

1. IE 361 students Demerath, Gottschalk, Rodgers and Watson worked with a manufacturer on improving the consistency of several critical dimensions of a part. One of their first steps toward process improvement was to do some gauge studies. An R&R study involving $I = 4$ parts, $J = 3$ operators, and $m = 3$ measurements of a particular dimension of each part by each operator (in undisclosed units) produced the values

$$\sqrt{MSE} = 38.46 \quad SSA = 10880 \quad SSB = 8855 \quad SSAB = 6343$$

for measurements made with a gauge used on the production line. Specifications on the dimension being gauged here were ± 10 (in the undisclosed units).

a) Without doing any calculations here at all, what does comparison of $\sqrt{MSE} = 38.46$ to the engineering specifications of ± 10 indicate about production-line measurement? (Use no more than 20 words.)

b) Give 95% confidence limits for the standard deviation that would result from many repeat measurements of a single part by a single operator. (Plug into a correct formula, but no need to simplify.)

c) Find an estimate of the standard deviation that would result from many operators each making a single measurement on the same part, in the absence of repeatability variation. (No need to simplify.)

d) It is possible to use the information given above to determine that $\hat{\sigma}_{\text{R\&R}} = 40.24$ and that the "Satterthwaite approximate degrees of freedom for R&R" are $\nu_{\text{R\&R}} = 21.81$. Use these facts and make approximate 95% confidence limits for the GCR (gauge capability ratio) here. (No need to simplify.)

e) Suppose that the actual/true values of the dimension of these parts that was the subject of this gauge study vary according to a standard deviation of $\sigma_x = 1.5$. Suppose that many of these parts are each measured once each by different operators (so that it makes sense to think of $\sigma_{\text{measurement}}$ being the same as $\sigma_{\text{R\&R}}$). In light of part d), what value do you expect to obtain as a standard deviation for these many measurements? (Give a number.)

2. A so-called resistance temperature device (RTD) can be used to measure temperature. Some calibration data were collected on one of these by placing it in a water bath with a high quality (NIST certified) thermometer and recording both temperature (x , in $^{\circ}\text{F}$ from the thermometer) and resistance (y , in ohms from the RTD) as the bath temperature was raised. A JMP linear regression printout for these data (including a plot with 95% prediction limits for y) is attached to this exam. Use it in what follows.

a) Give an estimate of standard deviation of RTD resistance you would expect to observe for repeated observations at $x = 43^{\circ}\text{F}$.

b) Tomorrow this RTD is used to determine temperature at a point in an HVAC system. If its resistance is read as $1600\ \Omega$, give 95% confidence limits for the actual temperature. Indicate carefully on the attached printout how you got this interval.

3. IE 361 students Bander, Brenner, Kerdus and Tierney worked with a manufacturer of plastic containers. Their primary concern was with "burst strength" of these containers (containers were pressurized until bursting and the percentage increase in volume at burst was recorded). Below are summary statistics from tests on 11 samples of $n = 3$ of these containers (that we will suppose were taken hourly).

Sample	1	2	3	4	5	6	7	8	9	10	11	
\bar{x}	16.16	16.60	16.87	16.17	16.27	15.90	17.53	18.00	16.37	17.20	17.00	$\sum \bar{x} = 184.07$
R	1.0	1.5	1.4	1.2	1.7	1.1	0.7	3.5	0.6	1.4	1.3	$\sum R = 15.4$

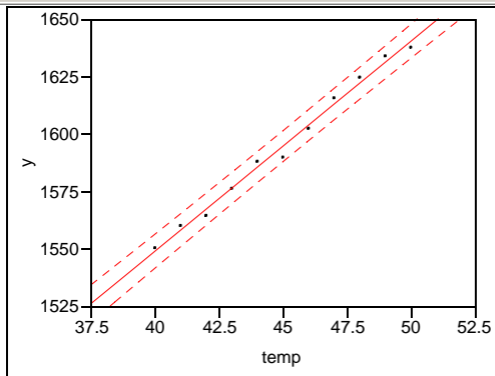
a) Find retrospective control limits for **both** means and ranges and apply them to the values in the table. **Is** there evidence of process instability in the values in the table? **Explain.**

b) Find an estimate of "process short term standard deviation" based on the values in the table above.

c) Why is it impossible to know in this context how much of your answer to part b) is due to real variation in container strength and how much is due to measurement error? (Use no more than 20 words.)

d) Suppose that in the future I wish to monitor burst strength using \bar{x} and s charts based on samples of size $n = 4$. (Note I'm changing both the type of chart I use for process short term variation and the sample size.) Propose sensible control limits for **both** these new charts based on the data in hand.

