Stat 447 Exam 1

February 21, 2002 **Prof. Vardeman**

1. Diseases A and B in sheep have similar symptoms, but never afflict the same animal. B is more rare and is harder to diagnose. 1 in 1000 sheep have disease A, and 1 in 10,000 have disease B. In routine exams, veterinarians fail to diagnose A in 50% of diseased animals and fail to diagnose B in 80% of diseased animals. Animals without diseases A and B are never diagnosed as having one of the diseases. No animal with A (B) is ever diagnosed as having B (A).

For parts (a) through (c) consider a single sheep given a routine exam by a veterinarian.

a) What is $P[\text{sheep has disease A or B}]$?

b) What is $P[\text{sheep is diagnosed with either A or B}]$?

c) What is $P[\text{sheep is free of both A\&B} \mid \text{sheep is diagnosed as free of A and B}]$?

d) Suppose that one samples 100 sheep at random with replacement (from all sheep in the world) and subjects them to a routine exam by a veterinarian. With

$$X = \text{the number of sheep diagnosed with A or B}$$

use your answer to (b) (or the number .00064 if you weren't able to do (b)) and find EX and $P[X=0]$.

EX = ______________ $P[X=0]=$ _________________

e) In contrast to part (d), suppose that the 100 sheep in the Iowa flock of Farmer Jones are examined by a veterinarian. With X as before, why would it now be unwise to use the calculations of (d) to describe X?

2. Iowa vehicle license plates have identifiers of the form XXX-YYY where the X's are digits 0-9 and the Y's are letters A-Z. (There are 26 letters in the alphabet.)

a) How many different identifiers of this type are possible?

b) I remember that my plate has the digits 2-5-7 in some order and letters A-D-Y in some order but beyond this recall nothing about the identifier. Loving things random, I "randomly" order the digits and then the letters. What is the probability that doing so, I recreate my license plate number?

3. At one stage of the manufacture of certain fiber-optic cable, serious flaws occur at a rate of .2 flaws/mile of cable.

a) Find a sensible value for the probability that 20 miles of this cable has less than 2 serious flaws.

b) Suppose that one buys 2000 miles of this cable and $X =$ the total number of serious flaws. "Exact" calculation of $P[X \le 380]$ is not pleasant "by hand." However, the (intrinsically discrete) distribution of X can be approximated with a (continuous) normal distribution. Find an appropriate mean and standard deviation for X (μ and σ), treat X as if it were normal, and approximate $P[X \le 380]$.

4. Suppose that U is Uniform$(0,1)$. (Note then that $\mathrm{E}U = \frac{1}{2}$, $\mathrm{E}U^2 = \frac{1}{3}$, $\mathrm{E}U^3 = \frac{1}{4}$ and $\mathrm{E}U^4 = \frac{1}{5}$.)

a) $W = 3U^2 + 5$ is a random variable with a mean $\mathrm{E}W$ and variance $\mathrm{V}(W)$. Find these.

$\mathrm{E}W =$ ____________ $\mathrm{V}(W) =$ ____________

b) It is even possible to work out probabilities for W defined in part (a). For example, find $P[W \leq 6]$. (Hint: What is this probability in terms of U?)

5. Suppose that X is a continuous random variable with pdf

$$f(x) = \begin{cases} 1/4 & \text{if } x \in (0,1) \\ 3/4 & \text{if } x \in (1,2) \\ 0 & \text{otherwise} \end{cases}$$

a) Find $P[X \leq 1.2]$.

b) Find the mean and variance of X.

$\mathrm{E}X =$______________ $\mathrm{V}(X) =$_________________