

1. IE 361 students Ives, Boldt, Cookingham and Ihlefeld did a gage R&R study on an optical gage used to measure a diameter in injection molded plastic parts of a certain type. They estimated  $\sigma_{\text{repeatability}}$  to be  $1.05 \times 10^{-3}$  inches and  $\sigma_{\text{reproducibility}}$  to be  $1.29 \times 10^{-3}$  inches (based on  $I = 15$  parts,  $J = 2$  operators and  $m = 3$  measurements for each part  $\times$  operator combination).

(a) One of many possible company operators is going to measure a particular part of interest one time. Give a standard deviation to attach to the resulting measurement as describing "overall" measurement uncertainty in the part's true diameter.

(b) Specifications on the diameter of interest were in fact  $.502 \text{ inch} \pm .002 \text{ inch}$ . Compute and interpret an estimated gage capability ratio here. Do you think the optical gage is adequate to check conformance to these specifications? Explain.

(c) Suppose that in a later study one confines attention to a single operator (so that "measurement" variation consists entirely of "repeatability" variation) and that single measurements on each of 20 parts produces a sample standard deviation of  $3.59 \times 10^{-3}$  inches. Now this figure represents BOTH "part variation" and measurement variation. Give an estimated standard deviation describing differences between parts only (not inflated by measurement variation).

The following table gives means and ranges of measured diameters for 10 samples of  $m = 5$  of the parts taken at roughly 2 hour intervals.

| sample    | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\bar{x}$ | .5044 | .4992 | .4975 | .4962 | .4954 | .4989 | .4968 | .4957 | .4976 | .4948 |
| $R$       | .0145 | .0160 | .0025 | .0010 | .0115 | .0065 | .0025 | .0025 | .0020 | .0025 |

( $\sum \bar{x}_i = 4.9765$  and  $\sum R_i = .0615$  here.)

(d) Find retrospective control limits for the 10 sample means and ranges. What do they indicate about the stability of the injection molding process during the study period?

$\bar{x}$  chart limits:

$R$  chart limits:

what is indicated about the molding process:

(e) Note that none of the  $\bar{x}$  values in the table are inside the engineering specifications of  $.5020 \pm .0020$  mentioned in part (b) of this problem. Is that conclusive evidence that essentially none of the parts from this period have diameters inside specifications? Explain.

(f) If (after substantial attention) the injection molding process is brought to stability with  $\mu = .5020$  and  $\sigma = .0005$ , what are appropriate control limits for  $\bar{x}$  and  $s$  based on  $n = 4$ ?

limits for  $\bar{x}$ :

limits for  $s$  :

(g) What is the ARL for your  $\bar{x}$  chart from (f) if the process mean remains on target at .5020, but the standard deviation degrades to  $\sigma = .0010$ ?

(h) If the sample size in parts (f) and (g) had been  $n = 5$  instead of  $n = 4$ , would your ARL in (g) have been bigger or would it have been smaller? Say which and explain.

2. IE 361 students Boulaevskaia, Fair and Seniva did a study of "defect detection rates" for the visual inspection of some glass vials. Vials known to be visually identifiable as defective were marked with invisible ink, placed among other vials, and run through a visual inspection process at 10 different time periods. The numbers of marked defective vials that were detected/captured, the numbers placed into the inspection process, and the corresponding ratios for the 10 periods are below.

|                                |    |     |    |    |     |     |     |     |    |     |
|--------------------------------|----|-----|----|----|-----|-----|-----|-----|----|-----|
| $X =$ number detected/captured | 6  | 10  | 15 | 18 | 17  | 2   | 7   | 5   | 6  | 5   |
| $n =$ number placed            | 30 | 30  | 30 | 30 | 30  | 15  | 15  | 15  | 15 | 15  |
| $X/n$                          | .2 | .33 | .5 | .6 | .57 | .13 | .47 | .33 | .4 | .33 |

(Overall, 91 of the 225 marked vials placed into the inspection process were detected/captured.)

(a) Carefully investigate (and say clearly) whether there is evidence in these data of instability in the defect detection rate.

(b)  $91/225 = .404$ . Do you think that the company these students worked with was likely satisfied with the 40.4% detection rate? What, if anything, does your answer here have to do with the analysis in (a)?

3. Miscellaneous short answer. (Write no more than 2 or 3 short sentences per question!)

(a) Why are most modern quality assurance programs "process-oriented"?