Bivariate Fit of y By temp



— Linear Fit

Linear Fit

$$y = 1183.2095 + 9.1396864 \text{ temp}$$

Summary of Fit

RSquare	0.992118
RSquare Adj	0.991242
Root Mean Square Error	2.848
Mean of Response	1594.495
Observations (or Sum Wgts)	11

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	9188.7254	9188.73	1132.858
Error	9	72.9999	8.11	Prob > F
C. Total	10	9261.7254		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	1183.2095	12.24971	96.59	<.0001
temp	9.1396864	0.271546	33.66	<.0001

1. Limestone blocks of length approximately $l = 1$ m, width approximately $w = .5$ m, and height approximately $h = .25$ m are used in a construction project. The actual sizes of the blocks and the density, d , of the limestone vary somewhat block to block. Suppose that the mean density of the blocks used in the project is $\mu_d = 2600$ kg/m³, while the standard deviation of density is $\sigma_d = 200$ kg/m³. If the standard deviations of block dimensions (length, width, and height) are all about $\sigma_l = \sigma_w = \sigma_h = .005$ m, find an approximation for the standard deviation of block weight

$$\text{weight} = l \cdot w \cdot h \cdot d$$

2. A packager of spices and seasonings checks moisture content of lots of pepper that it receives from a supplier. The company's specifications on moisture content are from 9% to 12%. Attached to this exam is a JMP report for measured moisture for $n = 22$ incoming pepper lots.

a) On what basis could you argue that it is plausible to treat moisture content for lots like these as normally distributed?

Henceforth assume that the supplier's process is stable as regards the moisture content of lots of pepper that it delivers.

b) Give limits that you are "95% sure" will bracket the measured moisture content of a new (23rd) lot.

c) Among the $n = 22$ lots tested, the smallest and largest measured moisture contents are respectively 9.30% and 11.51%. How "sure" can one be that 95% of all additional lots will have measured contents between these two values?

d) Give a single-number estimate of C_{pk} for measured moisture content.

e) Give 95% confidence limits for a capability index that measures (*not potential but*) current process performance.

3. A software vendor monitors the effectiveness of its development process by keeping track of both
 k = the number of new lines of code involved

and

X = the number of source code errors found

every time a new "build" of a program is attempted. Suppose that the vendor's historical records indicate that when all is well, it can nevertheless expect about 1 source code error per 100 lines of new code to remain at the build stage.

a) Tomorrow, a new build of a company program will be made involving 530 new lines of code. What are appropriate Shewhart control limits for both X and for the rate $X / 530$?

$$UCL_X = \underline{\hspace{2cm}}$$

$$UCL_{X/530} = \underline{\hspace{2cm}}$$

$$LCL_X = \underline{\hspace{2cm}}$$

$$LCL_{X/530} = \underline{\hspace{2cm}}$$

b) Suppose that (because of high turnover rate of programmers), the company is actually currently experiencing about 3 source code errors per 100 lines of new code at the build stage. Evaluate the probability that an "out of control" situation is signaled on the basis of tomorrow's attempt to build (using 530 new lines of code). (**Hint:** a Poisson distribution with a large mean can be treated as approximately normal (with mean and standard deviation of the original Poisson distribution).)

4. In a company analytical lab, the pH of a sample of a "standard" solution is measured once each morning with a particular pH meter. On $n = 9$ consecutive mornings the following values were obtained (in the order listed).

6.02, 5.97, 6.14, 6.16, 6.03, 6.11, 6.09, 6.27, 6.39

The simple standard deviation of these numbers is $s \approx .13$. There is a reasonably clear positive trend in the values that probably inflates this sample standard deviation. As a result, .13 is probably NOT a good value to use as representing variability in pH readings produced by this meter on a given day. What is a more sensible estimate of "standard deviation" to use here (that is not so strongly affected by the trend)? (Give a number and show an appropriate formula leading to it. A number alone is worth 0 points.)

