1. A data set in Brownlee's *Statistical Theory and Methods in Science and Engineering* was collected to compare two methods of measuring blood flow. Flows $x$ were measured by a "gold standard" direct method and corresponding values $y$ were measured by a new indirect method (units were not given). $n = 18$ pairs $(x, y)$ produced

$$\hat{y} = 15.4 + .975x \quad \text{with} \quad \sqrt{MSE} = 56.4 \quad \text{for} \quad \bar{x} = 2165 \quad \text{and} \quad \sum_{i=1}^{18} (x_i - \bar{x})^2 = 5,024,850$$

Give an approximate 90% confidence interval for the blood flow $x$ corresponding to an additional indirect measurement $y_{new} = 3000$. (No need to simplify.)

2. IE 361 students Bander, Brenner, Kerdis and Tierney studied a testing machine used by a manufacturer to monitor the strength of plastic beverage bottles. Roughly, a bottle is pressurized to a fixed high level and the percent expansion it undergoes, $y$, is measured.

(a) Why is it probably impossible to really determine the "repeatability" of this testing device?

(b) The students tested (in a randomly determined order) each member of a batch of 50 bottles produced under what the manufacturer believed to be the most homogeneous physical conditions possible. How would you have interpreted a plot of measured percent expansion against testing order if it showed a strong trend? (What cause do you suspect?)

c) In fact, a plot of measured percent expansion versus testing order showed no obvious trends or patterns. The measurements gave $\bar{y} = 13.73\%$ and $s_y = .56\%$. Suppose that the maker of the tester claims that the measurement error of the tester can be characterized by a standard deviation of $\sigma = .1\%$. Accepting the maker's claim, what do you estimate to be the standard deviation of actual bottle strengths, $\sigma_x$?
3. IE 361 students Chen, Susanto and Wu worked on process monitoring for a machined dimension of a high precision metal part. Engineering specifications on this dimension were .3765 ± .0005 inch. The "process σ" (including both part to part and measurement variation) for parts produced over a short period was around .00021 inch.

(a) Find standards given control limits for ranges of samples of $n = 4$ measured part dimensions.

(b) Find standards given control limits for sample means of $n = 4$ measured part dimensions.

(c) How, if at all, are the limits from (b) related to or to be compared to the specifications mentioned in the introduction to this problem?

(d) If in fact the process mean changes from the ideal value of .3765 to a point where 10% of measurements are above the upper specification, how many samples of size 4 do you expect to have to take in order to detect this change?
4. IE 361 students Klocke, Tan and Chai worked with the ISU Press on monitoring and improving the operation of a large collator machine. Under what they judged to be their best set of operating conditions, the machine experienced 0.0111 jams per second of running time. (Running time doesn't include the downtime associated with fixing the jams, so every instant of running time could be thought of as providing an opportunity for a jam to occur.)

Suppose that on the next 2 jobs run on this machine there are first 11 jams in 490 seconds, and then 15 jams in 474 seconds. Do either of these results provide clear evidence of a change from the standard jam rate? Show some calculations and explain clearly. (You might think of a "second of running time" as a "unit of product").

5. A cutting tool on a lathe wears at a rate of about 0.0001 inch per part cut. This lathe is controlled by a computer that automatically adjusts the tool position "in" by 0.0001 inch every 10th part cut. When approached about monitoring part diameters via "SPC" a colleague says that the computer is already doing SPC on the lathe. What do you have to say to this person?

6. What is the purpose of Shewhart control charting?

7. You've just been hired as an IE in charge of the quality assurance effort at Company V. You find that there is no one at Company V responsible for gage maintenance. In fact there is no budget for that activity at all. What do you say to your new boss about this circumstance?
1. Below is a two-dimensional sketch of a mass-produced part that has 2 holes drilled in it. The ideal positions of the holes are indicated by the grey-filled circles and the actual positions (as drilled) are indicated by the empty circles.

