

October 5, 2005

Prof. Vardeman

1. IE 361 students Wilhelm, Chow, Kim and Villareal worked with a company checking conformance of several critical dimensions of a machined part to engineering requirements. This question is based on their work on the measured (maximum) diameter of a nominally circular hole through the part.

An operator measures the hole in one part 5 times, producing a sample standard deviation of .003 mm. She then measures the hole in 10 other parts once each, producing a sample standard deviation of .010 mm.

a) **Give** 95% confidence limits for a "repeatability" standard deviation for this diameter measurement. (Plug in, but no need to simplify.)

b) **Give** a single-number estimate of the standard deviation of actual hole diameters. Then suppose that engineering specifications on hole diameters are *nominal*  $\pm .05$  mm. **Does** it appear that the hole drilling process can meet these specifications (putting essentially all hole diameters inside engineering specifications if properly "aimed")? **Explain.**

c) **What** "approximate degrees of freedom" should be associated with your single-number estimate from b)? (Plug in, but no need to simplify.)

d) **Do** the data from the 15 measurements made by the operator contain any information about accuracy of the measurements? **Answer** "yes" or "no" and then **explain.**

Subsequent to the small measurement study described above, the students conducted a Gauge R&R study using 10 parts. There is part of a JMP summary of the data collected in this study attached to the back of this exam. Use it as needed in answering questions e) and f) below.

e) **Which** appears (on the basis of single-number estimates) to be more important, repeatability variation in measurement, or operator-to-operator variability in measurement of these diameters? **Say** which ("repeatability" or "operator") you think is largest and **show** appropriate supporting calculations.

f) As it turns out, an ANOVA-based estimated "R&R standard deviation" calculated from these data is .002 mm with an associated approximate degrees of freedom 77. Lower and upper 5% points of the  $\chi^2_{77}$  distribution are 57.8 and 98.5 respectively. If the gauge in question is used to check conformance to *nominal*  $\pm .05$  mm specifications, **give** approximate 90% confidence limits for the gauge capability ratio. (Plug in, but no need to simplify.)

2. Attached to the back of this exam you will find a JMP report for a calibration data set of Prof. Wm. Switzer of the Chemistry Department at NCSU. Several standard solutions of Riboflavin were run through a chemical analyzer and "reflectance" was measured. (Concentration,  $x$ , was in micrograms/mL, and reflectance,  $y$ , was measured in units particular to the machine.) The JMP report includes the fitted least squares line and 95% prediction limits for  $y_{\text{new}}$  at each  $x_{\text{new}}$ .

a) Your first job is to tell a user how consistent the reflectance measurements are for a fixed concentration of Riboflavin. A single reflectance value read from the analyzer is "good to within" roughly  $\pm$  **how many** machine units? **Explain.**

b) **Give** 95% confidence limits for concentration if the machine reads a reflectance of 50 units.

3. In a mechanical assembly operation, a particular bolts is tightened with pneumatic tool. It is essential that the actual torque required to loosen this bolt be carefully monitored. Every hour,  $n = 6$  assemblies are taken from the production line and the torque required to loosen the bolt is measured.

Attached to the back of this exam are charts for  $\bar{x}$  and  $s$  based on 20 hourly samples. As it turns out, these 20 samples have  $\bar{\bar{x}} = 26.78$  ft lbs and  $\bar{s} = 2.86$  ft lbs .

a) The target for mean measured torque is  $\mu = 35$  ft lbs and past experience with the process and the torque wrench used to do measuring suggest that  $\sigma \approx 3.00$  ft lbs is about the best one can hope for with this process. **Compute** standards given control limits for  $\bar{x}$  and  $s$  , **apply** them to the values plotted on the JMP report, and **say** what the plot indicates about bolt torque.

Control Limits for  $\bar{x}$  :

Control Limits for  $s$  :



Interpretation:

b) **Find** retrospective control limits for  $\bar{x}$  and  $s$  and apply them to the values plotted on the JMP report, and **say** what the plot indicates about bolt torque.

