Confidence Intervals are data-based intervals meant to bracket some unknown population or model parameter that carry a probability-based reliability/confidence figure.

Basic idea used: sampling dsns for various that involve parameters of interest can lead to intervals for those parameters — e.g. the normal dsn for $\bar{x}$ leads to a CI for $\mu$.

$95\%$ of samples will produce $\bar{x}$’s so that:

$$(\bar{x} - 2\frac{\sigma}{\sqrt{n}}, \bar{x} + 2\frac{\sigma}{\sqrt{n}})$$

lands on top of $\mu$.

Call $\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}$ $95\%$ confidence limits for $\mu$.

(These limits involve the typically unknown $\sigma$ — typically useless in practice.)
Example: Suppose I'm interested in the average 2BR apartment rent in Ames. Perhaps historical information suggests that \( \mu = 80 \). If today a sample of \( n=25 \) units gives \( \bar{x} = 688.20 \) at 95% C.I. for \( \mu \), is what? Use:

\[
\bar{x} \pm Z \frac{\sigma}{\sqrt{n}} = 688.2 \pm 2 \frac{80}{\sqrt{25}} = 688.2 \pm 16
\]

Interval: (650.2, 726.2)

Demonstration Brown Bag:

- Approximately normal, \( \sigma = 1.715 \)
- Make some 80% C.I.'s for \( \mu \):

\[
\bar{x} \pm 1.282 \frac{1.715}{\sqrt{n}} = .58
\]

Where \( Z \) is chosen so that

\[
P(-Z < \text{normal} < Z) = \text{desired reliability/confidence}
\]

<table>
<thead>
<tr>
<th>Confidence</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>55%</td>
<td>1.56</td>
</tr>
<tr>
<td>90%</td>
<td>1.55</td>
</tr>
<tr>
<td>95%</td>
<td>1.65</td>
</tr>
<tr>
<td>99%</td>
<td>2.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Values</th>
<th>( Z )</th>
<th>Interval Work?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,6,7</td>
<td>5.8</td>
<td>5.8 ± .98</td>
</tr>
<tr>
<td>2</td>
<td>3,4,5</td>
<td>4.2</td>
<td>4.2 ± .98</td>
</tr>
<tr>
<td>3</td>
<td>5,5,5,2,7</td>
<td>4.8</td>
<td>4.8 ± .98</td>
</tr>
<tr>
<td>4</td>
<td>1,2,3,4,5</td>
<td>4.6</td>
<td>4.6 ± .98</td>
</tr>
<tr>
<td>5</td>
<td>4,6,2,8,9</td>
<td>5.8</td>
<td>5.8 ± .98</td>
</tr>
<tr>
<td>6</td>
<td>3,3,7,7,5</td>
<td>5.0</td>
<td>5.0 ± .98</td>
</tr>
</tbody>
</table>

\( \mu = 5 \) exactly

80% is a lifetime batting average...
Interpretation? The demonstration should help think about this...

To say — to — is a 90% CI for \( \mu \) is to say that in obtaining it I've used a method that works in about 90% of applications — whether it has or has not worked in my particular application is typically unknown — regardless. After plugging data into the formula, there is no probability left in the problem.

The quantity \( \frac{z}{\sqrt{n}} \) is called "the margin of error." Larger confidence requires larger \( z \); looser limits, larger "margin of error." Larger \( n \) produces tighter limits, smaller "margin of error." 

It's possible to choose sample sizes to get desired margin of error (for estimating \( \mu \)) by solving

\[
\frac{z}{\sqrt{n}} = \text{desired margin}
\]

for \( n \)

Example: Ames average rent -
\[
\bar{x} = 800, \quad 95\% \text{ confidence, target margin of error} 25
\]
25 = 2.576 \frac{80}{\sqrt{n}}

25\sqrt{n} = 2.576(80)

\sqrt{n} = \frac{2.576(80)}{25}

n = 68

General formula (for the sample size n)

\[ n = \left( \frac{Z}{\text{margin of error}} \right)^2 \]

The practical problem of prediction is that I don’t know μ.
(or usually σ) for population — I might think

\[ \bar{x} \pm 1.96 \sigma \]

but somehow I have hedges for the fact that \( \bar{x} \) isn’t exactly μ.
The prediction interval idea is its answer to “how do I hedge for this incomplete knowledge?”

