

Fall 1998 IE 361 Exam Solutions

Exam 1

1.

$$(a) \hat{\sigma}_{R\&R} = \sqrt{\hat{\sigma}_{\text{repeatability}}^2 + \hat{\sigma}_{\text{reproducibility}}^2} = \sqrt{(1.05)^2 + (1.29)^2} = 1.66 \times 10^{-3} \text{ inch.}$$

(b) $G\hat{C}R = \frac{6\hat{\sigma}_{R\&R}}{U-L} = \frac{6(1.66 \times 10^{-3})}{4 \times 10^{-3}} = 2.49$ This is awful. We want GCR's like .1 or .01. This gage is NOT adequate to check conformance to these specifications. Measurement uncertainty more than uses up the allowable part variation. Using this gage, one is "just guessing" whether parts meet spec.s.

(c) With x = part "true diameter", y = part measured diameter, and ϵ = measurement error, the usual measurement model implies that $\sigma_y^2 = \sigma_x^2 + \sigma_{\text{measurement}}^2$ so that

$$\sigma_x^2 = \sigma_y^2 - \sigma_{\text{measurement}}^2 \text{ and thus that}$$

$$\sigma_x = \sqrt{\sigma_y^2 - \sigma_{\text{measurement}}^2} \approx \sqrt{(3.29)^2 - (1.05)^2} = 3.44 \times 10^{-3} \text{ inch.}$$

(d) For the \bar{x} chart, use display (3.7) with $\bar{\bar{x}} = .49765$ and $\bar{R} = .00615$.

$$UCL_{\bar{x}} = .49765 + (.577)(.00615) = .5012 \text{ and}$$

$$LCL_{\bar{x}} = .49765 - (.577)(.00615) = .4941$$

For the R chart, use display (3.18). $UCL_R = 2.115(.00615) = .0130$ with no LCL_R .

There is evidence of process instability/change. The first sample mean and first two sample ranges are outside control limits.

(e) This is NOT conclusive evidence that essentially all individuals are outside of spec.s. To have a mean outside of (L, U) requires at least one item in the sample outside of (L, U) , but nothing more. Spec.s refer to individuals!!! Do NOT apply them to means.

(f) For the \bar{x} chart, use display (3.5). $UCL_{\bar{x}} = .5020 + 3\frac{.0005}{\sqrt{4}} = .50275$ and

$$LCL_{\bar{x}} = .5020 - 3\frac{.0005}{\sqrt{4}} = .50125. \text{ For the } s \text{ chart, use display (3.23).}$$

$$UCL_s = 2.088(.0005) = .001044.$$

(g) \bar{x} has mean .5020 and standard deviation $.0010/\sqrt{4}$. $UCL_{\bar{x}}$ has

$$z = \frac{.50275 - .5020}{.0010/\sqrt{4}} = 1.5 \text{ and } LCL_{\bar{x}} \text{ has } z = -1.5. \text{ (In all actuality, one is using not a 3}$$

sigma chart, but a 1.5 sigma chart.) So using displays (3.46) and (3.47),

$$q = P[Z < -1.5 \text{ or } Z > 1.5] \approx 2(.0668) = .1336 \text{ and } ARL = 1/q = 7.49.$$

(h) This is a poor question. Actually the ARL s turn out to be exactly the same. If this problem involved a *mean* change instead of an increase in σ , the larger sample size would produce more information and sensitivity, and therefore a decreased ARL .

2.

(a) This is a p chart problem. Use display (3.34) and get $\hat{p}_{\text{pooled}} = .404$. Now use display (3.33). When $n = 30$, $UCL_{\hat{p}} = .404 + 3\sqrt{\frac{.404(.596)}{30}} = .673$ and $LCL_{\hat{p}} = .404 - 3\sqrt{\frac{.404(.596)}{30}} = .153$. When $n = 15$, $UCL_{\hat{p}} = .404 + 3\sqrt{\frac{.404(.596)}{15}} = .784$ and $LCL_{\hat{p}} = .404 - 3\sqrt{\frac{.404(.596)}{15}} = .024$. No \hat{p} 's are outside the control limits. There is no evidence of change in the detection rate.

(b) Surely the company was NOT satisfied with a 40% detection rate! Part (a) says only that the inspection process was consistent/stable. It does NOT guarantee that the process was satisfactory.

3.

