EWMA Charts

(Section 4.1 of Vardeman and Jobe)

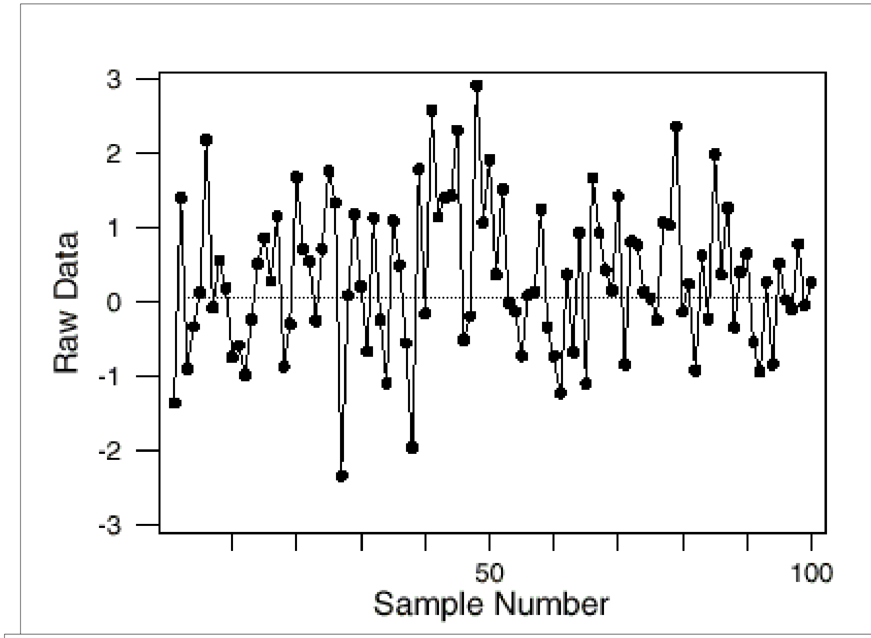
Basic Motivation

- It may be difficult to see small changes in the distribution of a plotted statistic Q on a Shewhart chart (Q is my generic name for a plotted value ... it could be $x, \overline{x}, R, s, \hat{p}, \hat{u}$ or something else)
- It can be easier to see those changes if one does some "smoothing" before plotting
- One form of smoothing is to make Exponentially Weighted Moving Averages