2nd type of Inference:

Prediction Intervals

object here is to bracket \( x_{\text{new}} \)
(not, e.g., to bracket μ)
notice that if I had complete information this would be a probability problem

Example: Brown bug has \( \mu = 5 \)
\( \sigma = 1.715 \) and is normal. 
\( x_{\text{new}} \) has \( 95\% \) chance of catching \( x_{\text{new}} \)

Here we use a 2nd sampling distribution for the population.
(additional \( x_{\text{new}} \) — sample much based on \( n \))

If the population is normal, I can tell you about the sampling distribution.
Example: Ames Apartment rents

$\sigma = 80 \quad n = 29 \quad \bar{x} = 688.20$

make a 95% prediction interval for one more rental

$688.20 \pm (1.96)(80)\sqrt{1 + \frac{1}{29}}$

162.70

Note that (of course) those limits are wider than confidence limits for $\mu$ - they must account for $\sigma = 80$ in predicting an additional observation.

$95\%$ of all experiences give

$-1.960\sqrt{1 + \frac{1}{n}} < x_{\text{new}} - \bar{x} < 1.960\sqrt{1 + \frac{1}{n}}$

So I call

$\bar{x} \pm z_0 \sqrt{1 + \frac{1}{n}}$

prediction limits for $x_{\text{new}}$

**Demonstration:** Brown bag -

make some 80% P.I.'s using $n=5$

$\bar{x} \pm (z_0)\sqrt{1 + \frac{1}{5}} \quad z_0 = 1.715$

1.282

1.715

i.e. $\bar{x} \pm 2.41$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Values</th>
<th>$\bar{x}$</th>
<th>$x_{\text{new}}$</th>
<th>Winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5, 5, 4, 3, 2</td>
<td>4.0</td>
<td>4</td>
<td>[1]</td>
</tr>
<tr>
<td>2</td>
<td>8, 4, 4, 3, 1</td>
<td>4.0</td>
<td>5</td>
<td>[1]</td>
</tr>
<tr>
<td>3</td>
<td>3, 3, 5, 5, 6</td>
<td>4.4</td>
<td>3</td>
<td>[1]</td>
</tr>
</tbody>
</table>

80% is a lifetime batting average
batting average refers to the whole business of (1) selecting the sample of n and (2) selecting one more, X, and seeing if the interval captures X

---

**For the situation of problem 6.11**

MMDOYS makes 95% prediction interval for one more statistics study time

---

3rd type of "standard inference" is **Significance Testing** (Hypothesis Testing)

---

**Def. An alternative hypothesis is** statement about the parameter that embodies the departures from the null hypothesis that are of interest (that we want to catch) —

$H_a: \text{parameter} \neq \#$

---

**Example** Filling 16oz pop bottles

$H_0: \mu = 16.0$

---

**Basic Idea**: Assessing the plausibility of a statement about a population or model parameter

**Def. A null hypothesis is** a statement about a parameter of the form

$H_0: \text{parameter} = \$

that represents a status quo or pre-data viewpoint

---

A consumer advocate might use

$H_a: \mu < 16.0$

A production supervisor might use

$H_a: \mu \neq 16.0$

**Example** Ames rent... suppose that last year's population mean was 660, and by standards of increase in CPI, 680 is "fair". This year a consumer advocate chucking a sample of apartments might use
$H_0: \mu = 680$
$H_a: \mu > 680$

How to assess the plausibility of $H_0$?

Defining a test statistic is the data summary to be used.