(a) The thinking is "make the process work well and good product will result." On the other hand, no amount of post-production attention to *product* will make a process work better.

(b) Such a shape suggests that there are multiple versions of some process element "upstream." If those different versions can be identified and eliminated, overall variation reduction may result.

(c) It must involve physical intervention/action to eliminate assignable/special causes.

(d) $R_1 = 3, R_2 = 4, R_3 = 3, R_4 = 3$ so that $\bar{R} = 3.25$.

There are many possible estimates of a common σ . One of them is $\bar{R}/d_2(3) = 3.25/1.693 = 1.92$.

The "grand sample standard deviation" unfortunately includes *variation between the means* (i.e. it includes "group-to-group variation" in addition to "within-group-variation").

Exam 2

1.

(a) $\frac{i-.5}{20} = .25$ implies that $i = 5.5$, so that $Q(.25)$ is the "5.5th ordered x data point," i.e. $Q(.25) = .49425$. $\frac{i-.5}{20} = .75$ implies that $i = 15.5$ so that $Q(.75) = .49675$. $Q(.5) = .496$. These form the left and right edges of the box and middle dividing line of the box. $IQR = .0025$ and there are no points more than $1.5IQR$ from the "box," so that "whiskers" extend down to .4925 and up to .498.

(b) No measurements are in specifications. The process is mis-aimed to the low side. The process "spread" appears to be so large that even with re-aiming it will not be possible to get all parts in spec.s.

(c) This is a very linear-looking plot. It suggests that a normal model is a reasonable one. (.4925, -1.96) and (.4940, -1.44).

(d) Use display (5.11), i.e. $.49567 \pm 1.729(.00151)\sqrt{1 + \frac{1}{20}}$ (i.e. $\pm .00268$.)

(e) Use display (5.10). $\hat{C}_{pk} = \frac{.004 - 2|.49567 - .5021|}{6(.00151)} = -.9559$. Thus the limit is $-.9559 - 1.282\sqrt{\frac{1}{9(20)} + \frac{(-.9559)^2}{2(20)^2}} = -1.18$.

(f) $\frac{\overline{MR}}{d_2(2)} = \frac{.0016}{1.128} = .00142$ This is not much different from $s = .00151$. The sample standard deviation doesn't seem to be much inflated by movement of the mean. The assumption seems plausible.

(g) Use display (4.5) with $\mu_Q = .502$ and $\sigma_Q = \sigma_{\bar{x}} = \frac{.00151}{\sqrt{2}}$. So

$$UCL_{EWMA} = .502 + (2.70)\frac{.00151}{\sqrt{2}}\sqrt{\frac{.1}{1.9}} = .50266 \text{ and}$$

$$LCL_{EWMA} = .502 - (2.70)\frac{.00151}{\sqrt{2}}\sqrt{\frac{.1}{1.9}} = .50134$$

(h) $r = .618$ This positive correlation says that x_1 and x_2 tend to be "big together" or "small together." That suggests that as feature *size* varies, the *shape* does **not** vary.

2. $V = LWD$ Use display (5.27).

$$\begin{aligned} \text{Var}V &\approx \left(\frac{\partial V}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial V}{\partial W}\right)^2 \sigma_W^2 + \left(\frac{\partial V}{\partial D}\right)^2 \sigma_D^2 = (4.5)^2(.01)^2 + (10.5)^2(.01)^2 + (10.4)^2(.01)^2 \\ &= 4.5 \text{ So } \sigma_V \approx \sqrt{.45} = .67 \text{ ft}^3 \end{aligned}$$

3.

(a) Plot the \bar{y} 's above the nominal resistances and connect the metal resistor values, then connect the carbon resistor values.

(b) $s_{\text{Pooled}}^2 = \frac{(10-1)((.16)^2 + (.26)^2 + (.48)^2) + (5-1)((.23)^2 + (.67)^2 + (.24)^2)}{3(10-1) + 3(5-1)} = .132$ so $s_{\text{Pooled}} = .363$ with $d.f. = 27 + 12 = 39$

(c) Use display (6.9). For carbon resistors, since $n_{ij} = 10$, this is approximately $2.022(.363)\frac{1}{\sqrt{10}} = .23$. For metal resistors, since $n_{ij} = 5$, this is approximately $2.022(.363)\frac{1}{\sqrt{5}} = .33$.

