6.2 The student is incorrect. It means 95% probability the confidence interval (267.8, 276.2) will capture the true mean NAEP quantitative score for men of ages 21 to 25 in repeated samples.

6.3 (a) standard deviation of \( \overline{X} = \sigma / \sqrt{n} = \frac{60}{\sqrt{1000}} = 1.90 \)

(b) 

(d) 

(c) \( m = 2 \times 1.90 = 3.80 \)

(e) 95%

6.4 \( \overline{X} = 11.78 \quad Z_{95.3\%} = 2.576 \quad \sigma = 3.2 \quad n = 114 \)

99% CI: 
\[ \overline{X} \pm Z_{95.3\%} \times \left( \frac{\sigma}{\sqrt{n}} \right) \]
\[ = 11.78 \pm 2.576 \times \left( \frac{3.2}{\sqrt{114}} \right) \]
\[ = 11.78 \pm 0.77 = (11.01, 12.55) \]

6.5 (a) stemplot:
7 | 7
8 | 1 6 9
9 | 1 1 3 6 8
10 | 0 2 3 3 4 5 7 8
11 | 1 2 2 4 4 4 9
12 | 0 8
13 | 0 2
The two low scores (72 and 74) are both possible outliers, but there are no other apparent deviations from normality.

(b) From JMP, Xbar = 105.8

\[
Z_{95\%} = 2.576 \quad \sigma = 15 \quad n = 31
\]

99% CI: \[Xbar \pm Z_{95\%} \times (\sigma/\sqrt{n})\]
\[= 105.8 \pm 2.576 \times (15/\sqrt{31})\]
\[= 105.8 \pm 6.9 = (98.9, 112.7)\]

(c) You want to make inference on 7th grade girls in the school district. But you have a sample of 7th grade girls in from only one of the schools in the district. This school could be the “best” or “worst” one in the district. Thus, your results would be biased.

Additional part:

99% PI: \[Xbar \pm Z_{95\%} \times (\sigma/\sqrt{n})\]
\[= 105.8 \pm 2.576 \times (15/\sqrt{31})\]
\[= 105.8 \pm 39.3 = (66.5, 145.1)\]

6.8 \[Xbar = 275 \quad \sigma = 60 \quad n = 1077\]

(a) \[Z_{95\%} = 1.96\]

95% CI: \[Xbar \pm Z_{95\%} \times (\sigma/\sqrt{n})\]
\[= 275 \pm 1.96 \times (60/\sqrt{1077})\]
\[= 275 \pm 3.6 = (271.4, 278.6)\]

(b) \[Z_{95\%} = 1.645\]

90% CI: \[Xbar \pm Z_{90\%} \times (\sigma/\sqrt{n})\]
\[= 275 \pm 1.645 \times (60/\sqrt{1077})\]
\[= 275 \pm 3.0 = (272, 278)\]

\[Z_{99\%} = 2.576\]

99% CI: \[Xbar \pm Z_{99\%} \times (\sigma/\sqrt{n})\]
\[= 275 \pm 2.576 \times (60/\sqrt{1077})\]
\[= 275 \pm 4.7 = (270.3, 279.7)\]

(c) Confidence Level: 90% 95% 99%
Margin of Error: 3.0 3.6 4.7
Therefore, increasing confidence level increases margin of error.

6.9 (a) \[Xbar = 275 \quad \sigma = 60 \quad Z_{97.5\%} = 1.96\]

\[n = 1077\]

95% CI: \[Xbar \pm Z_{97.5\%} \times (\sigma/\sqrt{n})\]
\[= 275 \pm 1.96 \times (60/\sqrt{1077})\]
\[= 275 \pm 3.6 = (271.4, 278.6)\] [same as 6.8 (a)]
(b) \( n = 250 \)
95% CI: \( \bar{X} \pm Z_{0.025} \cdot (\sigma/\sqrt{n}) \)
\[ = 275 \pm 1.96 \cdot (60/\sqrt{250}) \]
\[ = 275 \pm 7.4 = (267.6, 282.4) \]

(c) \( n = 4000 \)
95% CI: \( \bar{X} \pm Z_{0.025} \cdot (\sigma/\sqrt{n}) \)
\[ = 275 \pm 1.96 \cdot (60/\sqrt{4000}) \]
\[ = 275 \pm 1.9 = (273.1, 276.9) \]

(d) Sample Size: 250 1077 4000
Margin of Error: 7.4 3.6 1.9
Therefore, increasing sample size decreases margin of error.