On a coordinate system with origin at the ideal position of hole A, ideal A is at $(0, 0)$ and ideal B is at $(5, 0)$. Suppose hole A is actually positioned at $(x_1, y_1)$ and that hole B is at actually at $(5 + x_2, y_2)$. (The numbers $x_1$ and $x_2$ are horizontal positioning errors and the numbers $y_1$ and $y_2$ are vertical positioning errors.) With $u = x_2 - x_1$ and $v = y_2 - y_1$

the actual distance between the positions of holes A and B is

$$D = \sqrt{(5 + u)^2 + v^2}$$

(a) A design engineer believes that the holes can be drilled with $x_1$ and $x_2$ independent, with respective standard deviations $\sigma_{x_1} = \sigma_{x_2} = .01$. What does this person then expect will be the standard deviation of $u = x_2 - x_1$?

$$\sigma_u = \sigma_{x_1-x_2} = \text{____________________}$$

(b) Suppose that $u$ and $v$ can be described as independent variables (with 0 means and) standard deviations $\sigma_u = \sigma_v = .02$. Find an approximate standard deviation for $D$.

$$\sigma_D \approx \text{____________________}$$
The above parts a) and b) of this problem are based on theoretical/"what if" calculations. The rest of the problem deals with empirical data from actual manufactured parts. \( n = 20 \) actual parts are made and values of \( D \) determined using highly precise and accurate gauging. These have sample mean \( \bar{D} = 5.0017 \) and sample standard deviation \( s = .0437 \) and the normal plot below.

\[ \text{Normal Plot of Distances} \]

(c) Does it appear that treating \( D \) as normally distributed is sensible? Say YES or NO and then explain why or why not. What vertical plotting positions were used for the two leftmost points on the plot? (Give numbers.)

YES or NO
Explanation:

\[
\begin{align*}
1\text{st plotting position} & = \_\_\_\_\_\_ \\
2\text{nd plotting position} & = \_\_\_\_\_\_
\end{align*}
\]

(d) Give endpoints of an interval that you are "90% sure" will contain the distance between hole positions for a single additional part. (No need to simplify.)

Suppose that engineering specifications on \( D \) are set at 5.0 ± .1.

(e) Give two-sided 90% confidence limits for the "process capability" for the distance between hole positions, \( C_p \). (No need to simplify.)
(f) Give an approximate 90% lower confidence bound for the process capability index $C_{pk}$. (No need to simplify.)

2. Miscellaneous Short Answers

(a) 10 consecutive parts are machined on a particular lathe. Their diameters are measured and two possible measures of variability are proposed. One might simply put the 10 measurements into a calculator and hit the "s" button. Or one might compute 9 moving ranges and use the estimate $\bar{MR}/d_2$ (the $d_2$ value being for $n = 2$). Under what circumstances would you expect to find $s \approx \bar{MR}/d_2$ and under what circumstances would you expect to find $s$ much larger than $\bar{MR}/d_2$?

\[
s \approx \bar{MR}/d_2 \text{ when:} \quad s \gg \bar{MR}/d_2 \text{ when:}
\]

(b) In coded units, $x_1$ and $x_2$ have ideal means 0, ideal standard deviations 1 and ideal correlation .2. Does a sample of $n = 4$ with $\bar{x}_1 = 2$ and $\bar{x}_2 = 2$ cause an out-of-control signal on a multivariate Shewhart chart? (Show the calculations.) Note that

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix}
d & -b \\
-c & a
\end{pmatrix}.
\]
3. Quality Culture Short Answers

(a) In J.P.Kouri's discussion of the "Baldridge process" at 3M, he said that beyond finding out what best performance is in a business sector, "benchmarking" activities should do what?

(b) The term "six sigma" is tossed around very loosely in business circles these days. Sometimes it refers to a goal for process performance. Other times it refers to a corporate problem-solving/variance-reduction strategy. Other times it refers to a training and recognition system instituted to support this strategy. Circle one of the aspects listed below and briefly describe it:

<table>
<thead>
<tr>
<th>description:</th>
</tr>
</thead>
<tbody>
<tr>
<td>performance goal</td>
</tr>
<tr>
<td>problem-solving strategy</td>
</tr>
<tr>
<td>training/recognition system</td>
</tr>
</tbody>
</table>

(c) What must you do if after taking IE 361 you decide that it would be a good idea to become a "Certified Quality Engineer"? (What organization must you approach and what is required for certification?)