(b) What (relevant to quality assurance efforts) does a multimodal shape of a histogram for a part dimension suggest?

(c) An effective real application of control charting must involve *what* beyond the simple plotting of points and the naming of those outside of control limits?

(d) Below are 4 samples of size  $n = 3$ . Suppose that they are all from normal populations with the same standard deviation ( $\sigma$ ) and give any sensible estimate of that standard deviation based on all 4 samples. If one simply ignores the fact that there are 4 samples and computes " $s$ " based on 12 data points, the value 5.66 is obtained. This should be much different from your answer. Why?

3, 4, 6

14, 10, 12

1, 4, 1

15, 13, 16

estimate of a common  $\sigma$ :

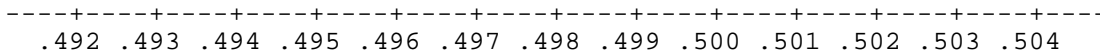
reasoning:

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Prof. Vardeman

1. An important feature of an injection molded part consists of "a circle with a flat bottom."  $n = 20$  of these parts produced under what we will assume were stable process conditions were inspected and  $x_1 =$  the circle diameter (in inches) and  $x_2 =$  distance from the flat bottom to the top of the circular arc (in inches) were measured. Attached to this exam are some parts of a Minitab analysis of the resulting data pairs. Use these in what follows.

(a) Make a box plot for the  $x_1$  values above the scale below.



(b) Specifications on  $x_1$  were  $.502 \pm .002$ . What, in rough/qualitative terms, does your plot in (a) say about how this process is doing relative to these engineering requirements on the parts?

(c) There is a Minitab-generated normal plot of the  $x_1$  values in the analysis attached to this exam. Say what this plot indicates about the shape of the diameter distribution. If you needed to make a normal plot "by hand" (**plotting 20 points**) using regular graph paper, what would be the coordinates for the two "lower left" points on your plot?

Interpretation of Plot:

Coordinates of 2 Hand-Plotted Points:

\_\_\_\_\_ , \_\_\_\_\_  
 \_\_\_\_\_ , \_\_\_\_\_

(d) Give 90% two-sided prediction limits for a single additional value of  $x_1$  from this process as it was running during the collection of these data. (No need to simplify.)

(e) Give an approximate 90% lower confidence bound for  $C_{pk}$  for the dimension  $x_1$ . (Again, no need to simplify.)

(f) As a matter of fact, the average moving range of the  $n = 20$  consecutive  $x_1$  values in the data set here was  $\overline{MR} = .0016$ . Compare an estimate of the "process sigma" derived from this figure to the sample standard deviation of the  $x_1$  values. What does this comparison suggest about the plausibility of the "stable process" assumption made at the outset of this problem? Explain.

(g) Suppose that one wishes to do EWMA monitoring of averages of  $n = 2$  values  $x_1$  using  $\lambda = .1$  and an "all-OK" ARL of about 370. If the sample standard deviation of these 20 values  $x_1$  is a reliable estimate of the "process sigma" for this variable, what control limits should be used? (Recall that the mid-specification is .502.)

$$LCL_{EWMA} = \underline{\hspace{2cm}}$$

$$UCL_{EWMA} = \underline{\hspace{2cm}}$$

(h) Consider the value of the sample correlation between  $x_1$  and  $x_2$  given on the printout. What does it indicate about the "shapes" of the parts. (Do you expect "big" ones to have a different appearance than "small" ones? Or as size varies do you expect shape to remain roughly the same?) Explain.

2. Tanks of a certain design are nominally  $10 \times 4 \times 5$  (the units of measure are feet). Manufacturing variation means that

$L =$  tank length has  $\mu_L = 10$  and  $\sigma_L = .01$

$W =$  tank width has  $\mu_W = 4$  and  $\sigma_W = .01$

and  $D =$  tank depth has  $\mu_D = 5$  and  $\sigma_D = .01$ .

If it is sensible to think of the tank dimensions as independent, find an approximate standard deviation to associate with the volume of the tank,  $V$ .

