IE 361 students Brown, Freyer, and Parayno worked on the placement of "center" buttons on some circular disks. They quantified the goodness of button placement by measuring \( y = \text{disk radial run-out} \) (in inches). The first step in the students' work was a measurement study.

a) One student took a single disk, placed it in the measuring device and measured \( y \). She then removed the disk, placed it a second time in the device, and measured \( y \). She repeated this (with the same disk) a total of 10 times. The values she obtained had \( R = .0045 \text{ in} \), \( s = .0012 \text{ in} \), and \( \bar{y} = .0350 \text{ in} \).

i) The values \( R \) and \( s \) measure what kind of variation? (Circle exactly one of the following):
- [ ] repeatability variation
- [ ] reproducibility variation
- [ ] both repeatability and reproducibility

ii) Here, \( R \) is several times \( s \). This should be no surprise. Explain why in 20 words or less.

For \( n = 10 \), \( MR = d_2 s = 3.078 \sigma \) while \( MS = c_4 \sigma = .9727 \sigma \). I expect \( R \) to be about \((3.078/0.9727) \times s \).

iii) Use the value of \( s \) above and find 90% confidence limits for the "population" standard deviation, \( \sigma \), corresponding to \( s \). (Plug correct numbers into a correct formula, but don't bother to simplify.)

\[
\left(0.012 \sqrt{\frac{10-1}{16.919}}, 0.012 \sqrt{\frac{10-1}{3.325}}\right) \text{ i.e. } (0.0009, 0.0020)
\]

b) Each of 15 company technicians individually placed a second disk (all used the same one) in the measuring device and measured \( y \). The sample standard deviation of the values they obtained was .0050 in.

i) In terms of the variances \( \sigma_\beta^2, \sigma_\alpha^2, \sigma_{\alpha\beta} \), and \( \sigma^2 \) that are parameters of the two-way random effects model often used to describe a standard gauge R&R study, the standard deviation above is an empirical approximation of what? (Give an expression involving some or all of these that .0050 in estimates.)

\[
\sigma_{\text{R&R}} = \sqrt{\sigma_\beta^2 + \sigma_\alpha^2 + \sigma^2}
\]

ii) If engineering specifications on run-out are 0 to .0600 in and one takes the .0050 in value as indicating "gauge R&R variability," what is an estimated gauge capability ratio (or "precision to tolerance ratio") here? Is it a "good" one? (Give a number and say whether or not it is favorable.)

\[
\hat{GCR} = \frac{6 \sigma_{\text{R&R}}}{U-L} = \frac{6(0.0050)}{0.0600-0} = s
\]

This is a terrible (estimated) GCR. We really want one that is no larger than .1 at worst.
At a later date, run-out values were measured once (by a fixed technician) on each of 20 other disks produced over a very short period. These values had a sample standard deviation of .0040 in.

**c)** Based on the .0040 in figure and the results of the measurement study, give an estimate of the actual disk-to-disk short term standard deviation of run-out for this process (not including measurement error).

\[
\hat{\sigma}_x = \sqrt{\max(0, s_y^2 - s^2)} = \sqrt{(0.0040)^2 - (0.0012)^2} = 0.0038 \text{ in}
\]

**d)** Your answer to c) is itself subject to uncertainty, that can be quantified in terms of a standard error (an estimated standard deviation). Give this standard error for your answer to c). (Plug in, but you need not simplify.)

\[
\text{Use } \sqrt{\frac{1}{2} \left( \frac{1}{s_y^2} - \frac{1}{s^2} \right) \left( \frac{s^4}{m-1} + \frac{s_y^4}{n-1} \right)}
\]

This is \[ \sqrt{\frac{1}{2} \left( \frac{1}{(0.0040)^2 - (0.0012)^2} \right) \left( \frac{(0.0012)^4}{10 - 1} + \frac{(0.0040)^4}{20 - 1} \right)} \]

\[ = 0.0007 \text{ in} \]

Suppose now that it is desirable to set up a process monitoring scheme for disk run-out. Samples of \( n = 5 \) disks will be taken hourly from production and run-out measured. Under a scenario in which there is a single technician that does all the measuring, a “stable process” model (stable production process and stable measuring process model) with “\( \sigma \)” of about .0040 in is perhaps appropriate. Suppose further that a standard mean run-out of \( \mu = 0.0350 \) in is realistic.