Control Limits for  $\bar{x}$  :

Control Limits for  $s$  :



Interpretation:

c) It's pretty clear that mean measured bolt torque is below the target of 35 ft lbs. **Explain** why it is or isn't clear from the plotted data that mean *actual* bolt torque is below 35 ft lbs. (**Say** whether it is clear or not.)

# JMP Printout for Problem 1

## Response Diameter

### Summary of Fit

RSquare	0.990248
RSquare Adj	0.985534
Root Mean Square Error	0.002183
Mean of Response	0.9923
Observations (or Sum Wgts)	90

### Analysis of Variance

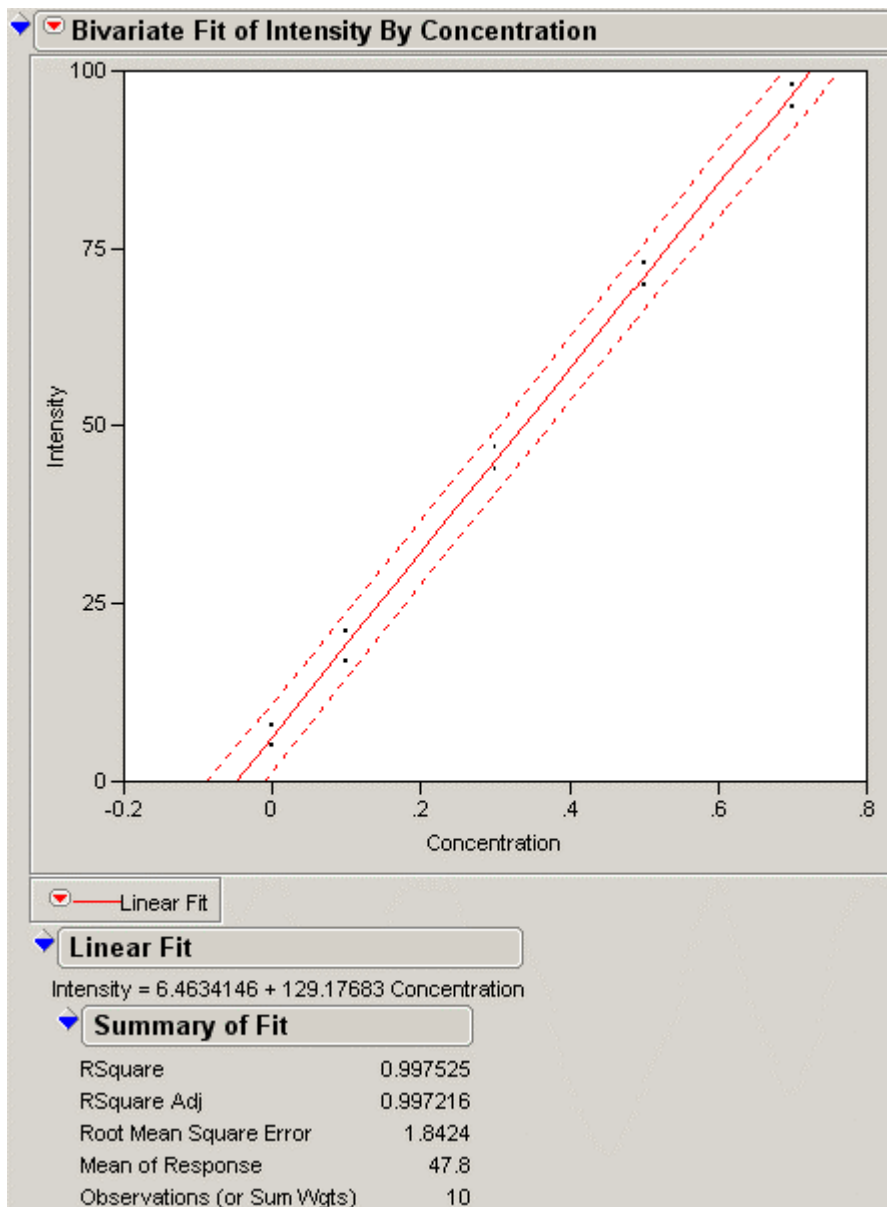
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	29	0.02904090	0.001001	210.0861
Error	60	0.00028600	0.000005	Prob > F
C. Total	89	0.02932690		<.0001