6.11 \( m = 1 \quad Z_{0.95} = 2.576 \quad \sigma = 3.2 \)
\( n = (Z_{0.95} \cdot \sigma / m)^2 = (2.576 \cdot 3.2 / 1)^2 = 67.95 = 68 \)

6.18 (a) The authors wish to draw inference on all US consumers. However, their sample only can be fully trusted for inference on all consumers in Indianapolis (listed in the phone book).

(b) 95% CI: (Here, use "s" in place of the unknown \( \sigma \))
Food Stores: \( \bar{X} = 18.67 \quad s = 24.95 \quad Z_{0.025} = 1.96 \quad n = 201 \)
\( \bar{X} \pm Z_{0.025} \cdot (s/\sqrt{n}) \)
\[ = 18.67 \pm 1.96 \cdot (24.95/\sqrt{201}) \]
\[ = 18.67 \pm 3.45 = (15.22, 22.12) \]

Mass Merchandisers: \( \bar{X} = 32.38 \quad s = 33.37 \quad Z_{0.025} = 1.96 \quad n = 201 \)
\( \bar{X} \pm Z_{0.025} \cdot (s/\sqrt{n}) \)
\[ = 32.38 \pm 1.96 \cdot (33.37/\sqrt{201}) \]
\[ = 32.38 \pm 4.61 = (27.77, 36.99) \]

Pharmacies: \( \bar{X} = 48.60 \quad s = 35.62 \quad Z_{0.025} = 1.96 \quad n = 201 \)
\( \bar{X} \pm Z_{0.025} \cdot (s/\sqrt{n}) \)
\[ = 48.60 \pm 1.96 \cdot (35.62/\sqrt{201}) \]
\[ = 48.60 \pm 4.92 = (43.68, 53.52) \]

(c) Yes, pharmacies offer higher performance than the other types of stores. The 95% CI for \( \mu \) of pharmacies is entirely above that of both food store and mass merchandisers, i.e. 22.12 < 43.68 and 36.99 < 43.68).
Additional Part:

95% PI: (Here, use "s" in place of the unknown \( \sigma \))

**Food Stores:**

\[
\bar{x} = 18.67 \quad s = 24.95 \quad Z_{0.5\%} = 1.96 \quad n = 201
\]

\[
\bar{x} \pm Z_{0.5\%} \frac{s}{\sqrt{n}} = 18.67 \pm 1.96 \times \frac{24.95}{\sqrt{201}}
\]

\[= 18.67 \pm 1.96 \times 1.93 = 18.67 \pm 3.75 = (14.92, 22.42)\]

**Mass Merchandisers:**

\[
\bar{x} = 32.38 \quad s = 33.37 \quad Z_{0.5\%} = 1.96 \quad n = 201
\]

\[
\bar{x} \pm Z_{0.5\%} \frac{s}{\sqrt{n}} = 32.38 \pm 1.96 \times \frac{33.37}{\sqrt{201}}
\]

\[= 32.38 \pm 1.96 \times 5.75 = 32.38 \pm 11.25 = (21.13, 43.63)\]

**Pharmacies:**

\[
\bar{x} = 48.60 \quad s = 35.62 \quad Z_{0.5\%} = 1.96 \quad n = 201
\]

\[
\bar{x} \pm Z_{0.5\%} \frac{s}{\sqrt{n}} = 48.60 \pm 1.96 \times \frac{35.62}{\sqrt{201}}
\]

\[= 48.60 \pm 1.96 \times 6.89 = 48.60 \pm 13.49 = (35.11, 62.09)\]

(Assuming scores must be positive)

6.25 \( \mu = 115 \quad \sigma = 30 \quad n = 25 \)

Ho: \( \mu = 115 \)

Ha: \( \mu > 115 \)

(a) \( \mu = 115 \)

standard deviation of \( \bar{x} = \sigma_{\bar{x}} = \sigma / \sqrt{n} = 30 / \sqrt{25} = 6 \)

So if Ho is true, the distribution of mean score is approximately normal.