**e)** Find appropriate standards-given control chart limits for \( \overline{x} \) and \( s \) under this scenario.

\( \overline{x} \) chart:  
- Center line at \( \overline{x} = \mu = 0.0350 \)
- \( \text{UCL}_x = \mu + 3 \frac{s}{\sqrt{n}} = 0.0350 + 3 \cdot \frac{0.0040}{\sqrt{5}} = 0.0404 \)
- \( \text{LCL}_x = \mu - 3 \frac{s}{\sqrt{n}} = 0.0296 \)

\( s \) chart:  
- Center line at \( \overline{s} = \sigma = 0.0400 \cdot (0.0040) = 0.0038 \)
- \( \text{UCL}_s = B_6 \overline{s} = 1.864 \cdot (0.0040) = 0.0073 \)
- \( \text{LCL}_s = B_4 \overline{s} = 0 \) for this sample size.

\( \text{UCL}_x = 0.0404 \)  
\( \text{LCL}_x = 0.0296 \)  
\( \text{UCL}_s = 0.0073 \)  
\( \text{LCL}_s = \)
Here are \( \bar{x} \) and \( s \) values for run-outs from 10 consecutive periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>.0257</td>
<td>.0412</td>
<td>.0428</td>
<td>.0324</td>
<td>.0416</td>
<td>.0348</td>
<td>.0347</td>
<td>.0340</td>
<td>.0420</td>
<td>( \Sigma \bar{x} = .3615 )</td>
</tr>
<tr>
<td>( s )</td>
<td>.0052</td>
<td>.0048</td>
<td>.0010</td>
<td>.0046</td>
<td>.0077</td>
<td>.0044</td>
<td>.0050</td>
<td>.0028</td>
<td>.0030</td>
<td>( \Sigma s = .0401 )</td>
</tr>
</tbody>
</table>

f) What do the limits of part e) applied to the values above indicate about the stability of the (production plus measuring) process?

There are 5 "out of control" \( \bar{x} \) values. There is clear evidence of some kind of physical instability in the process.

\[
UCL_{\bar{x}} = \bar{x} + A_3 s = .03615 + (1.427)(.00401) = .0419 \\
LCL_{\bar{x}} = - .0304 \\
\]

There are still 3 "out of control" \( \bar{x} \) values. The fundamental conclusion remains unchanged.

\[
UCL_{\bar{x}} = .0419 \text{ in} \\
LCL_{\bar{x}} = - .0304 \text{ in} \\
\]

h) As a matter of fact, a different technician made the measurements in each of the 10 periods represented in the values in the table. How might this help explain the results in parts f) and g)?

Technician-to-technician differences in measurement technique could be the physical source of the "lack of control" evident in the \( \bar{x} \) values.

i) The ideal is to eliminate technician differences in measuring technique. But if it is impossible to do so, one can take them into account in setting up process monitoring schemes. If \( "\sigma" \) represents stable "production process plus single-technician measuring process" variability, and \( \sigma_{\text{reproducibility}} \) represents technician-to-technician variability, a sensible standard deviation for \( \bar{y} \) becomes \( \sqrt{\frac{\sigma_{\text{reproducibility}}^2 + \sigma^2}{n}} \). In light of this, what limits for an \( \bar{x} \) chart would use in place of limits from part e) if \( \sigma_{\text{reproducibility}} = .0049 \) in?

\[
UCL_{\bar{x}} = \mu + 3 \bar{x} = .0350 + 3 \sqrt{(.0049)^2 + \frac{(.0040)^2}{5}} = .0506 \text{ in} \\
LCL_{\bar{x}} = - .0194 \text{ in} \\
\]
1. A case study in *Creating Quality* by Kolarik concerns the monitoring of a die-casting process for a 35-
mm camera body. Over 20 shifts of production, a total of 2735 camera bodies were made and inspected. A
total of 215 nonconformities were identified on those 2735 bodies, and 97 of the bodies were judged to be
nonconforming. A retrospective analysis of the data from the 20 shifts found no evidence of process
instability over that period, and so these historical rates are taken as standard values.