### Effect Tests

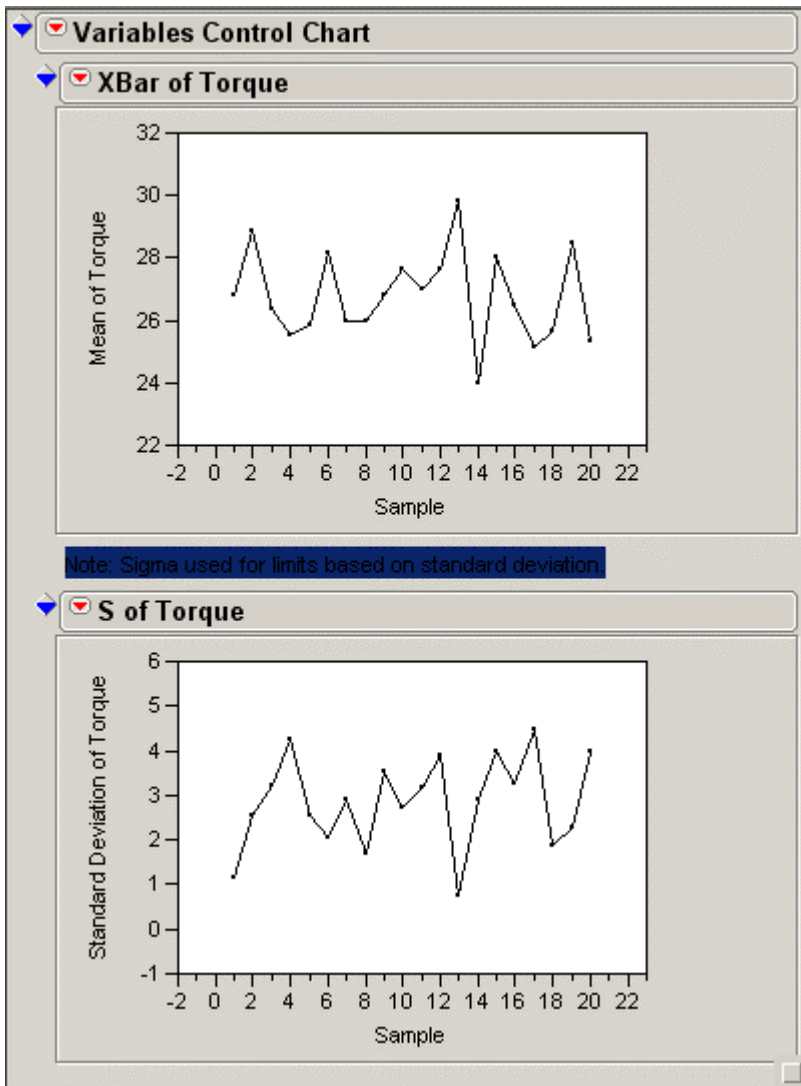
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Part	9	9	0.02899934	675.9754	<.0001
Operator	2	2	0.00000127	0.1329	0.8758
Operator*Part	18	18	0.00004029	0.4696	0.9615

## JMP Printout for Problem 2

ChemistryCalibrationData			
ChemistryCalibrati		Concentration	Intensity
	1	0	8
	2	0	5
	3	0.1	17
	4	0.1	21
	5	0.3	47
	6	0.3	44
	7	0.5	70
	8	0.5	73
	9	0.7	95
	10	0.7	98



### JMP Printout for Problem 3



1. A physically stable metal turning process is producing cylinders with diameters,  $y$ , and is being monitored by an SPC system. In this context there are a variety of "limits" related to diameter that one might be concerned with. These include at least:

**A**-specification limits, **B**-confidence limits, **C**-control limits, **D**-tolerance limits, **E**-prediction limits  
**Identify** (by writing exactly one letter from the list above in each blank next to a description below) **which types of limits do which jobs.**

- \_\_\_\_\_ These limits attempt to locate "most" cylinder diameters on the basis of a sample of diameters.  
 \_\_\_\_\_ These limits say what diameter is required in order for a cylinder to be functional.  
 \_\_\_\_\_ These limits attempt to locate an additional cylinder diameter on the basis of a sample.  
 \_\_\_\_\_ These limits attempt to locate a process parameter (like mean diameter) on the basis of a sample.  
 \_\_\_\_\_ These limits are applied to a sample statistic in order to check for process stability.

2. Historically, .1% (a fraction .001) of the cylinders referred to in Problem 1 above have been non-conforming in terms of diameters that are either too large or too small to be functional. Suppose that one decides to attempt attributes control charting based on weekly samples of  $n = 50$  cylinders.

a) **What** are standards given control limits for the number non-conforming in such samples?

$UCL =$  \_\_\_\_\_       $LCL =$  \_\_\_\_\_

b) If there is no change in process performance, **on average, how many weeks** do you expect to wait for the first "false alarm" from the chart from a)? (In case either is useful: the Binomial probability function is

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ and the Poisson probability function is } f(x) = \exp(-\lambda) \lambda^x / x!.)$$

c) Suppose that the fraction of non-conforming cylinders changes from .1% to .5%. **How many weeks** do you expect to wait until the chart from a) signals this change?

3. IE 361 Students Malviya, Gustafson, Joe, and Natalia worked with a manufacturer on quantifying the capability of a machining process to produce metal parts meeting certain engineering requirements. A particular diameter was specified to be between  $L = 1.5358$  inch and  $U = 1.5368$  inch. A sample of  $n = 35$  of these parts had measured diameters with  $\bar{x} = 1.53633$  and  $s = .00010$  and  $\min x_i = 1.5362$  and  $\max x_i = 1.5366$ . (If any place below you need a tabled value that isn't available, just pick one above or below it and say what you are doing.)

a) **Give** an interval that you are 95% sure will contain a 36<sup>th</sup> diameter from this process. (Plug, but no need to simplify.)

b) **How sure** can one be that at least 80% of diameters produced by this process are between 1.5362 inch and 1.5366 inch? (Again, plug in, but there is no need to simplify.)

c) **Give** two-sided 95% confidence limits for a process capability ratio that is a measure of current performance. (Plug in, but no need to simplify.)

d) **Give** two-sided 95% confidence limits for a process capability ratio that is a measure of potential performance. (Plug in, but no need to simplify.)

4. A manufacturer of a measuring instrument guarantees that it is accurate. In repeat measuring of the same object with this instrument, one gets approximately normally distributed measurements. An appropriate estimated "signal to noise ratio" for re-measuring in this context is

$$R = \frac{\bar{x}}{s}$$

Use the facts that for  $n = 10$ ,  $\bar{x}$  and  $s$  are independent with  $\mu_{\bar{x}} = \mu$  and  $\sigma_{\bar{x}} = .316\sigma$  and with  $\mu_s = .973\sigma$  and  $\sigma_s = .232\sigma$  and **find an approximate variance** for  $R$  based on a sample of size 10. (Your answer will depend upon the unspecified actual signal to noise ratio  $\mu / \sigma$ .)

5. Miscellaneous Short Answer

a) **Under what circumstances** is multivariate monitoring of two quality variables  $x_1$  and  $x_2$  **more effective than separate monitoring** of the variables and also **practically feasible**?

b) An injection molding machine has 24 supposedly equivalent cavities in a single die that simultaneously produce a part with each cycle of the machine. **Why** is it typically NOT a good idea to treat 24 parts from a cycle of the machine as a single "sample from a single molding process."

c) **Why** does Vardeman prefer the terminology "Statistical Process Monitoring" to the terminology "Statistical Process Control"?