(d) The term "ISO 9000" is also tossed around very loosely in business circles these days. Sometimes it refers to a series of documents issued by the International Standards Organization. Other times it refers to a process by which a business is "certified" to conform to the elements laid out in those documents. If you are supplying me widgets, why do I care if you are "ISO 9000" certified? (After all ISO 9000 certification is NOT certification that the particular widgets you send me are good ones, is it?)
This exam consists of 22 multiple choice questions that will be scored at 4 points apiece (plus 2 points absolutely free!). For each problem select a single best answer and fill in the corresponding circle on the Answer Sheet attached as the final page of this exam. (No credit will be given if more than one circle is marked.)

1. In a simple experiment involving \( r = 3 \) different setups of a process and a response \( y \), sample sizes and sample standard deviations are \( n_1 = 3 \) and \( s_1 = 2.0 \), \( n_2 = 4 \) and \( s_2 = 1.0 \), while \( n_3 = 2 \) and \( s_3 = 3.0 \). A pooled sample standard deviation is then
   a) 1.37
   b) 1.67
   c) 1.83
   d) 2.00
   e) 2.16

2. A pooled sample standard deviation is
   i) an estimate of a supposedly common "\( \sigma \)"
   ii) a measure of observed variation for any fixed set of process conditions
   a) both i) and ii) are true
   b) neither i) nor ii) is true
   c) only i) is true
   d) only ii) is true
   e) i) or ii) can be true, but only in unusual circumstances

3. In the "Gage R&R" context at the beginning of the term, we used \( \frac{\bar{R}}{d_2(n)} \) as an estimate of "\( \sigma \)." From those same (balanced 2-way factorial) data we could have computed \( s_p \) (though at the time we had not discussed this quantity). In a Gage R&R context, \( \frac{\bar{R}}{d_2(n)} \) and \( s_p \)
   a) differ because the first includes part-to-part (Factor A) variability and the second doesn't
   b) differ because the second includes part-to-part (Factor A) variability and the first doesn't
   c) both measure the accuracy of the gage
   d) measure the same kind of variability and will typically be comparable
   e) none of a)-d)

4. In a study involving \( r = 5 \) different process setups and a total of 14 observations, a sample mean response, \( \bar{y}_i \), based on \( n_i = 3 \) observations has associated with it "error limits" based on 95% confidence limits of
   a) \( \pm 1.306 s_p \)
   b) \( \pm 1.847 s_p \)
   c) \( \pm 1.883 s_p \)
   d) \( \pm 2.262 s_p \)
   e) \( \pm 0.605 s_p \)

5. In a "balanced data" study involving several different process setups, 95% confidence limits (based on the pooled standard deviation) for two means \( \mu_i \) and \( \mu_i \) are respectively 5.1 ± .5 and 4.2 ± .5. 95% two-sided confidence limits for the difference in the two means \( (\mu_i - \mu_i) \)
   a) can not be determined from the information supplied
   b) are .9 ± .4
   c) are .9 ± .5
   d) are .9 ± .7
   e) are .9 ± 1.0
6. In a balanced data experiment involving \( r = 4 \) different process setups and a common sample size \( m = 3 \), 95% confidence limits for \( L = \frac{1}{2}(\mu_1 + \mu_2 + \mu_3) - \mu_4 \) are \( \hat{L} \pm \Delta \) for
   a) \( \Delta = 2.306 s_p \)
   b) \( \Delta = 1.537 s_p \)
   c) \( \Delta = s_p \)
   d) \( \Delta = .354 s_p \)
   e) \( \Delta = .333 s_p \)

7. In a 2-way factorial study where AB interactions turn out to be negligible,
   i) an interaction plot will show "relatively parallel traces" of means
   ii) changing level of B produces about the same change in mean \( y \) for all levels of A
   iii) one can think of the two factors as acting "separately" on the response
   a) all of i)-iii) are true
   b) exactly two of i)-iii) are true
   c) only i) is true
   d) only ii) is true
   e) only iii) is true

8. In one particular \( 3 \times 3 \) study, AB interactions turn out to be negligible, \( a_1 = 2, a_2 = 3, b_1 = 7 \) and \( b_2 = -5 \). Suppose it is desirable to minimize the response, \( y \). The best combination of levels of A and B then seems to be
   a) level 1 of A and level 2 of B
   b) level 1 of A and level 3 of B
   c) level 2 of A and level 1 of B
   d) level 3 of A and level 3 of B
   e) level 3 of A and level 2 of B