a) In a subsequent shift, 150 camera bodies are produced. A total of 12 nonconformities are identified on
these, and 9 of the bodies are judged to be nonconforming. Do either of these counts provide clear evidence
of process change? (Compare the two counts to appropriate control limits and interpret your result.)

\[
\begin{align*}
\text{12 nonconformities} & \quad \hat{\lambda} = \frac{215}{2735} = 0.0786 \\
\hat{p} &= \frac{12}{150} = 0.08 \\
\hat{u} &= 3 \sqrt{\frac{0.08}{150}} = 0.0786 \pm 3 \sqrt{0.08} \\
\text{9 nonconforming} & \quad \hat{\lambda} = \frac{87}{2735} = 0.0355 \\
\hat{p} &= \frac{9}{150} = 0.06 \\
\hat{u} &= 3 \sqrt{\frac{0.08}{150}} = 0.0786 \pm 3 \sqrt{0.08} \\
\end{align*}
\]

\(\hat{u}\) is not out of control

Interpretation: There is no clear evidence of process change. Both values are inside control limits.

b) Suppose that at some point in the future, the standard nonconformity rate is reduced, production is
increased to 200 bodies per shift, and a process monitoring system is adopted that signals “out of
control”/“intervention warranted” if there are 3 or more nonconformities observed on a shift. Evaluate the
ARL of this monitoring system if in fact the nonconformity rate is 0.01 per camera body. Then interpret the
value in 15 words or less in the context of the application. (In case either is relevant, the binomial pmf is
\(f(x) = \binom{n}{x} p^x (1-p)^{n-x}\) and the Poisson pmf is \(f(x) = \frac{e^{-\lambda} \lambda^x}{x!}\).

\[
\begin{align*}
\text{"out of control" } \iff & \quad X \neq \# \text{ nonconformities } \geq 3 \\
q &= P[X \geq 3] = 1 - P[X \leq 2] = 1 - f(0) - f(1) - f(2) \\
\text{mean} = 2 \times (0.01) &= 2 \times 0.01 \\
\text{use Poisson } & \quad \text{ARL} = \frac{1}{q} = 3.09 \\
\end{align*}
\]

\(ARL = 3.09\)

Interpretation: If the rate is 0.01 per body there will be an "out of control" alarm on average every 3.09 shifts.
2. A hole with a circular cross section of diameter \( D \) is to be drilled in a rectangular metal block of thickness \( T \). While the hole axis is supposed to be perpendicular to the top face of the block, it may in fact make an angle of \( \theta \) (different from \( \pi/2 \)) with the faces of the block. This is pictured below.

![Diagram of a hole drilled in a block](image)

The amount of metal removed in drilling is then

\[
R = \left( \frac{\pi}{4} \right) \frac{D^2 T}{\sin \theta}
\]

In drilling, all of \( D, T \), and \( \theta \) are subject to variation. Suppose that means and standard deviations are \( \mu_D = 3 \) and \( \sigma_D = .3 \), \( \mu_T = 50 \) and \( \sigma_T = .5 \), while \( \mu_\theta = \frac{\pi}{2} \) and \( \sigma_\theta = .01\pi \) (the linear measures are in mm and the angle is in radians). Find an approximate standard deviation of \( R \).

\[
\frac{\partial R}{\partial D} = \left( \frac{\pi}{2} \right) \frac{DT}{\sin \theta} \quad \frac{\partial R}{\partial T} = \left( \frac{\pi}{4} \right) \frac{D^2}{\sin \theta} \quad \frac{\partial R}{\partial \theta} = -\left( \frac{\pi}{4} \right) \frac{D^2 T \cos \theta}{\sin^2 \theta}
\]

\[
\sigma_R \approx \sqrt{\left[ \left( \frac{\pi}{4} \right)^2 \frac{(3)^2}{\sin \pi/2} \cdot (1.5)^2 + \left( \frac{\pi}{2} \right) \frac{(50)^2}{\sin \pi/2} \cdot (0.3)^2 + \left( \frac{\pi}{4} \right)^2 \frac{(0.1\pi)^2}{\sin^2 \pi/2} \right]} (0.1)^2
\]

\[
= \sqrt{12.49 + 9,996.49} = 70.77 \text{ mm}^3
\]

