that (conditioned on θ) X_1 and X_2 are iid with marginal pmf $f\left(x\,|\,\theta\right)$ and that $\left(X_1, X_2\right) = (1,1)$ is observed. Give a pmf for the posterior distribution of θ.

2. In a Bayes analysis, a parameter θ has a continuous posterior with pdf

$$h(\theta) = \begin{cases} C\exp(-\theta) & \text{for } 1 < \theta < 4 \\ 0 & \text{otherwise} \end{cases}$$

for an appropriate constant C.

a) Find (two-sided "percentile") 90% credible limits for θ.

b) Find a 90% "HPD" credible set for θ.

3. Consider the following model for an observable Y and a parameter λ.

- conditioned on λ and an unobservable variable X, Y is $\mathrm{N}(X,1)$
- conditioned on the parameter λ, X is $\mathrm{Poi}(\lambda)$
- λ is $\mathrm{Exp}(1)$

a) Write out a formula for a joint "density" for (X,Y,λ), $h(x,y,\lambda)$ for $x=0,1,2,\ldots$, $y\in\Re$, and $\lambda>0$.

Based on your answer to a), consider a SSS/Gibbs algorithm for generating from the joint distribution of (X,Y,λ).

b) Name (distribution type and parameters) the conditional distributions that will be needed in the "Y" and "λ" updates. (Exactly what univariate distributions will you need to generate from?)

c) Completely describe a particular "rejection algorithm" that could be used to simulate from the conditional distribution of X given Y and λ.

4. Attached to this exam is a summary of a `WinBUGS` session conducted in the analysis of some data collected by a manufacturer that uses a pneumatic torque gun (set during a laboratory "calibration" to deliver 370 ft-lbs of torque) to tighten some bolts on an assembly line. Recorded are some measurements of peak torque made (at considerable inconvenience) as bolts are tightened, and some torques read later as bolts are loosened using one of two company torque wrenches (wrench A and wrench B). Since a tightened bolt can be loosened only once, only one of "A" and "B" measurements is available for a given bolt. Data for 51 bolts are given below in tabular form.

Tightening	Wrench A	Wrench B
343	463	
332	478	
365	378	
332	448	
353	465	
357	546	
346	473	
343	461	
378		381
345		372
354		365
347		362
351		349
340		330
365		365
345		338
360		382
357		383
345		376
347		370
	495	
	509	
	521	
	529	
	568	
	554	
	494	
	573	
	523	
	496	
	535	
	509	
	442	
	481	
	481	
	468	
		333
		336
		331
		320
		361
		370
		387
		385
		342
		364
		315
		430
		365
		338
		382

The "tightening" torque is the best available indication of how well a given bolt is tightened, but the corresponding measurement method cannot be regularly used to monitor the work on the assembly line. Instead, the company tries to use wrenches A and B to check bolt tightness "after the fact."

a) When tested in a lab, the torque gun delivers an average of 370 ft-lbs of torque. According to the Bayes analysis here, what is a 95% credible interval for the difference between average lab torque and average tightening torque during production?

b) What are 95% credible intervals for the mean "loosening torques" measured by wrenches A and B? Which of these intervals is wider, and what about the data set makes it "obvious in retrospect" that the interval should be the wider of the two?

Wrench A: Wrench B:

Rationale for the wider interval:

c) The company wishes to know which wrench could be used most effectively to detect real differences in torque delivered when tightening these bolts. Which wrench do you recommend and why? (Be careful, means don't tell the story.) Is it clear that *either* wrench really provides effective indication of differences in tightening torque? Explain.

d) The present analysis uses fairly "flat"/"diffuse" priors. Suppose that the present data are used to establish a prior distribution for analysis of data from a subsequent study similar to the present one. Think of the present data as perhaps equivalent to 6 (unattainable) complete vectors of 3 measurements. Exactly what do you recommend for an "informative" prior for the analysis of data from the later study? (Be sure to completely specify a joint distribution for a mean vector and covariance matrix, giving numerical values for all parameters.)