\( \bar{x} \sim N(115, 6) \)

(b) The actual result lies out toward the right tail of the curve, while 118.6 is

fairly close to the middle. If Ho were true, observing a value like 118.6 would

not be too surprising, but 125.7 is less likely, and it therefore provides some

evidence against Ho.

(c) See the sketch.

6.28 \( \mu = 52,500 \)

Ho: \( \mu = 52,500 \)

Ha: \( \mu > 52,500 \)
6.30  \[ \text{Ho: } \mu = 2.6 \]
\[ \text{Ha: } \mu \neq 2.6 \]

6.31  \[ \mu = 115 \quad \sigma = 30 \quad n = 25 \]
\[ \text{Ho: } \mu = 115 \]
\[ \text{Ha: } \mu > 115 \]

(a) \[ X\text{bar} = 118.6 \]
\[ Z = \frac{(X\text{bar} - \mu)}{(\sigma \sqrt{n})} = \frac{(118.6 - 115)}{(30/\sqrt{25})} = 3.6/6 = 0.6 \]
\[ P(X\text{bar} > 118.6) = P(Z > 0.6) = 1 - 0.7257 = 0.2743 \]

\[ X\text{bar} = 125.7 \]
\[ Z = \frac{(X\text{bar} - \mu)}{(\sigma \sqrt{n})} = \frac{(125.7 - 115)}{(30/\sqrt{25})} = 10.7/6 = 1.78 \]
\[ P(X\text{bar} > 125.7) = P(Z > 1.78) = 1 - 0.9625 = 0.0375 \]

Therefore, these two P-value express consistent results with 6.25 (b) that we made informally.

(b) \( X\text{bar} = 118.6 \) is not significant at either \( \alpha = 0.05 \) or \( \alpha = 0.01 \).
\( X\text{bar} = 127.5 \) is statistically significant at \( \alpha = 0.05 \), but not \( \alpha = 0.01 \).

6.34  \[ \mu = 354 \quad \sigma = 33 \quad n = 3 \]
\[ \text{Ho: } \mu = 354 \]
\[ \text{Ha: } \mu > 354 \]

(a) \[ X\text{bar} = \frac{(405 + 378 + 411)}{3} = 398 \]
\[ Z = \frac{(X\text{bar} - \mu)}{(\sigma \sqrt{n})} = \frac{(398 - 354)}{(33/\sqrt{3})} = 2.31 \]

(b) \( P\text{-value} = 1 - 0.9886 = 0.0104 < 0.05 \)

(c) The result is statistically significant at \( \alpha = 0.05 \). But not at \( \alpha = 0.01 \). So there is convincing evidence that mean sales are higher.

6.52  When a test is significant at the 1% level, it means that if the null hypothesis is true, outcomes similar to those seen are expected to occur less than once in 100 repetitions of the experiment or sampling. "Significant at the 5% level" means we have observed something that occurs in less than 5 out of 100 repetitions (when Ho is true). Something that occurs "less than once in 100 repetitions" also occurs "less than 5% level" (or any higher level). The opposite statement does not hold: Something that occurs "less than 5 times in 100 repetitions" is not necessarily as rare as something that occurs "less than once in 100 repetitions," so a test that is significant at 5% is not necessarily significant at 1%. 
6.53 This explanation is incorrect. Because it means that if Ho is true, we have observed outcomes that occur less than 5% of the time.

6.77 (a)

The distribution is right-skewed.

(b) From JMP, Xbar = 7.08

\[ Z_{0.05} = 2.576 \quad \sigma = 2.75 \quad n = 27 \]

90% CI:

\[ \text{Xbar} \pm Z_{0.05} \times \left( \frac{\sigma}{\sqrt{n}} \right) \]

\[ = 7.08 \pm 1.645 \times \left( \frac{2.75}{\sqrt{27}} \right) \]

\[ = 7.08 \pm 0.87 = (6.21, 7.95) \]

(c) Ho: \( \mu = 5.5 \)

Ha: \( \mu > 5.5 \)

\[ Z = \frac{(X\text{bar} - \mu)/\left(\sigma/\sqrt{n}\right)}{1} = \frac{(7.08 - 5.5)/\left(2.75/\sqrt{27}\right)}{1} = 2.99 \]

P-value = \( P(Z > 2.99) = 1 - 0.9986 = 0.0014 < 0.05 \)

So reject Ho: \( \mu = 5.5 \) at \( \alpha = 0.05 \). i.e. treasury bills have a mean return higher than 5.5%.