This exam consists of 20 multiple choice questions. Write (in pencil) the letter for the single best response for each question in the corresponding blank on the attached answer sheet. (Write ONLY ONE LETTER in each blank.)

1. The pooled sample standard deviation in an experiment for comparing  $r$  different experimental conditions,  $s_{\text{pooled}}$ ,

- a) serves as a measure of "baseline" or "background" variation/experimental error
- b) is guaranteed to lie between the smallest and largest of the sample standard deviations for the different experimental conditions
- c) serves as an estimate of variability in response for any fixed one of the experimental conditions
- d) all of a)-c) are true
- e) exactly 2 of a)-c) are true

2.  $r = 3$  conditions in an experiment have sample sizes and produce sample standard deviations in the table below. What is the corresponding value of  $s_{\text{pooled}}$ ?

$n_1 = 4, s_1 = 2$	$n_2 = 3, s_2 = 6$	$n_3 = 5, s_3 = 4$
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- a) 3.78
- b) 4.00
- c) 4.06
- d) 4.12
- e) 4.32

3. The one-way normal model used as a basis for inference in Chapter 6 and Section 7.1 of Vardeman and Jobe says that responses for  $r$  different experimental conditions

- a) are normally distributed for each given condition
- b) have a single mean that is common across all  $r$  different conditions
- c) have a single standard deviation that is common across all  $r$  different conditions
- d) all of a)-c) are true
- e) exactly 2 of a)-c) are true

4. If 95% confidence limits for  $\mu_1$  are  $\bar{y}_1 \pm 3$  while 95% confidence limits for  $\mu_2$  in the same study are  $\bar{y}_2 \pm 4$ , 95% confidence limits for the difference  $\mu_1 - \mu_2$  (all limits based on the same  $s_{\text{pooled}}$ ) are

- a)  $\bar{y}_1 - \bar{y}_2 \pm 7$
- b)  $\bar{y}_1 - \bar{y}_2 \pm 6$
- c)  $\bar{y}_1 - \bar{y}_2 \pm 5$
- d)  $\bar{y}_1 - \bar{y}_2 \pm 4$
- e)  $\bar{y}_1 - \bar{y}_2 \pm 3$

5. In an experiment where  $r = 4$  conditions are each represented by samples of size  $m = 5$  and  $s_{\text{pooled}} = 3$ , 95% confidence limits for each individual mean response,  $\mu_i$ , are  $\bar{y}_i \pm \Delta$  for what value of  $\Delta$ ?

- a) 6.36
- b) 3.72
- c) 2.84
- d) 2.80
- e) 2.43

6. In a  $3 \times 3$  two-way factorial study, fitted A main effects are  $a_1 = 2$  and  $a_2 = 3$ , fitted B main effects are  $b_1 = -1$  and  $b_2 = 2$ , and four of the fitted AB interactions are  $ab_{11} = 1, ab_{12} = -1, ab_{21} = 2$  and  $ab_{22} = -3$ . If the average of the 9 sample means is  $\bar{y}_{..} = 10$ , what was the sample mean response where both A and B were at their 3<sup>rd</sup> levels?

- a) 3
- b) 4
- c) 6
- d) 7
- e) 10

7. In the context of question 6, suppose that all 9 sample sizes were  $m = 2$  and that  $s_{\text{pooled}} = 2$ . Consider the matter of lack of parallelism on an "interaction plot" (a plot traces of sample means against level of A, one for each level of B). If 95% confidence intervals are used to judge the statistical detectability of lack of parallelism

- a) lack of parallelism is detectable because some fitted interactions are larger in magnitude than their corresponding "margins of error" (used in making confidence intervals for  $\alpha\beta_{ij}$ 's)
- b) lack of parallelism is not detectable because all fitted interactions are smaller than their corresponding "margins of error"
- c) lack of parallelism is detectable because some fitted interactions are larger in magnitude than  $t s_{\text{pooled}}$
- d) lack of parallelism is not detectable because all fitted interactions are smaller in magnitude than  $t s_{\text{pooled}}$
- e) exactly 2 of a)-d) are true