WinBUGS Session Summary

```
model

    {
        for(i in 1:51)
        {
            Y[i, 1:3] ~ dmnorm(mu[], R[ , ])
        }

        mu[1:3] ~ dmnorm(alpha[],Tau[ , ])

        R[1:3 , 1:3] ~ dwish(Lambda[ , ], nu)

        D[1:3, 1:3]<-inverse(R[1:3, 1:3])

  sig1<-sqrt(D[1,1])

  sig2<-sqrt(D[2,2])

  sig3<-sqrt(D[3,3])

  rho12<-D[1,2]/(sig1*sig2)

        rho13<-D[1,3]/(sig1*sig3)

        rho23<-D[2,3]/(sig2*sig3)

        diff21<-mu[2]-mu[1]

        diff31<-mu[3]-mu[1]

    }

list(nu=5, alpha=c(0,0,0),

 Tau = structure(.Data = c(0.000001, 0, 0, 0, 0.000001, 0, 0, 0, 0.000001), .Dim = c(3, 3)),

 Lambda = structure(.Data = c(2000, 0, 0, 0, 2000, 0, 0, 0, 2000), .Dim = c(3, 3)),

 Y = structure(.Data = c(
                    343,463,NA,
                    332,478,NA,
                    365,378,NA,
                    332,448,NA,
                    353,465,NA,
                    357,546,NA,

                      ⋮

                    NA,NA,315,
                    NA,NA,430,
                    NA,NA,365,
                    NA,NA,338,
                    NA,NA,382), .Dim = c(51, 3)))
```

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
D[1,1]	228.3	77.62	0.3855	122.3	213.5	420.5	50000	200004
D[1,2]	9.358	181.6	1.489	-364.5	12.49	364.8	50000	200004
D[1,3]	109.5	112.9	0.7208	-106.3	106.0	344.6	50000	200004
D[2,1]	9.358	181.6	1.489	-364.5	12.49	364.8	50000	200004
D[2,2]	1944.0	572.9	2.517	1124.0	1845.0	3330.0	50000	200004
D[2,3]	18.58	618.8	8.992	-1086.0	29.68	1099.0	50000	200004
D[3,1]	109.5	112.9	0.7208	-106.3	106.0	344.6	50000	200004
D[3,2]	18.58	618.8	8.992	-1086.0	29.68	1099.0	50000	200004
D[3,3]	702.0	197.9	0.8733	414.9	669.3	1180.0	50000	200004
R[1,1]	0.006025	0.001792	8.249E-6	0.003071	0.005835	0.01004	50000	200004
R[1,2]	-2.326E-5	5.911E-4	3.445E-6	-0.00121	-2.562E-5	0.001175	50000	200004
R[1,3]	-0.001022	0.001168	7.622E-6	-0.003473	-9.706E-4	0.001172	50000	200004
R[2,1]	-2.326E-5	5.911E-4	3.445E-6	-0.00121	-2.562E-5	0.001175	50000	200004
R[2,2]	8.939E-4	4.312E-4	3.465E-6	3.877E-4	7.836E-4	0.002041	50000	200004
R[2,3]	-2.707E-5	0.00103	1.473E-5	-0.002141	-2.588E-5	0.002104	50000	200004
R[3,1]	-0.001022	0.001168	7.622E-6	-0.003473	-9.706E-4	0.001172	50000	200004
R[3,2]	-2.707E-5	0.00103	1.473E-5	-0.002141	-2.588E-5	0.002104	50000	200004
R[3,3]	0.002646	0.001094	7.929E-6	0.001204	0.002411	0.005416	50000	200004
diff21	145.1	9.526	0.04738	126.3	145.0	163.9	50000	200004
diff31	9.996	5.614	0.02938	-1.065	9.996	21.09	50000	200004
mu[1]	350.0	3.479	0.02053	343.0	350.0	356.8	50000	200004
mu[2]	495.0	9.03	0.04543	477.3	495.0	512.9	50000	200004
mu[3]	360.0	5.064	0.02464	349.9	360.0	370.0	50000	200004
rho12	0.01504	0.2525	0.002117	-0.4819	0.02099	0.4832	50000	200004
rho13	0.2701	0.2486	0.001654	-0.2646	0.2927	0.6834	50000	200004
rho23	0.01593	0.5097	0.007665	-0.8067	0.02798	0.8107	50000	200004
sig1	14.91	2.423	0.012	11.06	14.61	20.51	50000	200004
sig2	43.65	6.2	0.02719	33.53	42.95	57.7	50000	200004
sig3	26.25	3.577	0.01569	20.37	25.87	34.35	50000	200004