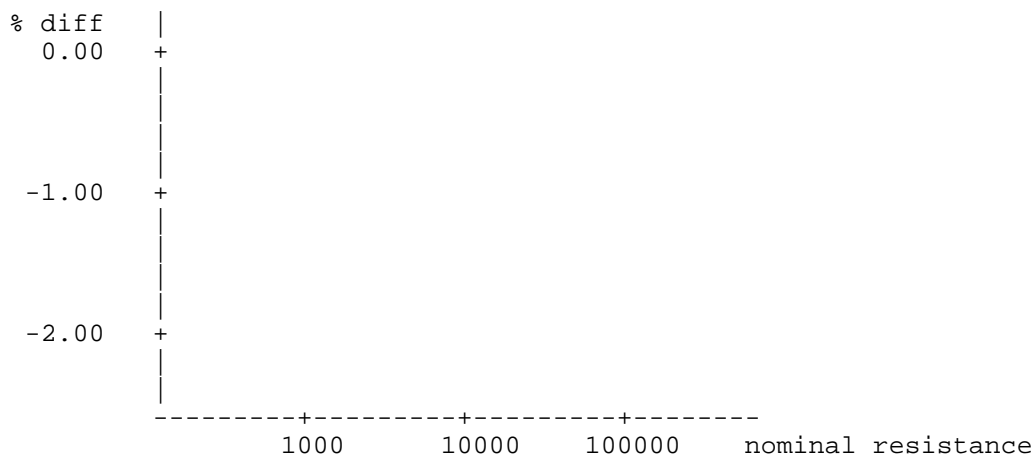
3. An ISU student group measured resistances of some commercially available resistors of two different types (carbon and metal) and three different nominal resistances ( $1000\Omega$ ,  $10000\Omega$  and  $100000\Omega$ ). They recorded for each resistor

$$y = (\text{signed}) \text{ percent difference from nominal}$$

(so that a nominally  $1000\Omega$  resistor that measures  $980\Omega$  has  $y = -2$ ). Below is a summary of their data.

|        | 1000 $\Omega$          | 10000 $\Omega$         | 100000 $\Omega$        |                        |
|--------|------------------------|------------------------|------------------------|------------------------|
| Carbon | $n_{11} = 10$          | $n_{12} = 10$          | $n_{13} = 10$          | $\bar{y}_{1.} = -1.95$ |
|        | $\bar{y}_{11} = -2.13$ | $\bar{y}_{12} = -2.26$ | $\bar{y}_{13} = -1.46$ |                        |
|        | $s_{11} = .16$         | $s_{12} = .26$         | $s_{13} = .48$         |                        |
| Metal  | $n_{21} = 5$           | $n_{22} = 5$           | $n_{23} = 5$           | $\bar{y}_{2.} = .02$   |
|        | $\bar{y}_{21} = .03$   | $\bar{y}_{22} = .02$   | $\bar{y}_{23} = .01$   |                        |
|        | $s_{21} = .23$         | $s_{22} = .67$         | $s_{23} = .24$         |                        |
|        | $\bar{y}_{.1} = -1.05$ | $\bar{y}_{.2} = -1.12$ | $\bar{y}_{.3} = -.725$ |                        |

(a) Sketch an interaction plot for these data on the axes below.



(b) Find a pooled estimate of a common "within cell" standard deviation of  $y$ . What degrees of freedom should be associated with it?

estimate = \_\_\_\_\_

$d.f.$  = \_\_\_\_\_

(c) Suppose one determines to use 95% (individual) two-sided confidence limits to put "error bars" around the means on the interaction plot. What  $\pm$  values are appropriate for first the carbon resistors and then for the metal resistors?