(d) Untitled 1-Time Series

Additional Part:

Because returns are skewed right, not normally distributed. But predict intervals need a normal distributed population.
### Distributions

#### Column 1

<table>
<thead>
<tr>
<th>Quantiles</th>
<th>Value</th>
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<td>14.710</td>
</tr>
<tr>
<td>99.5%</td>
<td>14.710</td>
</tr>
<tr>
<td>97.5%</td>
<td>14.710</td>
</tr>
<tr>
<td>90.0%</td>
<td>11.108</td>
</tr>
<tr>
<td>75.0% quartile</td>
<td>8.430</td>
</tr>
<tr>
<td>50.0% median</td>
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</tr>
<tr>
<td>25.0% quartile</td>
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<tr>
<td>10.0%</td>
<td>4.036</td>
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<td>2.5%</td>
<td>3.040</td>
</tr>
<tr>
<td>0.5%</td>
<td>3.040</td>
</tr>
<tr>
<td>0.0% minimum</td>
<td>3.040</td>
</tr>
</tbody>
</table>

#### Moments

- Mean: 7.0751852
- Std Dev: 2.7558572
- Std Err Mean: 0.530365
- upper 95% Mean: 8.165366
- lower 95% Mean: 5.985044
- N: 27
7.2 (a) d.f. = 5

7.3 (a) d.f. = 15 - 1 = 14

7.11 (b) Subject | Dc: Depression(caffeine) | Dp: Depression(placebo) | Dc-DP
--- | --- | --- | ---
1 | 5 | 16 | -11
2 | 5 | 23 | -18
3 | 4 | 5 | -1
4 | 3 | 7 | -4
5 | 8 | 14 | -6
6 | 5 | 24 | -19
7 | 0 | 6 | -6
8 | 0 | 3 | -3
9 | 2 | 15 | -13
10 | 11 | 12 | -1
11 | 1 | 0 | 1

Stemplot:
-1198
-1131
-0166
-014311
011

From stemplot, it is basically normal with a slight skew left.

X-bar (for difference) = -7.36  S = 6.92  n = 11

Ho: μ = 0
Ha: μ < 0

t = (X̄ - μ)(S/√n) = (-7.36 - 0)/6.92/√11) = -3.53  d.f. = 11-1=10
0.0025 < P-value < 0.005

So reject Ho: μ = 0 at α = 0.05, i.e. this matched pairs study give evidence that being deprived of caffeine raises depression scores.
7.12 (a) \( \bar{X} \)bar(for difference) = 332  \( S = 108 \)  \( n = 200 \)

Ho:  \( \mu = 0 \)

Ha:  \( \mu > 0 \)

\[ t = \frac{(\bar{X} - \mu)}{(S/\sqrt{n})} = \frac{(332 - 0)}{(108/\sqrt{200})} = 43.47 \quad \text{d.f.} = 200-1=199 \]

P-value < 0.0005

So reject Ho:  \( \mu = 0 \) at \( \alpha = 0.01 \), i.e. there is significant evidence at the 1% level that the mean amount charged increases under the no-fee offer.

(b) 99% CI:  
\[ \bar{X} \pm 29.5\% \cdot (S/\sqrt{n}) \]
\[ = \bar{X} \pm Z_{0.95} \cdot (S/\sqrt{n}) \quad \text{for large sample size} \]
\[ = 332 \pm 2.576 \cdot (108/\sqrt{200}) \]
\[ = 332 \pm 19.7 = (312.3, 351.7) \]

(b) The \( t \) procedure is OK, because 1) the sample size is very large (here: \( n=200 \gg 40 \)), then the \( t \) distribution is fairly good in spite of the skewness. 2) The credit limit enforces by the bank. So outliers are prevented though the distribution of charges is skewed right, non-normality.