Defining a p-value is the probability that
the sampling distribution of test statistic $T$ assigned to values
"more extreme" than the one observed

to get p-value, I need a normal area... I need a $z$-value

$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{688.20 - 680}{\frac{80}{\sqrt{25}}} = 0.55$

a table look-up gives .7088 as a
tabled area, so

$p$-value $= 1 - .7088 = .2912$

i.e. I've seen an outcome $(\bar{x})$
such that things this extreme
occur 29% of the time if $H_0$ is
true --- $H_0$ is not particularly implausible.

Example: Amos rent

$T$ will be the test statistic
(we've seen $\bar{x} = 688.20$)

$T_{\bar{x}} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{680}{15} = 4.53$

den of $T$ is
if $H_0$ is true
is from $H_0$

"small" p-values $\rightarrow$ "strong" evidence
against $H_0$
$H_0$ implausible

"big" p-values $\rightarrow$ "weak" evidence
against $H_0$
$H_0$ not implausible

In our example we used

$H_0: \mu = \#$
$H_a: \mu > \#$

$\bar{x} = \alpha$ right tail
area for
$p$-value

20
I might as well admit that to get a p-value I'll have to compute a z-value and call the z-value the test statistic — i.e., the test statistic for \( H_0: \mu = \mu_0 \) will be

\[
 z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}
\]

and using this we get 3 sets of hypotheses and pictures for p-values.

Before you don't hear what isn't true — that is, statistical significance = practical importance

you have enough data to see that \( H_0 \) is wrong

2) By Varnden's standards CIs are much more useful than tests

- They attempt to answer “The right” question ("what is the parameter value")

- Tests try to answer "the wrong" question ("is parameter = ?")

<table>
<thead>
<tr>
<th>( H_0 ): ( \mu = \mu_0 )</th>
<th>( H_a: \mu &gt; \mu_0 )</th>
<th>( H_a: \mu &lt; \mu_0 )</th>
<th>( H_a: \mu \neq \mu_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Comments/Philosophy/Caveat (Rant)

1. Common terminology is p-value < .05 \( \rightarrow \) statistically significant

2. p-value < .01 \( \rightarrow \) highly statistically significant

Besides CIs carry significance testing information any way — e.g., if a 95% CI for \( \mu \) is

\( (5.3, 7.2) \)

p-value for testing \( H_0: \mu = 8 \)

\( H_a: \mu \neq 8 \)

will be smaller than .01 (1 -.99)

3. A p-value is not a "probability that \( H_0 \) is true" (nor is it a "probability \( H_0 \) is false") — it is a measure of "implausibility"
Problem 6.11

\[ n = 25, \; \bar{x} = 110, \; \sigma = 40 \]

55% confidence limits for \( \mu \)

\[ \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \]

\[ 110 \pm 1.96 \frac{40}{\sqrt{25}} = 16.68 \]

This calls this "the margin of error."

Problem 6.11 Revisited

95% prediction limits for \( x \) new

\[ \bar{x} \pm 1.96 \sqrt{\frac{\sigma^2}{n} + \frac{1}{n}} \]

\[ 110 \pm 1.96 (40) \left( 1 + \frac{1}{25} \right) = 75.95 \]

As far as I can tell the answer to problem 1.9 is wrong. I get

\[ \frac{475 - 250}{\sqrt{2}} = 12.82 \]

\[ \sqrt{\frac{475 - 250}{12.82}} = 175.5 \]

I presume Duckworth needs 1.96 instead of 1.282.

Problem 6.54a) page 402

\[ H_0: \mu = 115 \]

\[ H_a: \mu > 115 \]

\[ z = \frac{\bar{x} - 115}{\sigma/\sqrt{n}} = \frac{135.2 - 115}{30/\sqrt{20}} > 3.01 \]

P-value is right tail area = small

BTW I would phrase this as

"Assess the strength of the evidence that the older students have a mean above 115."
Basic Probability Fact: when sampling from a normal distribution (normal universe/population)

\[ t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \]

was tabulated \( t \) den with \( \nu \) d.f.