carbon " $\pm$  value" = \_\_\_\_\_

metal " $\pm$  value" = \_\_\_\_\_

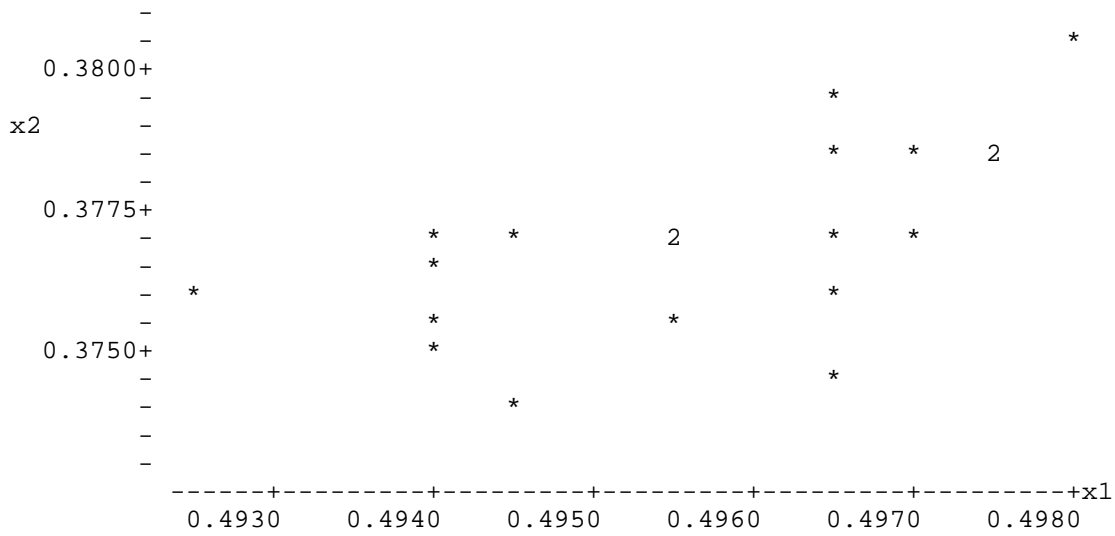
(d) Make and interpret a 95% two-sided confidence interval for the difference in resistor type main effects,  $\alpha_1 - \alpha_2$ . (Note that all the coefficients in the " $L$ " are  $\pm \frac{1}{3}$ .)

computation of limits:

interpretation of the result (both in terms of "statistics" and in terms of the original problem):

MTB > plot c1 c2

**Character Plot**



MTB > describe c1 c2

**Descriptive Statistics**

| Variable | N  | Mean    | Median  | TrMean  | StDev   | SEMean  |
|----------|----|---------|---------|---------|---------|---------|
| x1       | 20 | 0.49567 | 0.49600 | 0.49572 | 0.00151 | 0.00034 |
| x2       | 20 | 0.37695 | 0.37700 | 0.37692 | 0.00168 | 0.00038 |

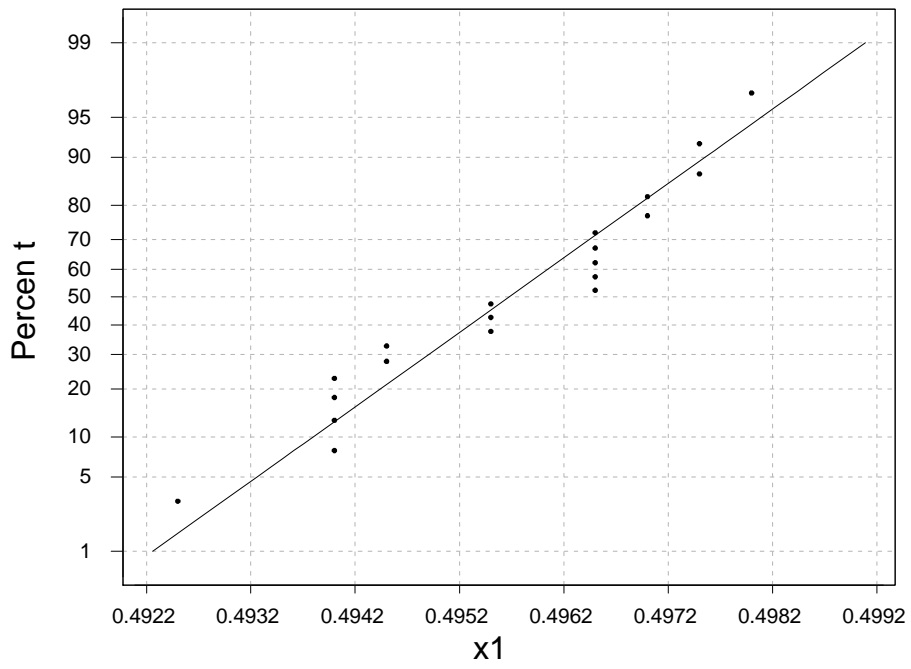
| Variable | Min     | Max     | Q1      | Q3      |
|----------|---------|---------|---------|---------|
| x1       | 0.49250 | 0.49800 | 0.49413 | 0.49687 |
| x2       | 0.37400 | 0.38050 | 0.37562 | 0.37850 |

MTB > corr c1 c2

**Correlations (Pearson)**

Correlation of x1 and x2 = 0.618

Normal Plot for x1



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Prof. Vardeman

1. Miscellaneous Short Answer

(a) Suppose that in an experiment on an industrial process to determine the effects of factors A, B, C and D on a response  $y$ , one determines that factor A is the most important determiner of response.