8. If in a two-way factorial study, the fitted interactions  $ab_{ij}$  are both statistically detectable and large in a practical sense (for example, they are NOT an order of magnitude smaller than main effects)

- a) one can not think of Factors A and B acting "separately" on the response variable
- b) it is fairly certain that somewhere in the data collection an error has been made, producing an outlying observation
- c) the change in response that accompanies a change in level of Factor A can depend upon which level of Factor B is under discussion
- d) exactly 2 of a)-c) are true
- e) all of a)-c) are true

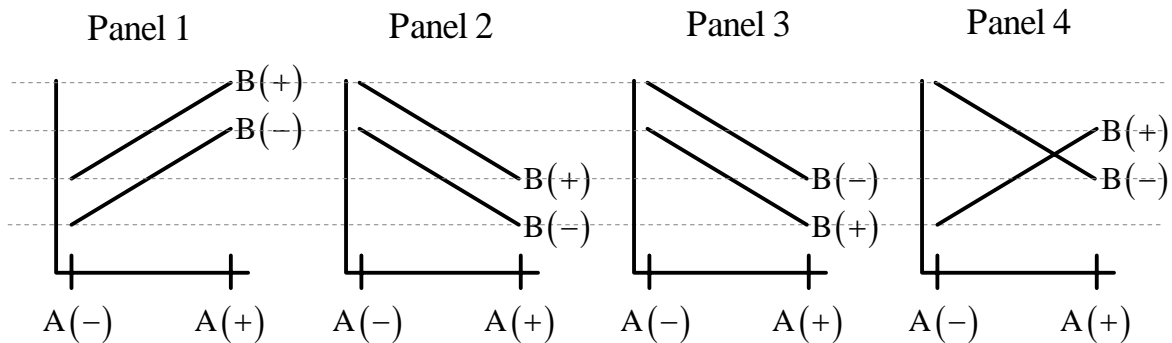
9. In a  $2 \times 2$  factorial study, sample means are  $\bar{y}_{(1)} = 5, \bar{y}_a = 9, \bar{y}_b = 3$ , and  $\bar{y}_{ab} = 7$ . The fitted main effects of Factors A and B at their low levels ( $a_1$  and  $b_1$ ) are respectively

- a) 2 and -1
- b) -2 and 1
- c) 6 and 2
- d) 6 and -2
- e) none of a)-d)

10. In a  $2 \times 2$  factorial study where two sample sizes are 3 and two are 4, we can say with 95% confidence that fitted effects from the Yates algorithm are "good to within" about

- a)  $2.23s_{\text{pooled}}$
- b)  $1.81s_{\text{pooled}}$
- c)  $.60s_{\text{pooled}}$
- d)  $.49s_{\text{pooled}}$
- e) none of a)-d)

Below is a "cartoon" giving plots of sample means against level of Factor A. Assume that the vertical scales on all are the same. (Only four different values of mean response are portrayed on the cartoon.)



11. Thinking of the four panels as different possible outcomes in a 2 factor study with Factors A and B, which portrays a situation where there are *no* Factor A main effects?

- a) Panel 1 only
- b) Panels 2 and 3 only
- c) Panel 4 only
- d) all of panels 1 through 4
- e) none of panels 1 through 4

12. Suppose that Panel 1 represents responses in a  $2^3$  study when Factor C is at its low (-) level. If, in fact, C has no main effects and no interactions with Factors A or B, responses for Factor C at its high (+) level

- a) must be as in Panel 1
- b) must be as in Panel 2
- c) must be as in Panel 3
- d) must be as in Panel 4
- e) are impossible to determine from the given information

13. Suppose that Panel 2 represents responses in a  $2^3$  study when Factor C is at its low (-) level and Panel 3 represents responses when C is at its high (+) level. The nature of C main effects and AC two factor interactions is then