\( \bar{X} \) is the sample mean.

For 1 sample, note application to paired data

For 2 samples, Bob's preferred/default method

Basic Probability Fact: when sampling from a normal distribution (normal universe/population)

\[ t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \]

was tabulated \( t \) den with \( \nu = n - 1 \) - see The inside back cover of text and Figure 7.1 page 434 of text

What we did Tuesday was build mostly on the fact

\[ z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \]

This propagates through the formulas.

It is (at least approximately) standard normal.

\[ z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \]

It would be nice if we could start with something that doesn't involve \( z \), perhaps I can replace \( z \) with \( t \) and still do something.

\[ t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \]

\( t \) is den with \( \nu = n - 1 \) - see The inside back cover of text and Figure 7.1 page 434 of text.

\[ t_0 \]

The \( t \) den are bell-shaped, centered at 0, and "flatter" than normal - but for large degrees of freedom they are nearly standard normal.

\[ t_0 \]

\[ \mu \]
Note that the $t$-table is set up differently than the std normal table - the information available for $t$-dists. is not as detailed as for std. normal.

Right tail area on margin

In the body of table enter the table on row corresponding to "degrees of freedom".

From the $t$-table

$P(-2.056 < t_{26} < 2.056) = .95$

Example

$n=27$ no load mutual funds 1999 rates of return ($\overline{x} = 13.3\%$, $s = 4.1\%$)

Suppose 1 rates of return for such funds in 1999 were approximately normal.

2 This is based on a random sample of such funds.

Based this we're going to do inference.

So for $95\%$ of all samples:

$-2.056 < \frac{\overline{x} - \mu}{s/\sqrt{n}} < 2.056$

is the same as

$\overline{x} - 2.056 \frac{s}{\sqrt{27}} < \mu < \overline{x} + 2.056 \frac{s}{\sqrt{27}}$

i.e. for $n=27$

\[ \overline{x} \pm 2.056 \frac{s}{\sqrt{27}} \]

are $95\%$ confidence limits for $\mu$. 

23
The text calls an empirical approximation to

\[ \frac{S}{\sqrt{n}} \]

The "estimated standard error" of the mean

Example:

\[ \frac{0.07}{\sqrt{127}} = 1.6 \]

95% confidence limits for mean rate of return

\[ \bar{X} \pm 1.96 \frac{S}{\sqrt{n}} \]

\[ \bar{X} \pm 1.96 \frac{0.07}{\sqrt{127}} = 1.6 \]

Do problem 7.31 page 456

- Make both 95% and 95% intervals

Note that 80% is the lifetime guarantee. Note length of interval agrees with the sample.

Demonstration: Brown Bag, n=3

Use \[ \bar{X} \pm 1.65 \frac{S}{\sqrt{n}} \]

\[ \bar{X} \pm 1.65 \frac{0.07}{\sqrt{3}} \]

\[ \bar{X} = 2, 1.24, 1.54, 1.69 \]

\[ \frac{S}{\sqrt{n}} = 0.24, 0.49, 0.4, 1.48, 1.02 \]

\[ 2, 58.46, 5.0, 2.24, 1.54 \]

\[ 2, 2.49, 3.9, 4.4, 2.30, 1.58, 1.69 \]
For large, the formula turns into
\[ \bar{x} \pm \frac{s}{\sqrt{n}} \]
and some authors call this the "large sample C.I. formula"

Fixing the problem with Tuesday's PI's ... I can do this for normal populations - I do need a fairly bell-shaped universe here ... "robustness" doesn't hold

Example: no load mutual funds... 1999 rates of return - make a 95% PI for a single additional fund
\[ n = 27, \bar{x} = 13.3, s = 4.1 \]
assuming the population is normal use
\[ \bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \]

