ASQ Workshop on Bayesian Statistics for Industry

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Module 8: Some Multi-Sample Examples

We finish up with a couple of interesting multi-sample examples of Bayes analyses. The first is a "one way random effects analysis" example.

**Example 7** In a paper "Calibration, Error Analysis, and Ongoing Measurement Process Monitoring for Mass Spectrometry," Vardeman, Wendelberger, and Wang discuss a Bayes analysis of 44 measurement of a spectrometer’s sensitivity to Argon gas, made across 3 days. With

\[ DS_{t,j} = \mu_S + \delta_t + \epsilon_{t,j} \]

they used a decomposition

- \( DS_{t,j} \) = the device sensitivity computed from specimen \( j \) on day \( t \)
for $\mu_S$ a fixed unknown "true" device sensitivity, $\delta_t$ a random "day $t$" deviation in sensitivity, and $\epsilon_{t,j}$ a random specimen deviation. Assuming that the $\delta_t$ are independent draws from a normal distribution with mean 0 and standard deviation $\sigma_\delta$, independent of the $\epsilon_{t,j}$ that are independent random draws from a normal distribution with mean 0 and standard deviation $\sigma$, this is a problem with parameters $\mu_S, \sigma_\delta, \text{ and } \sigma$.

The model can be rephrased as

$$\mu_t \equiv \mu_S + \delta_t = \text{the day } t \text{ sensitivity} \sim N(\mu_S, \sigma_\delta^2) \text{ for } t = 1, 2, 3$$

and given the values of $\mu_t$, the specimen sensitivities are

$$DS_{t,j} = \mu_S + \delta_t + \epsilon_{t,j} \sim N(\mu_t, \sigma)$$
We considered the specification of a prior for the parameters $\mu_S, \sigma_\delta$, and $\sigma$:

$$\mu_S \sim N(0, 10^6) \text{ independent of }$$

$$\frac{1}{\sigma_\delta^2} \sim \text{Gamma}(0.001, 0.001) \text{ independent of }$$

$$\frac{1}{\sigma^2} \sim \text{Gamma}(0.001, 0.001)$$

These were intended to be relatively non-informative (but nevertheless proper) priors for the parameters.

The WinBUGS code used in the paper is in the file

```
BayesASQEx7.odc
```

and listed below.

```r
list(sens=c(31.3, 31.0, 29.4, 29.2, 29.0, 28.8, 28.8, 27.7, 4
```
27.7, 27.8, 28.2, 28.4, 28.7, 29.7, 30.8, 30.1, 29.9, 32.5, 32.2, 
31.9, 30.2, 30.2, 29.5, 30.8, 30.5, 28.4, 28.5, 28.8, 28.8, 30.6, 
31.0, 31.7, 29.8, 29.6, 29.0, 28.8, 29.6, 28.9, 28.3, 28.3, 28.3, 
29.2, 29.7, 31.1), ind=c(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 
2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3), 
N=44)

list(mu=c(3, 3, 3), tau=1, muS=0, taudelta=1)

model {
  for(i in 1:N) {
    sens[i]~dnorm(mu[ind[i]], tau)
for(i in 1:3) {
  mu[i]~dnorm(muS,taudelta)
}

tau~dgamma(0.001,0.001)

sigma<-1/sqrt(tau)

muS~dnorm(0.0,1.0E-6)

taudelta~dgamma(0.001,0.001)

sigmadelta<-1/sqrt(tau)
Figure 1 shows some summaries from a WinBUGS session based on the code.

Notice that the mean(s) in this problem are far more precisely determined than are the standard deviations. That is a well known phenomenon. It takes a very large sample size to make definitive statements about variances or standard deviations ... and in the case of $\sigma_\delta$, the appropriate "sample size" is 3! (Tests were made on only 3 days.)
Figure 1: Summaries of a Bayes analysis of a spectrometer’s sensitivity to Argon gas