(d) Use display (6.25). $\hat{L} = -1.95 - .02 = -1.97$ Then limits are $-1.97 \pm 2.022(.363)\sqrt{\frac{1}{9}\left(\frac{3}{10} + \frac{3}{5}\right)}$ i.e. $-1.97 \pm .23$

There is a statistically detectable difference in main effects of resistor type. Carbon resistors have mean signed % difference from nominal substantially below the metal resistors. (In fact, the carbon resistors are consistently about 2% below nominal resistance.)

Exam 3

1.

(a) In looking for a maximum mean response, it may be adequate to simply "turn the A knob" until large (mean) response is reached.

In order to produce consistent y , it appears that one needs to have a constant/consistent level of A. (This suggests monitoring/control of A.)

(b) $I \leftrightarrow ABD \leftrightarrow BCE \leftrightarrow ACDE$. Combinations to include in the study are: de, ae, b, abd, cd, ac, bce, abcde

(c) One is 1) relying upon the (exact) appropriateness of the Gaussian/normal model (especially in the *tails* of the distribution of individuals) and 2) trying to treat all (μ, σ) pairs with a given $p(\mu, \sigma)$ equally.

(A further problem not really discussed in the text is that one is essentially assuming that there is ABSOLUTELY NO measurement error ... the possibility of even the tiniest measurement error destroys the possibility of treating all (μ, σ) pairs with a given $p(\mu, \sigma)$ equally!)

(d) Doing X may not be a bad thing. More Y is not innovation, while X may be innovative. Besides, we may have talent and interest for X and not Y.

2.

(a) Since all the sample sizes are the same, $s_p^2 = \frac{1^2+2^2+1^2+5^2+3^2+1^2+3^2+4^2}{8} = \frac{66}{8} = 2.87$ with $d.f. = 8(4 - 1) = 24$.

(b) Use equation (6.9) to estimate μ_c . For 95% two-sided limits (for example) this is $22 \pm 2.064 \frac{2.87}{\sqrt{4}}$ i.e. 22 ± 2.96 .

(c) Use the Yates algorithm. $\bar{y}_{...} = 25.0, a_2 = .5, b_2 = 4.0, ab_{22} = -.5, c_2 = -.5, ac_{22} = 0, bc_{22} = -2.0, abc_{222} = .5$

combination	\bar{y}	cycle #1	cycle #2	cycle #3	cycle #3/8
(1)	18	39	102	200	25
a	21	63	98	4	.5
b	32	45	2	32	4.0
ab	31	53	2	-4	-.5
c	22	3	24	-4	-.5
ac	23	-1	8	0	0
bc	26	1	-4	-16	-2.0
abc	27	1	0	4	.5

(d) Use equation (6.31) to estimate effects. For individual 95% two-sided confidence limits (for example) this is $\hat{E} \pm 2.064(2.87)^{\frac{1}{8}}\sqrt{\frac{8}{4}}$ i.e. $\hat{E} \pm 1.05$. By this standard, only the B main effects and BC two-factor interactions are statistically detectable.

(e) $\hat{y} = \bar{y}_{...} + b_2 + bc_{22} = 25 + 4 + (-2) = 27$

3.

(a) Gupta's plan confounds/aliases some 2 factor interactions with other 2 factor interactions (and some main effects with 3 factor interactions). The plan from class confounds/aliases 2 factor interactions with 3 factor interactions (and main effects with 4 factor interactions).

(b) $A < C < E < B < D$ (since $.187 < 1.062 < 2.063 < 2.437 < 7.437$)

Since the E main effect is aliased with the ABC 3 factor interaction, we consider that estimated sum of effects. Then assuming that any estimate of a sum involving a main effect represents primarily that main effect, we get the order above.

(c) The "fat pencil rule" suggests that only the -7.437 value is clearly estimating something beyond experimental noise. This is the D main effect (plus its aliases).

(d) To produce small y , use the 2nd level of D, i.e. .6% aerosol. The fitted/predicted y (assuming all other effects to be 0 and -7.437 is estimating primarily δ_2) is $\hat{y} = 37.563 + (-7.437) = 30.126$.

(e) No, there is no need to change A, B or D when switching levels of C. Only the D main effects is detectable (and in particular, there are no detectable interactions with C that could change this picture).

4. Use equations (8.17) and (8.18). $n \approx (1.645)^2 \left(\frac{(.01)(.99)}{(.005)^2} \right) = 1,072$ and $c \approx 1,072(.005) \approx 5$.