Briefly say how this information might be of practical use

if one wishes to maximize  $y$ :

if one wishes to reduce variation in  $y$ :

(b) Suppose that one determines to carry out a  $\frac{1}{4}$  fraction of a full  $2^5$  experiment, using the generators  $D \leftrightarrow AB$  and  $E \leftrightarrow BC$ . Write out the defining relation for this study and list (in Yates standard order for factors A through C) which 8 combinations of high and low levels of factors A through E will be run.

Defining relation:

$$I \leftrightarrow \underline{\hspace{1cm}} \leftrightarrow \underline{\hspace{1cm}} \leftrightarrow \underline{\hspace{1cm}}$$

Combinations included in the study:

(c) Variables acceptance sampling attempts to take measurements  $x_1, x_2, \dots, x_n$  on a sample from a lot and use them to decide whether the fraction of all measurements in the lot outside of specifications is small (and acceptable) or large (and unacceptable). What are two of the main limitations of the methods for variables acceptance sampling presented in Section 8.2?

(d) What do you say to a "TQM zealot" who says to you "Our company is trying to do X, while leaders in our industry are all doing Y! Clearly we must change!""? (Is this person surely right?)



(e) Based on your answer to (d) what value of  $y$  do you predict for a future run of this process with all factors set at their high levels? (Something other than  $\bar{y}_{abc}$  is needed here.)

3. The article "Establishing Optimum Process Levels of Suspending Agents for a Suspension Product" by A. Gupta, that appeared in *Quality Engineering* Vol. 10, discussed an unreplicated  $2^{5-1}$  fractional factorial experiment. The experimental factors and their levels in the study were:

|                          |                               |
|--------------------------|-------------------------------|
| A- Method of Preparation | Usual ( - ) vs Modified ( + ) |
| B- Sugar Content         | 50% ( - ) vs 60% ( + )        |
| C- Antibiotic Level      | 8% ( - ) vs 16% ( + )         |
| D- Aerosol               | .4% ( - ) vs .6% ( + )        |
| E- CMC                   | .2% ( - ) vs .4% ( + )        |

and the response variable was

$y$  = separated clear volume (%) for a suspension of antibiotic after 45 days

and the manufacturer hoped to find a way to make  $y$  small. The author of this paper failed to follow Vardeman's recommendation for choosing a best half fraction of the  $2^5$  factorial, and used the generator

$$E \leftrightarrow ABC$$

(instead of the "better one"  $E \leftrightarrow ABCD$  recommended in class and in the text).

(a) In what sense was the experimental plan used by Gupta inferior to the one prescribed in class? (How is the one from class "better"?)

The Yates algorithm applied to the 16 responses given in the paper produced the 16 fitted sums of effects:

|                  |         |                  |         |
|------------------|---------|------------------|---------|
| $mean + alias =$ | 37.563  | $D + alias =$    | - 7.437 |
| $A + alias =$    | .187    | $AD + alias =$   | .937    |
| $B + alias =$    | 2.437   | $BD + alias =$   | .678    |
| $AB + alias =$   | .312    | $ABD + alias =$  | .812    |
| $C + alias =$    | - 1.062 | $CD + alias =$   | 1.438   |
| $AC + alias =$   | .312    | $ACD + alias =$  | .062    |
| $BC + alias =$   | - 1.187 | $BCD + alias =$  | .062    |
| $ABC + alias =$  | - 2.063 | $ABCD + alias =$ | - .062  |

A normal plot of the last 15 of these fitted sums is attached to this exam.

(b) If you had to guess (based on the results of this experiment) the order of the magnitudes of the **5 main effects** (A, B, C, D and E) from smallest to largest, what would you guess? Explain.

\_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_

Explanation:

(c) Based on the normal plot, which sums of effects do you judge to be statistically detectable? Explain.

(d) Based on your answer to (c), how do you suggest that suspensions of this antibiotic be made in order to produce small  $y$ ? What mean  $y$  do you predict if your recommendations are followed?

(e) Actually, the company that ran this study planned to make suspensions using both high and low levels of antibiotic (factor C). Does your answer to (c) above suggest that the company needs to use different product formulations for the two levels of antibiotic? Explain.

4. Design an attributes acceptance sampling plan whose OC curve drops in the vicinity of  $p = .005$  and has  $Pa \approx .05$  at  $p = .010$ . (Give  $n$  and  $c$ .)

Normal Plot of Fitted Sums of Effects

