Stat 511 HW#4 Spring 2004 (Not to be Collected)

1. In the context of Problems 3 and 4 of HW #1, use R matrix calculations to do the following in the (non-full-rank) Gauss-Markov normal linear model.
   a) Find 90% two-sided confidence limits for σ.
   b) Find 90% two-sided confidence limits for $\mu + \tau_1$.
   c) Find 90% two-sided confidence limits for $\tau_1 - \tau_2$.
   d) Find a p-value for testing the null hypothesis $\mathrm{H}_0 : \tau_1 - \tau_2 = 0$ vs $\mathrm{H}_a : \tau_1 - \tau_2 \neq 0$.
   e) Find 90% two-sided prediction limits for the sample mean of $n = 10$ future observations from the first set of conditions.
   f) Find 90% two-sided prediction limits for the difference between a pair of future values, one from the first set of conditions (i.e. with mean $\mu + \tau_1$) and one from the second set of conditions (i.e. with mean $\mu + \tau_2$).
   g) Find a p-value for testing $\mathrm{H}_0 : \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \mu \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. What is this test in the terminology of your previous statistical methods courses?
   h) Find a p-value for testing $\mathrm{H}_0 : \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \mu \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

2. In the following, make use of the homes data from Problem 2 of HW #3. Consider a regression of y on $x_1, x_2, \ldots x_5$. Use R matrix calculations to do the following in a full rank Gauss-Markov normal linear model.
   a) Find 90% two-sided confidence limits for σ.
   b) Find 90% two-sided confidence limits for the mean response under the conditions of data point #1.
   c) Find 90% two-sided confidence limits for the difference in mean responses under the conditions of data points #1 and #2.
   d) Find a p-value for testing the hypothesis that the conditions of data points #1 and #2 produce the same mean response.
   e) Find 90% two-sided prediction limits for an additional response for the set of conditions $x_1 = 1500, x_2 = 3, x_3 = 1000, x_4 = 500, x_5 = 8000$.
   f) Find 90% prediction limits for the difference in two additional responses under the two sets of conditions $x_1 = 1500, x_2 = 3, x_3 = 1000, x_4 = 500, x_5 = 8000$ and $x_1 = 1800, x_2 = 3, x_3 = 1000, x_4 = 1000, x_5 = 10000$.

g) Find a *p*-value for testing the hypothesis that a model including only x_1, x_3 and x_5 is adequate for "explaining" home price.

3. In the context of Problem 1, part g), suppose that in fact $\tau_1 = \tau_2, \tau_3 = \tau_4 = \tau_1 - d\sigma$. What is the distribution of the F statistic? Use R to plot the power of an $\alpha = .05$ level test as a function of d for $d \in [-5, 5]$ (that is, plot $P[F > \text{the cut-off value}]$ against d). The R function `pf` will compute cumulative (non-central) F probabilities for you. The call `pf(q,df1,df2,ncp)` returns the cumulative non-central F probability corresponding to the value `q`, for degrees of freedom `df1` and `df2` when the non-centrality parameter is `ncp` .

4. Use the R function `dchisq(x,df,ncp)` and plot on the same set of axes the chi-square probability density functions for 3 degrees of freedom and non-centrality parameters $0, 1, 3, 5$.