- a) 0 C main effects and 0 AC two factor interactions
- b) 0 C main effects and non-zero AC two factor interactions
- c) non-zero C main effects and 0 AC two factor interactions
- d) non-zero C main effects and non-zero AC two factor interactions
- e) impossible to determine from the given information

A classic book on engineering statistics by Brownlee has data from a  $2^4$  factorial study on a chemical purification process. The response variable

$$y = \text{a measure of specimen purity}$$

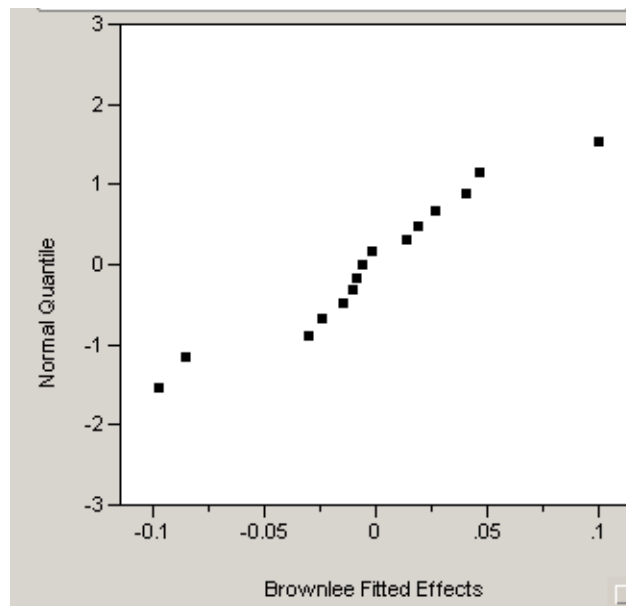
was potentially influenced by

A- wash of the crude material	hot (-)	vs	cold (+)
B- boiling of material	none (-)	vs	some (+)
C- solvent	first (-)	vs	second (+)
D- final wash	cold (-)	vs	hot (+)

Fitted effects (from the Yates algorithm) for Brownlee's data are

$$\begin{aligned} \bar{y}_{\dots} &= .4125, a_2 = -.0087, b_2 = .1000, ab_{22} = -.0238, \\ c_2 &= -.0850, ac_{22} = -.0013, bc_{22} = -.0975, abc_{222} = .0137, \\ d_2 &= -.0062, ad_{22} = -.0150, bd_{22} = .0263, abd_{222} = -.0300, \\ cd_{22} &= .0187, acd_{222} = .0400, bcd_{222} = .0463, abcd_{2222} = -.0100 \end{aligned}$$

and a normal plot of the last 15 of these is below.



14. There was apparently no replication in this study (all sample sizes were  $m = 1$ ).
- this means that confidence intervals for judging the statistical detectability of the fitted effects can not be made
  - this is justifiable because in order to include replication in the study, at least 32 observations would have been required instead of only 16
  - this a major weakness of the study
  - exactly 2 of a)-c) are true
  - all of a)-c) are true
15. Suppose that viewing the normal plot of fitted effects, one judges that even if most are explainable as "noise" there are 3 fitted effects that are clearly more than noise. Then one judges that
- effects of both "boiling" and "solvent" are discernable and the factors act separately on purity
  - effects of both "boiling" and "solvent" are discernable and the factors do not act separately on purity
  - the two wash temperatures have no clear effect on purity
  - both a) and c) are true
  - both b) and c) are true

16. If, as suggested in question 15, exactly three of the plotted fitted  $2^3$  factorial effects are judged to be detectable and *large* purity is desirable, how do you suggest setting levels of "boiling" and "solvent" and what purity do you predict for your choice?
- some boiling with the first solvent,  $\hat{y} = .6950$
  - some boiling with the first solvent,  $\hat{y} = .5975$
  - some boiling with either solvent,  $\hat{y} = .5125$
  - either boiling condition with either solvent,  $\hat{y} = .4125$
  - none of the above are appropriate answers since no levels for the wash temperatures are specified

Armed with data from a full factorial study like the Brownlee study, it is possible to consider what could have been learned if only a fractional factorial had been run (instead of the full factorial).

