Stat 511 HW#3 Spring 2003

Attention: Henceforth computing turned in as part of Stat 511 HW must be done in R. Becoming familiar with R is a course objective, and besides I don't want Norma to have to try to decipher code from some other system.

1. In class Vardeman claimed that that

$$\{\underset{\sim}{c} \mid \underset{\sim}{c} \text{ is "special"}\} \equiv \{\underset{\sim}{c} \mid \text{ if } \underset{\sim}{d} \in C(X')^{\wedge} \text{ then } \underset{\sim}{d} \perp \underset{\sim}{c}\} = C(X')$$

Prove this. (Argue that $\underset{\sim}{c} \in C(X') \Rightarrow \underset{\sim}{c}$ is "special", and then that $\underset{\sim}{c}$ is "special" $\Rightarrow \underset{\sim}{c} \in C(X')$. For the second of these, write $\underset{\sim}{c} = P_{X'}\underset{\sim}{c} + (I - P_{X'})\underset{\sim}{c}$ and note that if $\underset{\sim}{c} \notin C(X')$ then $\underset{\sim}{d}^* = (I - P_{X'})\underset{\sim}{c}$ is a nonzero element of $C(X')^{\wedge}$. Consider $\underset{\sim}{c}'\underset{\sim}{d}^*$.)

2. In the context of Christensen's Example 1.0.2 and the fake data vector used in Problem 6 of HW1, use R and weighted generalized least squares to find appropriate estimates for

$$\mathrm{E}Y \text{ and } \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{b}$$

in the Aitken models with

$$\text{first } V_1 = \mathrm{diag}(1,1,4,1,4,4) \text{ and then } V_2 = \begin{bmatrix} 1 & .5 & .5 & 0 & 0 & 0 \\ .5 & 1 & .5 & 0 & 0 & 0 \\ .5 & .5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & .5 \\ 0 & 0 & 0 & 0 & .5 & 1 \end{bmatrix}$$

For the first of these covariance structures, compare the (Aitken model) covariance matrices for generalized least squares estimators to (Aitken model) covariance matrices for the OLS estimators of $\mathrm{E}Y$ and the $C\underset{\sim}{b}$ above.

3. Continue in the context of Christensen's Example 1.0.2. For a given s^2, how do the covariance matrices for BLUEs of

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \underset{\sim}{b}$$

compare for the Gauss Markov model and in the Aitken model with $\mathrm{Cov}\,\underset{\sim}{e} = V_2$ for V_2 as in Problem 2? (I'm talking about the Gauss-Markov covariance matrix of the Gauss-Markov BLUE versus the Aitken covariance matrix of the Aitken BLUE.) In retrospect, why is this sensible in light of the nature of V_2?

4. Consider the (non-full-rank) "effects model" for the 2×2 factorial (with 2 observations per cell) called "Example d)" in the first lecture. Determine which of the parametric functions below are estimable, and for each that is, identify a vector $\underset{\sim}{a}$ such that $\mathrm{E}\underset{\sim}{a}'Y$ is equal to that function.

$$m, a_1, m + a_1, a_1 - a_2, m + a_1 + b_1, m + a_1 + b_1 + ab_{11}, ab_{11}, ab_{12} - ab_{11} - (ab_{22} - ab_{21})$$