Stat 447 Exam 2

April 4, 2002 Prof. Vardeman

1. Below is a table giving a joint probability function for discrete random variables X and Y.

$y \backslash x$	**3**	**4**	**5**	**6**
4	.1	.1	0	0
3	.05	.2	.05	0
2	0	.05	.2	.05
1	0	0	.1	.1

a) Evaluate $P[X = Y]$.

b) Find the marginal probability function for X, $p(x)$.

c) Find the conditional mean of Y given $X = 4$, $\mathrm{E}[Y \mid X = 4]$.

d) Are the random variables X and Y independent? Argue very carefully.

2. Suppose that U and Z are independent, $U \sim \text{Uniform}(0,1)$ and $Z \sim \text{Normal}(0,1)$.

a) Find the mean and variance of $X = U + Z$.

$\text{E}\,X =$ __________ $\text{V}(X) =$ __________

b) Completely set up (but do not try to evaluate) a double integral giving $P[U + Z < 2]$. (Use the abbreviation $f(z)$ for the standard normal pdf rather than writing out its formula.)

c) Find the correlation between $X = U + Z$ and $Y = UZ$.

3. Suppose that in describing waiting times between "hits" on a certain web page, it is sensible (in appropriate units) to model

   $X_1 =$ time till first hit

   $X_2 =$ time between first and second hits and

   $X_3 =$ time between second and third hits

   as independent $\text{Exp}(1)$ random variables.

a) Find the probability that the time till the first hit exceeds its mean, $P[X_1 > 1]$.

b) It turns out that $W = X_1/(X_1 + X_2 + X_3)$ has the $\text{Beta}(1,2)$ distribution. (Don't try to show this.) Use this fact to find $P[X_1 > X_2 + X_3]$.

c) Write out (but don't evaluate) an integral giving the expected value of the random variable $Y = \max(X_1, X_2, X_3)$.

4. Suppose that we model "true" IQs, *X*, as Normal with $m_X = 100$ and $s_X = 10$, and that when an IQ is measured, what is actually observed is $Y = X + e$ where the measurement error e is Normal with $m_e = 0$ and $s_e = 3$. A subject is selected and is tested twice, producing measured IQs $Y_1 = X + e_1$ and $Y_2 = X + e_2$. Suppose that X, e_1 and e_2 are independent.

a) What is the joint distribution of Y_1 and Y_2?

b) What is the correlation between Y_1 and Y_2?

c) What is the distribution of $\frac{1}{2}(Y_1 + Y_2)$?

5. Consider the cdf *F* below. If *X* has cdf *F*, find a pdf for $Y = \exp(X)$.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } 0 \le x \le 1 \\ \frac{1}{2}x & \text{if } 1 < x \le 2 \\ 1 & \text{if } 2 < x \end{cases}$$