9. The formulas like that in display (6.25) and in Tables 6.5 and 6.6 of Vardeman and Jobe
   i) provide quantitative means of judging detectability of the main effects and interactions
   ii) provide means of assessing \( \sigma \), the size of the "experimental/background noise"
   iii) provide means of assessing the "lack of fit" of the 2-way model
   a) only i) is true
   b) only ii) is true
   c) only iii) is true
   d) exactly two of i)-iii) are true
   e) none of a)-d)

10. In a balanced data \( 3 \times 3 \) factorial study with common sample size \( m = 3 \), 95% confidence limits for AB interactions are \( ab_{ij} \pm \Delta \) for
    a) \( \Delta = 2.101 s_p \)
    b) \( \Delta = s_p \)
    c) \( \Delta = .809 s_p \)
    d) \( \Delta = .311 s_p \)
    e) \( \Delta = .152 s_p \)

11. In a 2-way factorial study, if there are strong/important AB interactions, the quantity \( \alpha_1 - \alpha_2 \)
    i) still measures the difference between average means for levels 1 and 2 of Factor A
    ii) can still be estimated (provided there are data from all \( I \times J \) combinations of levels of A and B)
    iii) is of limited practical importance (because "lack of parallelism" says this isn't necessarily the change in mean response upon changing from level 1 to level 2 of A for any single level of B)
    a) only i) is true
    b) only ii) is true
    c) only iii) is true
    d) all of i)-iii) are true
    e) none of a)-d)
12. In a particular $2^2$ factorial, sample means are $\overline{y}_{(1)} = 8, \overline{y}_a = 10, \overline{y}_b = 6$ and $\overline{y}_{ab} = 16$. The fitted main effects of A and B at their "high" levels ($a_2$ and $b_2$) are respectively
   a) 10 and 6
   b) 3 and 1
   c) 12 and 4
   d) $-3$ and $-1$
   e) $-12$ and $-4$

13. For $2^5$ factorial data, the Yates algorithm requires how many cycles/columns of additions and subtractions, followed by division by what?
   a) 32 and 32
   b) 2 and 5
   c) 5 and 32
   d) 3 and 8
   e) 3 and 32

14. In a balanced $2^3$ factorial study with $m = 4$ observations $y$ per combination, 95% confidence limits for the factorial effects are of the form $\hat{E} \pm \Delta$ for
   a) $\Delta = 2.064s_p$
   b) $\Delta = 1.192s_p$
   c) $\Delta = s_p$
   d) $\Delta = .365s_p$
   e) none of a)-d)

15. In a particular $2^3$ factorial study, the only clearly detectable effects are the grand mean and the C main effect. This indicates that
   i) changing from the low to high level of C definitely changes mean response
   ii) changing levels of factors A or B has no clearly discernible effect on mean response
   iii) all of the conditions {$(1), a, b, ab$} have indistinguishable mean responses, as do all of the conditions {c, ac, bc, abc}
   a) none of i)-iii) are true
   b) all of i)-iii) are true
   c) only i) is true
   d) only ii) is true
   e) only iii) is true

16. In a particular $2^3$ factorial study, the only clearly detectable effects are the grand mean and the AB 2-factor interaction. (In fact $a_{b_{22}} = 3.$) This indicates
   a) that for large mean response one wants either {A high and B high} or {A low and B low}
   b) that as long as you keep level of A fixed, changing level of B doesn't change mean y
   c) that since factor C isn't involved, the low level of C is best
   d) that the conditions {$(1), a, b, ab$} have indistinguishable mean responses
   e) none of a)-d)

17. When there are detectable "high order effects" (like 5-factor interactions) in a $2^p$ study (with large $p$),
   i) the attempt to understand the system in simple terms has failed
   ii) more data are certainly needed to characterize the system
   iii) one might best think of the setups studied as simply "$r = 2^p$ different conditions"
   a) only i) is true
   b) only ii) is true
   c) only iii) is true
   d) all of i)-iii) are true
   e) exactly two of i)-iii) are true
18. In a $2^{4-1}$ study, three sample sizes are 2 and the other five are 1. 95% confidence limits for sums of pairs of factorial effects are then $\hat{E} \pm \Delta$ for 
   a) $\Delta = 3.182_{sp}$
   b) $\Delta = 2.250_{sp}$
   c) $\Delta = 2.028_{sp}$
   d) $\Delta = 1.014_{sp}$
   e) $\Delta = .959_{sp}$