5. $p = 4$ variables are to be monitored simultaneously. Suppose for simplicity that all 4 ideal means are 0, and that the standard covariance matrix is

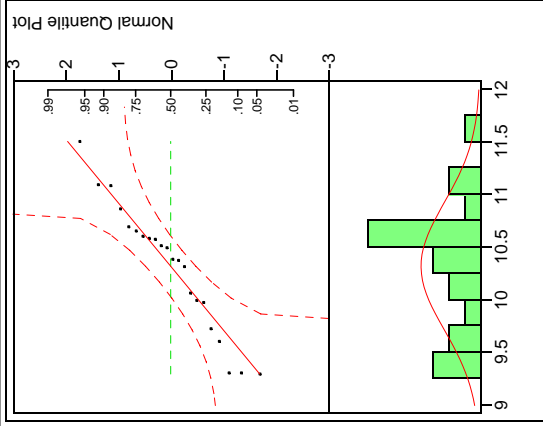
$$\mathbf{V} = \begin{pmatrix} 2 & 1 & -1 & -1 \\ 1 & 2 & -1 & -1 \\ -1 & -1 & 2 & 1 \\ -1 & -1 & 1 & 2 \end{pmatrix}$$

A vector of sample means is of the form $\bar{\mathbf{x}} = (+5, \pm 5, \pm 5, \pm 5)'$. Give a set of signs that produces the smallest possible value of X^2 . (Write a plus or minus sign in each blank. Note that the sign on \bar{x}_1 is assumed to be +.)

for smallest X^2 value possible: \bar{x}_2 sign _____, \bar{x}_3 sign _____, \bar{x}_4 sign _____

Distributions

Moisture



Quantiles

100.0%	maximum	11.510
99.5%		11.510
97.5%		11.510
90.0%		11.097
75.0%	quartile	10.670
50.0%	median	10.445
25.0%	quartile	9.918
10.0%		9.310
2.5%		9.300
0.5%		9.300
0.0%	minimum	9.300

Moments

Mean	10.324091
Std Dev	0.6054645
Std Err Mean	0.1290855
upper 95% Mean	10.592539
lower 95% Mean	10.055643
N	22

Stem and Leaf

Stem	Leaf	Count
11	5	1
11		
11	11	2
10	9	1
10	66677	5
10	4455	4
10	3	1
10	001	3
9		
9	67	2
9		
9	333	3

9|3 represents 9.3

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate
Location	Mu	10.32409
Dispersion	Sigma	0.60546

— Normal(10.3241,0.60546)

An IE 361 project group worked with a plastics manufacturer and experimented on an extrusion process. They conducted what was ultimately a (partially replicated) 2^{6-2} fractional factorial study aimed at optimizing a variable, y , that is a coded version of the amount of useful product produced per hour (you may think of this as process yield measured in some undisclosed units above some undisclosed baseline value). The 6 factors studied were:

- A- Bulk Density of Raw Material
- B- Amount of Water Added to the Raw Material Mix
- C- The Amperage Supplied to the Crammer-Auger
- D- Extruder Screw Speed
- E- Front-End Temperature
- F- Back-End Temperature

First consider an analysis of 4 of the conditions under which the machine was run. As regards factors C and D, these constituted a full 2×2 factorial study. The observed yields are below.

Set-up	Level of C	Level of D	Response(s), y	\bar{y}	s
1	-	-	13.99	13.990	
2	+	-	19.61	19.610	
3	-	+	24.94	24.940	
4	+	+	24.94, 25.21	25.075	.1909

a) Give 95% two sided confidence limits for the mean response for set-up 1. Then give 95% limits for the mean response for set-up 4. (No need to simplify.)

for set-up 1: _____ for set-up 4: _____

b) Do you judge there to be a statistically detectable difference between the mean production levels of set-ups 3 and 4? Explain, based on a *single* appropriate set of 95% confidence limits.

c) Treat the 4 set-ups listed in the table on page 1 as a 2^2 factorial in factors C and D. Find fitted main effects of C and D (c_2 and d_2) and the fitted CD interaction (cd_{22}) if one takes this point of view.

d) Which (if any) of the fitted/estimated effects you computed in part c) are "statistically significant"? (Base your answer on appropriate 95% confidence limits and explain.)

e) As a matter of fact, while levels of factors A,B, and E were held constant across set-ups 1 through 4 in listed in the table on page 1, set-ups 1 and 3 were made with the high (+) level of factor F and set-ups 2 and 4 were made with the low level (–) of factor F. Even limiting attention to the single set of levels of A,B and E represented in set-ups 1 through 4, this last fact (about factor F) should probably change your interpretation of the results of parts c) and d). Explain.