13.3 ± 1.96 \frac{4.1}{\sqrt{27}} = 2.056

\[ 13.3 \pm 0.8 \] (or much larger from estimating N)

(I'd only apply this if I "know" the population is normal or if I check a histogram of the sample and see a bell shape)

Basic Probability Fact: when sampling a normal universe
\[ \frac{x_{\text{new}} - \bar{x}}{s \sqrt{\frac{1}{n} + \frac{1}{N}}} \]
and this leads to prediction limits
\[ \bar{x} \pm s \sqrt{\frac{1}{n} + \frac{1}{N}} \]

Demonstration: Brown Bag
make an 80% PI for \( x_{\text{new}} \) based on a sample of size 5
3, 4, 5, 6, 7  \( \bar{x} = 5.0, s = 1.58 \)
prediction limits are
\[ 5.0 \pm 1.53 \frac{1.58}{\sqrt{1 + \frac{1}{5}}} \]
\[ 2.80 \]

\[ x_{\text{new}} = 5 \] - ... a winner yet again
The 80% guarantee is a lifetime batting average guarantee for the whole business of selecting 5, making an interval and selecting 1 more for one more corn price.

Significance testing for μ without the known σ assumption goes just as you should expect... replace S with S and z with t and operate as before.

H₀: μ = 15
Hₐ: μ < 15

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{13.3 - 15}{4.1 / \sqrt{27}} = -2.15 \]

p-value < 0.05

To test \( H₀: μ = \# \) use

\[ t = \frac{\bar{x} - \#}{s / \sqrt{n}} \]

and the t table (with df = n-1) to get p-values.

Example: No load mutual funds 1995
"Was the mean rate of return in 1995 clearly/definitely below 15%?"

For scenario of 7.31 page 456
- If the price is electably clearly below 2.10 policy makers want to intervene
- "Assess the strength of sample evidence that mean price is below 2.10"

(What follows is the alternative hypothesis)

An important application of the (one-sample) t methods of Section 7.1 is to
"paired data" -
Paired data arise when I have "before and after" or "with and without treatment" or "left side and right side" etc. observations on a single sample of objects/items.

A standard method of analysis is to take differences

\[ x - y = d \]

and to do inference for \( M_d \)

This possibility is fundamentally different from the content of Section 7.2 "2 sample" Methods.

The text's favorite/default method uses the variable

\[
\frac{\bar{x}_1 - \bar{x}_2 - (M_1 - M_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
\]

This is not \( t \)-distributed (even if populations are normal) but if you act as if it were (with d.f. the smaller of \( n_1 - 1 \) and \( n_2 - 1 \)) you get conservative inferences.
treats that variable as $t$ and leads to confidence limits for $(M_1 - M_2)$ of the form

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

df = smaller of $n_1 - 1$ and $n_2 - 1$

demonstration

80% confidence limits for $M_{blue} - M_{brown}$

\[ \bar{x}_{men} = 121.3, \quad s_{men} = 32.5 \]

my 80% limits are \[ 4.25 \pm 3.19 \]

(since I actually know that \[ M_{blue} - M_{brown} = 10 - 5 = 5 \], I know that yet again I'm a winner)

<table>
<thead>
<tr>
<th>Sample 4 from Brown, 5 from Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown: 5, 5, 3, 6 [ \bar{x}<em>{brown} = 4.75 ] [ s</em>{brown} = 1.26 ]</td>
</tr>
</tbody>
</table>

\[ \bar{x}_{blue} - \bar{x}_{brown} \pm t_{\alpha/2} \sqrt{\frac{s_{blue}^2}{n_{blue}} + \frac{s_{brown}^2}{n_{brown}}} \]

\[ 4.25 \leq 0 - 4.75 \pm 1.638 \sqrt{\frac{4.12^2}{5} + \frac{1.26^2}{5}} \]

\[ 3.13 \]

The significance testing version of this is to test

\[ H_0: M_1 - M_2 = 0 \quad H_1: M_1 - M_2 \neq 0 \]

with

\[ t = \frac{\bar{x}_1 - \bar{x}_2 - \#}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

and using $t$ table with df = \[ \text{smaller of } n_1 - 1 \text{ and } n_2 - 1 \]
An alternative to the book’s preferred method is based on an assumption that the 2 populations being compared have the same standard deviation ($\sigma_1 = \sigma_2 = \sigma$) (in practice this works fine if they aren’t radically different).