19. Suppose that in a $2^{4-1}$ study with generator D $\leftrightarrow$ ABC, only the "grand mean plus alias," "A main effect plus alias," "BC interaction plus alias" and "ABC interaction plus alias" sums of effects are clearly detectable. The simplest possible interpretation of this result is that 
   a) only factor A is important in determining mean $y$
   b) all of factors A, B and C have clearly discernible effects on mean $y$
   c) there is a lot of variation in $y$
   d) only factors A and D clearly influence mean $y$, and their effects must be understood jointly
   e) only factors A and D clearly influence $y$, and their effects can be understood separately

20. Suppose that in a $2^{4-1}$ study with generator D $\leftrightarrow$ ABC, data analysis leads to the conclusion that the detectable sums of effects are "grand mean plus alias," "A main effect plus alias," "B main effect plus alias," "C main effect plus alias" and "ABC 3-factor interaction plus alias." A plausible interpretation of this is that all of A, B, C and D have detectable main effects. Further, suppose that $\alpha_2$, $\beta_2$ and $\gamma_2$ appear to be positive, while $\delta_2$ appears to be negative. What combination of levels of factors A, B, C and D then has the largest fitted mean? Is this combination represented in the original data set?
   a) combination d, which is not in the original data set 
   b) combination abcd, which is in the original data set 
   c) combination abed, which is not in the original data set 
   d) combination abc, which is in the original data set 
   e) combination abc, which is not in the original data set

21. In a $2^{8-2}$ study, an engineer uses generators G $\leftrightarrow$ ABCD and H $\leftrightarrow$ CDEF. This plan aliases the A main effect with 
   a) two 5-factor interactions and the grand mean 
   b) one 4-factor interaction, one 5-factor interaction and one 6-factor interaction 
   c) two 3-factor interactions and one 5-factor interaction 
   d) one 5-factor interaction and one 6-factor interaction and the grand mean 
   e) none of a)-d) is correct

22. In a $2^{8-2}$ study, an engineer uses generators G $\leftrightarrow$ ABCD and H $\leftrightarrow$ CDEF. This plan involves taking samples for 64 different process setups. Two of those have respectively "all of A,B,C,D,E,F at their low levels" and then "A low but all of B,C,D,E,F high." What corresponding levels of G and H are prescribed for these two setups of A through F? 
   a) first "G low and H low" then "G high and H low"
   b) first "G low and H low" then "G low and H high"
   c) first "G high and H high" then "G low and H high"
   d) first "G high and H high" then "G high and H low"
   e) none of a)-d) is correct
Blacken the circle for EXACTLY ONE answer for each question.

1. a ○   b ○   c ○   d ○   e ○
2. a ○   b ○   c ○   d ○   e ○
3. a ○   b ○   c ○   d ○   e ○
4. a ○   b ○   c ○   d ○   e ○
5. a ○   b ○   c ○   d ○   e ○
6. a ○   b ○   c ○   d ○   e ○
7. a ○   b ○   c ○   d ○   e ○
8. a ○   b ○   c ○   d ○   e ○
9. a ○   b ○   c ○   d ○   e ○
10. a ○  b ○  c ○  d ○  e ○
11. a ○  b ○  c ○  d ○  e ○
12. a ○  b ○  c ○  d ○  e ○
13. a ○  b ○  c ○  d ○  e ○
14. a ○  b ○  c ○  d ○  e ○
15. a ○  b ○  c ○  d ○  e ○
16. a ○  b ○  c ○  d ○  e ○
17. a ○  b ○  c ○  d ○  e ○
18. a ○  b ○  c ○  d ○  e ○
19. a ○  b ○  c ○  d ○  e ○
20. a ○  b ○  c ○  d ○  e ○
21. a ○  b ○  c ○  d ○  e ○
22. a ○  b ○  c ○  d ○  e ○