We will henceforth consider the whole 2^{6-2} experiment run by the students. As a matter of fact, they used the two generators

$$E \leftrightarrow AB \text{ and } F \leftrightarrow AC$$

f) What levels of E and F were used with the following combinations of A,B,C and D? (Fill in the blanks.)

Level of A	Level of B	Level of C	Level of D	Level of E	Level of F
+	+	+	–		
+	+	–	–		
+	–	–	–		

The students reported a "4 degree of freedom" pooled sample standard deviation of .37. Further, the sum of the reciprocals of their 16 sample sizes was 14.

g) A "fitted sum of effects" computed via the Yates algorithm based on the 16 sample means can be used to make 95% confidence limits of the form

$$\overbrace{\text{fitted sum of effects}} \pm \Delta$$

for an appropriate value Δ . Find this value.

Below is a table of 16 ABCDEF combinations with corresponding sample means, and a final column that comes from use of the Yates algorithm on the sample means.

Combination	\bar{y}	Output of the Yates Algorithm
ef	13.99	21.42
a	6.76	-1.43
bf	20.71	2.04
abe	11.12	.10
ce	19.61	2.50
acf	15.73	.81
bc	23.45	.68
abcef	20.00	.43
def	24.94	4.99
ad	24.53	1.59
bdf	24.97	-.36
abde	24.29	.34
cde	25.08	-.77
acdf	24.40	-.37
bcd	30.00	1.05
abcdef	33.08	.08

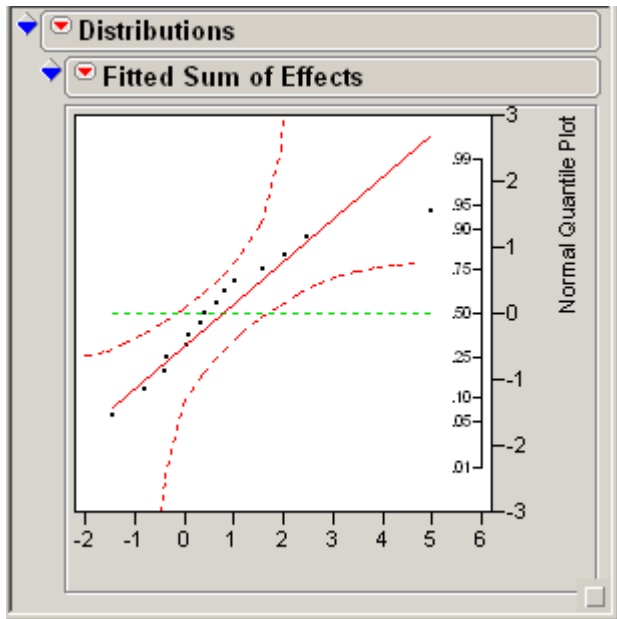
h) How many "cycles" of additions and subtractions and what final divisor were used to produce the last column in the above table?

number of cycles _____

final divisor _____

i) Circle values in the last column of the table that represent "statistically detectable sums" according to the criterion of part g).

As it turns out, the pooled sample standard deviation reported by the students is probably "too small" because their "replication" consisted of consecutive runs of the machine for a given set-up. Experience with the machine strongly suggested that for a given combination of levels of the factors A through F, consecutive runs for a given set-up were more homogeneous than were runs where the same combination of levels of A through F was used, but other (different) set-ups intervened between the runs. So the practical implications of part i) are suspect, and a second way of judging the statistical detectability of sums of effects was tried. A normal plot of the last 15 entries in the "Output of the Yates Algorithm" column in the table on the previous page was made. It is below.



j) On the basis of the plot above, what value(s) in the table on page 3 clearly represent statistically detectable sum(s) of effects? Explain.

k) What is the simplest possible interpretation of your answer to j)? If the object of running the machine is "large y " what does j) suggest about how the machine should be set up (which levels of which factors should be used)?

l) Say why, in retrospect, your answer to k) is clearly consistent with the list of sample means in the table on page 3.