17. Consider what could have been learned if only one solvent had been used. Two "half" datasets consisting only of the specimen purities for a fixed solvent produce fitted effects below:

First Solvent		Second Solvent	
Fitted Effect	Value	Fitted Effect	Value
$\bar{y}_{...}$	.4975	$\bar{y}_{...}$	.3275
$a_2$	-.0075	$a_2$	-.0100
$b_2$	.1975	$b_2$	.0025
$ab_{22}$	-.0375	$ab_{22}$	-.0100
$d_2$	-.0250	$d_2$	.0125
$ad_{22}$	-.0550	$ad_{22}$	.0250
$bd_{22}$	-.0200	$bd_{22}$	.0725
$abd_{222}$	-.0200	$abd_{222}$	-.0400

Suppose that *only one* of the two half datasets summarized above had been obtained.

- nothing about the effects of either "solvent" or "boiling" would have been learned
- everything important about the effects of both "solvent" and "boiling" would have been learned
- nothing about the effects of "solvent" would have been learned, but the important message about "boiling" would certainly have been learned
- nothing about the effects of "solvent" would have been learned, but the important message about "boiling" might have been learned if by chance the correct solvent was the one used
- it would have been immediately obvious that collection of the second half dataset was necessary

Suppose now that instead of fixing the solvent and collecting half of the Brownlee dataset, a standard half fraction defined by the generator  $D \leftrightarrow ABC$  had been used.

18. 8 out of the  $2^4 = 16$  combinations of levels of the 4 factors would have been included in the experiment. What levels of Factor D would have been used in combination with respectively

$$(A,B,C) = (+, +, -) \text{ and } (A,B,C) = (+, -, -)$$

- (-) and (-)
- (-) and (+)
- (+) and (-)
- (+) and (+)

If the 8 responses from the Brownlee dataset defined by the generator  $D \leftrightarrow ABC$  are listed in Yates standard order as regards Factors A,B and C and the (3 cycle) Yates algorithm is applied, the 8 numbers below are produced (listed in the same order).

.4025
-.0550
.1400
-.0051
-.1150
.0250
-.1125
-.0199

19. What does the value .1400 on the 3<sup>rd</sup> line of this table represent?
- an estimate of  $\beta_2$  (B main effect) in the full  $2^4$  factorial
  - an estimate of  $\beta_2 + \beta\delta_{22}$  (B main effect plus BD two factor interaction) in the full  $2^4$  factorial
  - an estimate of  $\beta_2 + \alpha\gamma\delta_{222}$  (B main effect plus ACD three factor interaction) in the full  $2^4$  factorial
  - none of the above
20. The simplest subject matter interpretation of the fact that the largest (in magnitude) values in the table above (ignoring the first) are the 3<sup>rd</sup>, the 5<sup>th</sup> and the 7<sup>th</sup>
- is exactly the same as the subject matter interpretation of the normal plot on page 4
  - is consistent with the fact that when all main effects and interactions associated with some factor in a  $2^4$  study are negligible, the standard half-fraction defined by the generator  $D \leftrightarrow ABC$  provides complete information on the main effects and interactions of the other 3 factors
  - both a) and b) are true
  - neither a) nor b) is true

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Name

**IE 361 Exam 3 Fall 2005  
Answer Sheet (Form A)**

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_
4. \_\_\_\_\_
5. \_\_\_\_\_
6. \_\_\_\_\_
7. \_\_\_\_\_
8. \_\_\_\_\_
9. \_\_\_\_\_
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15. \_\_\_\_\_
16. \_\_\_\_\_
17. \_\_\_\_\_
18. \_\_\_\_\_
19. \_\_\_\_\_
20. \_\_\_\_\_