

Stat 328 Lab #1 Summer 2000

ISU students Geerts, Ortiz and Patz worked with a company on a packaging problem. The basic issue was this. A particular consumer product unit consists of 1) a metered amount of a powdered material, 2) an empty plastic bag and 3) the packaging that holds the powdered material and the plastic bag. The company was experiencing problems ensuring that every consumer product unit contains both the powder and the plastic bag. Every unit is weighed at the end of the production line and "underweight" packages are kicked off for recycling of the powder. But too many consumer units without bags were passing this inspection. At this point the students joined the team looking for a solution to this problem.

The first thing the students did was to study the precision of the scale used to do final inspections of the units. Repeat measurements of a fixed object on that digital scale produced the values below (in grams):

95.1123, 95.1126, 95.1123, 95.1124, 95.1124, 95.1125
95.1123, 95.1122, 95.1125, 95.1123, 95.1122

(a) Find the sample mean and sample standard deviation of these values "by hand." Then check your calculations using JMP and attach a printout. (To get this printout enter these data in the worksheet and look under the **Analyze>Distribution of Y** menu, clicking on the "checkmark" at the lower left of the display and making sure that the "Text Report" option is checked. They are under the "moments" report.)

(b) Make a normal plot of these data as laid out on the formula sheet. You can do this by hand by finding 11 standard normal quantiles from a normal table. Or (and probably better) you can practice with JMP. Put the data into the worksheet. Use the **Tables>Sort** menu to get them in order. Then use **Cols>New Column** to get to the dialog box for entering a new column that will have in it the values $Q_z\left(\frac{i-.5}{n}\right)$. Make the "Data Source" be "Formula." Click "OK." Then build the expression

$$\text{normQuant}\left(\frac{i - .5}{11}\right)$$

in the calculator window. One way to do this is to begin with parends, select "i" from the **Terms** menu, subtract .5, highlight the whole expression and divide by 11, cut the whole expression to the clipboard, select "Normal Quantile" from the **Probability** menu and paste back inside the parends and hit the "Evaluate" button. You should now have a column of standard normal quantiles. Use the **Analyze>Fit Y by X** menus and plot the standard normal quantiles against the (ordered) data values. (This plot should rise consistently to the right.) (A slightly different kind of normal plot can be gotten much more directly in JMP by simply entering the data, going through the **Analyze>Distribution of Y** menu and clicking on the "checkmark" at the lower left of

the display you get and then adding checks to the "Horizontal Layout" and "Normal Quantile Plot" options.)

(c) Suppose that one grants that the normal plot is adequately linear to justify treating repeat measurements of this test object as approximately normal. Make 95% two-sided confidence intervals for the long run mean μ and standard deviation σ one should expect to get remeasuring this object. (You'll need to use the .975 quantile of the t_{10} distribution and the .025 and .975 quantiles of the χ^2_{10} distributions in the formulas provided on the course formula sheet.) Check your calculations for the first of these against the confidence limits provided by JMP under the **Analyze>Distribution of Y** menu, clicking on the "checkmark" at the lower left of the display and making sure that the "Text Report" option is checked. (The confidence interval is under the "moments" report.)

(d) Make a 95% prediction interval for a single additional weight obtained remeasuring this object. How does this compare to your confidence interval for μ ? Why does this comparison make sense?

(e) Suppose that, in fact, the item measured was known to have a "true" weight of 95.2000 grams. (Perhaps it was a standard test item.) In the light of such information which interval from (c) would say something about the accuracy of the scale? What does it say?

(f) The students subsequently weighed $n = 30$ complete consumer units and found them to have a standard deviation of around 5.96 grams. Discuss whether (in light of this and your confidence interval for σ in part (c)) you think the scale used to sort packages at the end of the production line is sufficiently precise.

(g) The smallest of the $n = 30$ weights referred to in part (f) was 100.8 grams and the largest was 130.8 grams. How "sure" are you that the median weight of complete consumer units is in the range 100.8 grams to 130.8 grams? How "sure" are you that a single additional package would weigh between 100.8 and 130.8 grams?

As a matter of fact, data collection and calculations done by the students led to the conclusions that measurement variability, variation in the weights of the empty plastic bags, and variation in the weights of the packaging was negligible in comparison to variation in the amount of powder being dispensed to a given consumer unit. In fact, it was reasonable to model weights of complete packages as normal with $\mu_{\text{complete}} = 117.12$ grams and $\sigma = 5.96$ grams and the weights of packages missing a plastic bag as normal with mean $\mu_{\text{missing}} = 104.42$ grams and $\sigma = 5.96$ grams. (Essentially all of $\sigma = 5.96$ is due to variation in the amount of powder dispensed to a unit.) Currently any package below 111.2 grams is kicked off the line as underweight.

(h) What fraction of complete consumer units will fail final inspection? What fraction of consumer units missing the plastic bag will pass the final inspection?

(i) Consider alternative final inspection limits, first 113.2 grams and then 109.2 grams. Answer part (h) for these limits.

(j) The final weight check is doing more than simply checking for missing plastic bags, but for the time being, consider only this aspect of its function. It is possible to think of the checking of a product unit as a hypothesis test of the null hypothesis $H_0 : \mu = 117.12$ (the unit is complete), with a test statistic $y = \text{measured weight}$ having a normal reference distribution with mean 117.12 grams and $\sigma = 5.96$ grams. (Equivalently we could think of $z = (y - 117.12)/5.96$ as a test statistic with a standard normal reference distribution.) In a hypothesis test, α is the probability (from the reference distribution) of rejecting the null hypothesis when in fact it is true. What α is the company currently using?

(k) Your calculations in (h) and (i) should have convinced you that currently one can not really separate complete packages from packages missing the plastic bag on the basis of final weight. The variation in the amount of powder dispensed masks whether or not the bag is present. Suppose that the company was willing to make a significant capital expenditure and obtain a more consistent powder dispensing mechanism. In fact, suppose that the current standard deviation $\sigma = 5.96$ grams could be cut in half. Would that make detection of missing bags at final inspection feasible? Explain carefully.

A possible alternative to the capital expenditure considered in part (k) is to use a heavier grade of plastic and therefore a heavier bag. I don't know the weight of the film currently used in the bag, but for purposes of discussion, suppose it is 2 mil plastic film. The mean weight of a plastic bag (of the same dimensions as the present one) is thus $6.35x$, for x the plastic film thickness in mils. So then, the mean complete customer unit weight is

$$\mu_{y|x} = 104.42 + 6.35x$$

And since essentially all of the variation in weight is still coming from the amount of powder dispensed, it is plausible to think that weights of complete customer units will be normal with this mean and standard deviation $\sigma = 5.96$ grams. In regression notation, the model for the weight, y , of a complete package becomes

$$y = 104.42 + 6.35x + \epsilon$$

where ϵ is normal with mean 0 and standard deviation $\sigma = 5.96$ grams.

(l) What fraction of complete consumer units will have weights below 111.2 grams if 3 mil bags are used? If 4 mil bags are used?

(m) Find a plastic thickness so that if the final inspection limit is set 5.92 grams below the mean weight of a complete consumer unit, only a fraction .001 of units missing a plastic bag will pass final inspection. (The .999 quantile of standard normal is about 3.08.)