3. The same air supply runs both heads of a pneumatic wrench used to simultaneously tighten two nominally identical bolts in the assembly of a hydraulic pump. Process standards for the torque required to loosen either of those bolts are \( \mu = 40 \text{ ft lbs} \) and \( \sigma = 2 \text{ ft lb} \), and the standard for correlation between the two torques is \( \rho = .8 \). A check of a single newly assembled pump produces a pair of measured torques of 37 ft lbs and 43 ft lbs. Does this clearly indicate a change from process standard values? (Show appropriate calculations.)

\[
\chi^2 = n \left( \frac{\bar{x}_i - \mu}{\sigma} \right)^2 \quad \chi^2 = 1 \left( \frac{37-40}{2} \right)^2 \cdot .8(2) = \frac{1}{16} \left( \frac{4(-3)^2 + 2(-3.2)(-3)(3)}{4} \right) + (37-40)^2
\]

\[
= \frac{1}{16} \left( \frac{4(-3)^2 + 2(-3.2)(-3)(3)}{4} \right) = \frac{1}{5.76} \left( \frac{4(-3)^2 + 2(-3.2)(-3)(3)}{4} \right)
\]

To be compared with NCL \( \chi^2 = 2 + 3\sqrt{2(2)} = 8 \) and we have \( 2 \) definitive evidence of process change.
4. Lengths of steel shafts are measured by a laser gauge that produces a coded voltage proportional to shaft length (above some reference length). In the units produced by the gauge, shafts of a particular type have length specifications 2300 ± 20. Below are measured lengths of n = 10 such shafts.

2298, 2301, 2298, 2289, 2291, 2290, 2290, 2287, 2280, 2289

Assume that these shafts were produced by a physically stable process.

a) How confident are you that an 11th shaft produced by this process would measure at least 2280? (Give a number.)

\[
\text{confidence level} = \frac{n}{n+1} = \frac{10}{11} = 90.9\%
\]

Henceforth assume that it is sensible to model shaft length as normally distributed. For the lengths listed above, \( \bar{x} = 2291.3 \) and \( s = 6.2 \).

b) Give 90% confidence limits for a process capability index that measures process potential.

\[
\text{Use limits (5.7)}
\]

\[
\left( \frac{40}{6(6.2)} \sqrt{3.325} \pm \frac{40}{6(6.2)} \sqrt{16.919} \right)
\]

\[
(0.65, 1.47)
\]

c) Give two-sided limits that you are 95% sure contain at least 99% of all shaft lengths.

\[
\text{Use limits (5.13)}
\]

\[
2291.3 \pm 4.437(6.2)
\]

\[
2291.3 \pm 27.5
\]

d) Give a 95% lower confidence bound for a process capability index that measures current process performance.

\[
\hat{C}_{pk} = \frac{2291.3 - 2280}{3(6.2)} = 0.6075
\]

\[
\text{Use limit (5.10)}
\]

\[
0.6075 - 1.645 \sqrt{\frac{1}{9(10)} + \frac{(0.6075)^2}{2(10)-2}}
\]

i.e. \( 0.315 \)
IE 361 Exam 3 Spring 2004 Key

Form A

1. e  
2. d  
3. d  
4. e  
5. b  
6. c  
7. e  
8. b  
9. e  
10. d  
11. d  
12. c  
13. d  
14. e  
15. b  
16. e  
17. c  
18. d  
19. e  
20. b

Form B

1. d  
2. e  
3. e  
4. d  
5. c  
6. c  
7. a  
8. d  
9. a  
10. a  
11. b  
12. c  
13. b  
14. a  
15. b  
16. d  
17. b  
18. e  
19. d  
20. d