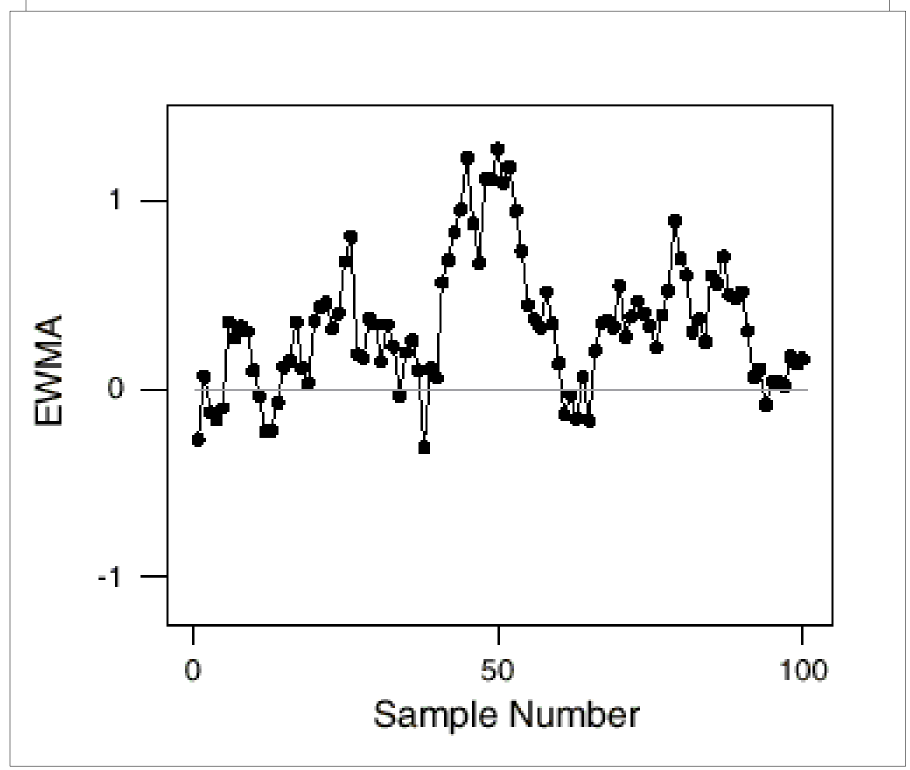
$$EWMA_i = l Q_i + (1 - l) EWMA_{i-1}$$

Example

- *Qs* Normal with mean *.5* and std dev 1.0



- *EWMA*s of *Q*s with $EWMA_0 = 0$ and $l = .2$



How to Use This in SPM?

- Somehow choose $EWMA_0$ and l
- Somehow set and use control limits on *EWMA*s instead of on *Q*s

- Suppose that an "all-OK" distribution of Q has mean m_Q and standard deviation s_Q
- Then (with $EWMA_0 = m_Q$)

$$m_{EWMA} = m_Q \text{ and } s_{EWMA} \rightarrow s_Q \sqrt{\frac{l}{2-l}}$$

Control Limits for *EWMA*s

- "k-sigma" standards-given limits for *EWMA*s are then

$$UCL_{EWMA} = m_Q + k s_Q \sqrt{\frac{l}{2 - l}} \quad \text{and}$$

$$LCL_{EWMA} = m_Q - k s_Q \sqrt{\frac{l}{2 - l}}$$

- Naïve choice of $k = 3$ is rational only when $l = 1$ (i.e. only when there is NO smoothing)

Choice of k (for a given l)

- For normal Q, Table 4.3 can be used to choose k giving a desired mean time between false-alarms (a desired "all-OK ARL")
  - For $all\text{-}OK\ ARL = 370$ (all-OK behavior like that of a Shewhart $\overline{x}$ chart), use

l

.05	.1	.2	.3	.4	.5	.75	1.0
2.49	2.70	2.86	2.93	2.96	2.98	3.00	3.00

  - Note that $k = 3.00$ only for large l

Choice of *l*

- Convenience
- Small values provide more smoothing and better ability to see small changes, *at the cost of slower reaction to big changes*
- "Optimal" choice is possible (for given all-OK ARL and potential shift in mean *Q*)
  - With $"shift" = d / s_Q$ and $all\text{-}OK\ ARL = 370$ "optimal" choices are

"*shift*"	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
	.05	.14	.25	.37	.54	.70	.82	.90

Predicted Behavior?

- For given l, UCL_{EWMA} and LCL_{EWMA}, with
  $$EWMA_0 = \left(UCL_{EWMA} + LCL_{EWMA}\right)/2$$
  and normal Q, it is possible to find (not-all-OK) ARLs (mean times to detection)
  - Use formulas (4.6) and (4.7) to find parameters needed to enter Table A.3 (one inputs the mean and standard deviation of Q)

Perspective

- Not a tool for plotting "by hand"
- EWMA charts provide quicker detection of small departures from standard conditions than corresponding Shewhart charts, *supposing those departures pertain from time 1 on*
- Big departures are typically caught more quickly by a Shewhart chart
- EWMA charts *have poor "worst case" behavior (possible inertia, should a shift occur some time after time 1)*

Workshop Exercises

- Find $l = .2$ *EWMA*s for the *Q*s below

i	Q_i	$EWMA_i$
0		0
1	1	
2	3	
3	0	
4	1	
5	20	

- For the case of the example on slide 3, find "$all\text{-}OK\ \ ARL = 370$" control limits for *EWMA*s and apply them (to the plotted *EWMA*s)