(d) Compare this sample of 200 customers on the no-fee offer to a control group of 200 other customers that are not on the no-fee offer.

7.15 (a) \( \bar{X} \)bar = 24  \( S = 11 \)  \( Z_{0.95} = 1.96 \)  \( n = 75 \)

95% CI:  
\[ \bar{X} \pm 1.96 \cdot (S/\sqrt{n}) \]
\[ = \bar{X} \pm Z_{0.975} \cdot (S/\sqrt{n}) \quad \text{for large sample size} \]
\[ = 24 \pm 1.96 \cdot (11/\sqrt{75}) \]
\[ = 24 \pm 2.5 = (21.5, 26.5) \]

(c) Because the \( t \) procedure is robust and according to the central limit theorem, with a larger sample size (here: \( n=75 \gg 40 \)), the \( t \) distribution is fairly good in spite of the skewness.

**Additional part:**

Although the \( t \) confidence interval for \( \mu \) is robust to non-normality, but the \( t \) prediction interval for an individual is not, when we are sampling form a skewed-right, non-normal population.

7.46

Last year: \( \bar{X} \)bar1 = 49  \( S1 = 11 \)  \( n1 = 53 \)
This year: \( \bar{X} \)bar2 = 52  \( S2 = 13 \)  \( n2 = 75 \)

(a) Ho:  \( \mu_1 = \mu_2 \)

Ha:  \( \mu_1 < \mu_2 \)

Moore's method:

\[ SE = \sqrt{\frac{S1^2}{n1} + \frac{S2^2}{n2}} = \sqrt{\frac{11^2}{53} + \frac{13^2}{75}} = 2.13 \]

\[ t = (\bar{X} \text{bar}_1 - \bar{X} \text{bar}_2)/SE = (49-52)/2.13 = -1.41 \quad \text{d.f.} = 53-1 = 52 \]

0.05 < P-value < 0.10
Dielman's method:
\[ Sp = \sqrt{((n_1-1)S_1^2 + (n_2-1)S_2^2)/(n_1+n_2-2)} \]
\[ = \sqrt{(53-1)*11^2 + (75-1)*13^2)/(53+75-2)} \]
\[ = 12.21 \]
\[ t = (X_{bar 1} - X_{bar 2})/(Sp\sqrt{1/n_1+1/n_2}) = (49-52)/(12.21\sqrt{1/53+1/75}) = -1.37 \]
d.f. = n_1 + n_2 - 2 = 53 + 75 - 2 = 126
0.05 < P-value < 0.10

Therefore, both method suggest that the difference between this year and last year in the mean number of units sold at all retail stores is significant at \( \alpha = 0.10 \), but not significant at \( \alpha = 0.05 \).

(b) 95% CI:
Moore's method:
\[ (X_{bar 1} - X_{bar 2}) \pm t_{32, 97.5\%} * (\sqrt{S_1^2/n_1 + S_2^2/n_2}) \]
\[ = 49 -52 \pm 2.009 * \sqrt{11^2/53 + 13^2/75} \]
\[ = -3 \pm 4.28 = (-7.28, 1.28) \]

Dielman's method:
\[ (X_{bar 1} - X_{bar 2}) \pm t_{126, 97.5\%} * Sp\sqrt{1/n_1 + 1/n_2} \]
\[ = 49 -52 \pm 1.984 * 12.21\sqrt{1/53 + 1/75} \]
\[ = -3 \pm 4.35 = (-7.35, 1.35) \]

JMP output exercise:
(a) \( \bar{X} = 0.76 \quad S = 0.256 \quad n = 40 \quad \text{d.f.} = 40-1 = 39 \quad t_{39, 95\%} = 1.684 \)
90% CI: \( \bar{X} \pm t_{39, 95\%} * (S/\sqrt{n}) \)
\[ = 0.76 \pm 1.684 * (0.256/\sqrt{40}) \]
\[ = 0.76 \pm 0.07 = (0.69, 0.83) \]

(b) 90% PI: \( \bar{X} \pm t_{39, 95\%} * (S/\sqrt{1 + 1/n}) \)
\[ = 0.76 \pm 1.684 * (0.256/\sqrt{1 + 1/40}) \]
\[ = 0.76 \pm 0.44 = (0.32, 1.20) \]