Example: Famous 60’s marketing study compared ages of purchasers and non-purchasers of Crest toothpaste.

<table>
<thead>
<tr>
<th></th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$\bar{y}_1$</th>
<th>$\bar{y}_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Non-Purchasers</em></td>
<td>20</td>
<td>20</td>
<td>47.2</td>
<td>39.8</td>
<td>13.62</td>
<td>10.04</td>
</tr>
<tr>
<td><em>Purchasers</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These aren’t terribly different and so perhaps a $\sigma_1 = \sigma_2 = \sigma$ assumption isn’t bad.

If I’m assuming $\sigma_1 = \sigma_2$, it makes sense to use a value compromising between $s_1$ and $s_2$ as an estimate of $\sigma$ (a “pooled” estimate of $\sigma$) is:

$$Spooled = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

Example: Crest

$Spooled = \sqrt{\frac{(20-1)(13.62)^2 + (20-1)(10.04)^2}{(20-1) + (20-1)}}$

$= 11.96$ years

Basic variable used is then:

$$\frac{\bar{x}_1 - \bar{x}_2 - (\bar{y}_1 - \bar{y}_2)}{Spooled \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \text{ has exactly a t-distribution with}$$

$$df = (n_1 - 1) + (n_2 - 1)$$
This reasoning leads to confidence limits for $\mu_1 - \mu_2$

$$\bar{x}_1 - \bar{x}_2 \pm t \frac{s}{\sqrt{n_1 + n_2}}$$

d.f. = $(n_1 - 1) + (n_2 - 1)$

and to testing $H_0: \mu_1 - \mu_2 = \pm$ using

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \pm}{s \sqrt{n_1 + n_2}}$$

and a $t$ table with d.f. = $(n_1 - 1) + (n_2 - 1)$

---

Example: Crest Make 95% confidence limits for the difference in mean ages.

$$\bar{x}_{\text{non}} - \bar{x}_{\text{crest}} \pm t \frac{s_{\text{non}}}{\sqrt{n_{\text{non}}} + \sqrt{n_{\text{crest}}}}$$

47.2 - 39.8 $\pm 2.09$ (11.96 $\sqrt{20} + \sqrt{20}$)

7.4

Upper 2.5% point of $t_{38}$ distribution: 2.024

7.66
Problem 7.31 page 456 95% and 99% intervals

2.08

2.080 for 95%
2.831 for 99%

Prob 7.31 (testing version)

\( H_0: \mu = 2.10 \)
\( H_a: \mu < 2.10 \)

\( t = \frac{\bar{x} - 2.10}{\frac{s}{\sqrt{n}}} = \frac{2.08 - 2.10}{.176} = -.11 \)

Not much evidence against \( H_0 \) in favor of mean price below 2.10

7.31 95% and 99% PI's for one more price

\[ \frac{.176}{\sqrt{22}} \]

\[ s = .176 \sqrt{22} = .8255 \]

\[ \bar{x} \pm t \cdot s \sqrt{\frac{1}{n} + \frac{1}{22}} \]

2.08

\[ .8255 \]

2.080 for 95% confidence
2.831 for 99% confidence

7.79 Default method 90% C.I for difference in means

\[ \bar{x}_1 - \bar{x}_2 \pm t \cdot \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}} \]

\[ \frac{14.1 - 12.3}{1.740} \sqrt{\frac{(26.4)^2}{18} + \frac{(32.9)^2}{20}} \]

15.8

16.7