Stat 447 Homework Sets, Spring 2002

HW #1 Due January 25

Chapter 2: 5, 8, 14, 21, 40, 42, 45, 59, 61, 70

HW #2 Due February 8

Chapter 2: 87, 90, 100, 101, 139

Chapter 3: 3, 10, 21, 24, 25, 26, 31, 34, 44, 57, 58, 60, 97, 98, 99, 111

HW #3 Due February 15

Chapter 4: 4, 13, 19, 25, 26, 39, 42, 45, 60, 61, 62, 66

HW#4 Due March 1

Chapter 4: 73, 74, 83, 84, 92, 100, 115, 117

Chapter 5: 2, 3, 8, 13, 14, 17, 18, 19, 24, 29, 30

HW#5 Due March 8

Chapter 5: 40, 46, 51, 52, 56, 59, 60, 62, 66, 68, 75, 83, 86, 87, 93

HW#6 Due March 15

Chapter 5: 113, 115, 116, 118

Consider Example 5.25 and suppose that the joint distribution of the Y's is multivariate normal. Show the matrix calculations that could be used to find the means, variances and covariance for U and W (that are obtained in the example without using matrices).

Do the following multivariate normal exercises.

1. (Lahiri) For $(X_1, X_2, X_3)' \sim \text{MVN}_3(m, \Sigma)$ with $m = (1,0,1)'$ and $\Sigma = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 10 \end{pmatrix}$, find the joint distribution of $U = X_1 - X_2 + X_3$ and $V = 3X_1 + X_2 + 1$.
2. Suppose that X_1, X_2, Y_1, Y_2, Y_3 are independent normal random variables with $\text{E}X_i = m$ and $\text{Var}\, X_i = s^2$ and $\text{E}Y_j = 0$ and $\text{Var}\, Y_j = h^2$. What is the joint distribution of $W_1 = X_1 + Y_1, W_2 = X_1 + Y_2$ and $W_3 = X_2 + Y_3$?
3. (Moore) Suppose that in appropriate units, the following is true. The length of a standard bar of steel is m . A copy of the bar is not perfect, and has length L_1 so that

$D_1 = L_1 - m \sim \mathrm{N}(0,1)$. A copy of this copy has length L_2 so that $D_2 = L_2 - L_1 \sim \mathrm{N}(0,1)$ and D_2 is independent of L_1. A copy of the copy of the copy has length L_3 so that $D_3 = L_3 - L_2 \sim \mathrm{N}(0,1)$ and D_3 is independent of L_1 and L_2.

a) What is the (joint) distribution of $L = (L_1, L_2, L_3)$?
b) Evaluate the correlation between L_1 and L_3.
c) What is the distribution of $L_3 - m$, the error in length of the last bar?

HW#7 Due March 29

Chapter 6: 1, 2, 6, 9, 11, 19, 25, 26, 60, 61, 65, 68

HW#8 Due April 12

Chapter 7: 3, 4, 7, 8, 11, 15, 18, 21, 25, 26, 27, 42, 75

Chapter 8: 4, 10, 12

HW#9 Due April 26

Chapter 8: 19, 33, 40, 41, 44, 45, 47, 58, 59, 69, 70, 76, 81, 82, 110

Chapter 9:76 abc, 80

HW#10 Not to be Collected (But to be Examined on the Final)

1. Suppose that $X_1, X_2, ..., X_n$ are iid Geometric (p).

a) Write out the likelihood function, $L(p)$ and its natural logarithm, $l(p)$. Then find the score function, $l'(p) = \frac{d}{dp} l(p)$. Set the score function equal to 0 and solve for a maximum likelihood estimator of *p*. (To be absolutely precise, you should treat separately the case that all X_i take the value 0.)

b) Find a standard error for your estimator in a) by finding $\hat{s}_{\hat{p}} = \sqrt{-\frac{1}{l''(\hat{p})}}$. Based on this, what would you report as an approximate 95% confidence interval for *p*?

c) Find the method of moments estimator for *p*. Is it different from the MLE?

d) Suppose now that one takes a Bayes view of inference in this problem and uses a Beta(a, b) prior distribution for *p*. Call the prior probability density $g(p)$. The product $L(p)g(p)$ (which is proportional to the posterior density of p given the observed values of $X_1, X_2, ..., X_n$) can be recognized as having what standard form? What are then the mean and variance of the posterior distribution? For large n, do you expect the posterior distribution to be concentrated "near the true value of p"? Why?

2. Suppose that $X_1, X_2, ..., X_n$ are iid Poisson(l) and that that one takes a Bayes view of inference for l. Further suppose one uses an Exponential(b) prior distribution for l. The product $L(l)g(l)$ (which is proportional to the posterior density of l given the observed values of $X_1, X_2, ..., X_n$) can be recognized as having what standard form? What are then the mean and variance of the posterior distribution? For large n, do you expect the posterior distribution to be concentrated "near the true value of l"? Why?

Chapter 10: 2, 3, 7, 11, 15, 17, 22, 35, 37, 51, 57, 73, 74 (Also find *p*-values in Exercises 7, 11, 15, 17, and say what you can about the size of the *p*-values in 51, 57, 73, 74)

Chapter 11: 4, 14, 29, 33, 41