Stat 511 Lecture 18

Note Title 2/25/2008

Recall ... 2 way factorial

Testing $H_0: \alpha\beta_{ij} = 0 \ \forall i,j$

Testing $H_0: \alpha_i = 0 \ \forall i$ (or $H_0: \beta_j = 0 \ \forall j$)

- in the cell means model
- in (restrictions of) the effects model

you and I need to be real careful

It's not "obvious" what to do except in cases where all $n_{ij} = m > 0$ (balanced data cases) – e.g. testing H_0: all $\alpha_i^* = 0 \ \forall i$ in one of these restricted effects models is typical NOT a test of $H_0: \alpha_i = 0 \ \forall i$

Example 2×3 effects model under SAS baseline restriction

for this model

B

A	1	2	3
1	$\mu^* + \alpha_1^* + \beta_1^* + \alpha\beta_{11}^*$	$\mu^* + \alpha_1^* + \beta_2^* + \alpha\beta_{12}^*$	$\mu^* + \alpha_1^*$
2	$\mu^* + \beta_1^*$	$\mu^* + \beta_2^*$	μ^*

$H_0: \alpha_1^* = 0$ is the hypothesis that $\mu_{13} = \mu_{23}$

So what makes sense? Talk 1st about balanced data contexts .. in such contexts

$$SS_{H_0 \alpha\text{'s}} = R(\alpha^*\text{'s} \mid \mu) \leftarrow$$

e.g. computed from a cell means model

$$= R(\alpha^*\text{'s} \mid \mu, \beta^*\text{'s}) \leftarrow$$

$$= R(\alpha^*\text{'s} \mid \mu, \beta^*\text{'s}, \alpha\beta^*\text{'s})$$

i.e.

$$SS_{H_0 \alpha\text{'s}} = Y'\left(P_{(\underset{\sim}{1}, X_{\alpha^*})} - P_{(\underset{\sim}{1})}\right)Y$$

$$= Y'\left(P_{(\underset{\sim}{1} | X_{\alpha^*} | X_{\beta^*})} - P_{(\underset{\sim}{1} | X_{\beta^*})}\right)Y$$

$$= Y'\left(P_X - P_{(\underset{\sim}{1} | X_{\beta^*} | X_{\alpha\beta^{**}})}\right)Y$$

on the other hand, what these R(|)'s might be appropriate for testing (i.e. for using as numerators in F statistics) is a bit bizarre (in particular involving sample sizes n_{ij}) – go look at Koehler's notes – Vardeman finds this unappealing. – one (in order to communicate) needs to know common jargon for these "SS's" –

"Source"	Type I SS	Type II SS	Type III SS
A × B	$\underbrace{R(\alpha\beta^*\text{'s} \mid \mu, \alpha^*\text{'s}, \beta^*\text{'s})}$		$SS_{H_0^{\alpha\beta}}$
	Same		
A	See 556K $R(\alpha^*\text{'s} \mid \mu)$	$R(\alpha^*\text{'s} \mid \mu, \beta^*\text{'s})$	$SS_{H_0^{\alpha}}$
B	See 560K $R(\beta^*\text{'s} \mid \mu, \alpha^*\text{'s})$	$R(\beta^*\text{'s} \mid \mu, \alpha^*\text{'s})$	$SS_{H_0^{\beta}}$

Related to the issue of "lack of balance" in a 2 way factorial is the possibility that factorial is not complete

A \ B	1	2	3
1	μ_{11}	μ_{12}	μ_{13}
2	μ_{21}	μ_{22}	μ_{23}

"real" / model means

Data

✓	✓	
✓	✓	✓

lacking data from cell (1,2) I know nothing about μ_{13} unless I'm willing to make strong model assumptions