

1997 Exam 1

1.  $\bar{R} = .0007/9 = .000078$ ,  $\Delta_1 = .0007$ ,  $\Delta_2 = .00065$ ,  $\Delta_3 = .00065$ ,  $\bar{\Delta} = .00067$

(a)  $\hat{\sigma} = \bar{R}/d_2(2) = .000078/1.128 = .000069$  inch

(b)  $\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \left(\frac{.00067}{1.693}\right)^2 - \frac{1}{2}(.000069)^2\right)} = .000391$  inch

(c)  $\hat{\sigma}_{\text{overall}} = \sqrt{\hat{\sigma}^2 + \hat{\sigma}_{\text{reproducibility}}^2} = \sqrt{(.000069)^2 + (.000391)^2} = .000399$  inch

This is an estimate of the standard deviation of many operators making a single measurement each on the same part.

(d) No I do not. This captures only measurement variation, not the unavoidable part-to-part manufacturing variation that should be included in " $\sigma$ ."

2. (a) A  $c$  chart for  $X =$  total nonconformities on 10 units uses control limits  $UCL_X = 10 + 3\sqrt{10} = 19.48$  and  $LCL_X = 10 - 3\sqrt{10} = .51$ . So  $q = P[1\text{st point plots outside limits}] = P[X_1 = 0] + P[X_1 \geq 20]$  where I use a Poisson distribution with mean 20 to get  $q$ . Finally,  $ARL = 1/q$ .

(b) If  $X =$  number of nonconformities on a single unit, then  $X$  is Poisson with mean 1.0. So  $P[X \geq 1] = 1 - P[X = 0] = 1 - \exp(-1) = .632$ .

(c)  $X$  here under standard conditions is binomial with  $n = 10$  and  $p = .632$ . Using  $np$  chart limits  $UCL_X = np + 3\sqrt{np(1-p)} = 6.32 + 3\sqrt{10(.632)(.367)} = 10.9$  (i.e. there is effectively no UCL) and  $LCL_X = np - 3\sqrt{np(1-p)} = 6.32 - 3\sqrt{10(.632)(.367)} = 1.7$ .

3. One must take physical action to track down and eliminate "assignable causes" that are producing (undesired) physical change.

4. (a)  $UCL_R = D_4\bar{R} = 2.115(9.25) = 19.56$ , and there is no LCL. There are no points outside of retrospective control limits and therefore no evidence of process instability.

(b)  $\bar{R}/d_2(5) = 9.25/2.326 = 3.98$ .

(c)  $UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 245.18 + .577(9.25) = 250.52$  and  $LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 245.18 - .577(9.25) = 239.84$  There are no points outside retrospective control limits and there is thus no evidence of process instability.

(d) Use  $target \pm 3\kappa\frac{\sigma}{\sqrt{n}}$ .  $UCL_{\bar{x}} = 244 + 3(1.160)(3.98)/\sqrt{3} = 244 + 8.0 = 252$  and  $LCL_{\bar{x}} = 244 - 3(1.160)(3.98)/\sqrt{3} = 244 - 8.0 = 236$ .

(e)  $\text{Var}x = \text{Var}t + \text{Var}e$ , so  $(3.98)^2 = \text{Var}t + (1.00)^2$ . This gives  
 $\sigma_t = \sqrt{\text{Var}t} = \sqrt{(3.98)^2 - (1.00)^2} = 3.85 \mu\text{m}$ . (See pages 19 and 20.)

(f) The one in part (e). My guess is that specifications are meant to apply to  $t$  (the "true" part dimension ... what you would expect to get by averaging many measurements of the same part). Otherwise engineering specifications would need to be quoted for a specific gage ( $\sigma_e$ ) AND a part could be "nonconforming" and then suddenly "conforming" on consecutive measurements (which seems more than slightly silly).

(g) If processes work well they produce good product. No amount of post-production attention to product will solve process problems and improve process behavior.

(h) I have told you nothing about either process stability or product acceptability. Regarding process stability: all  $\bar{x}$ 's could be inside control limits, but they could also all be outside them. Regarding product acceptability: all individuals could be inside (3, 7), but they could also all be outside (3, 7).

## 1997 Exam 2

1. (a)  $shift = 1.5/\sqrt{2.2} \approx 1$ , so use  $\lambda = .2$  (from Table 4.2). Then using Table 4.3 and formula (4.5),  $UCL_{EWMA} = 4.6 + (2.36)(\sqrt{2.2})\sqrt{\frac{.2}{2-.3}} = 5.77$ ,

$$LCL_{EWMA} = 4.6 - (2.36)(\sqrt{2.2})\sqrt{\frac{.2}{2-.3}} = 3.43, \text{ where } EWMA_0 = 4.6.$$

(b)  $\rho = -.7/\sqrt{2.2(4.9)} = -.21$

The negative correlation makes sense because as the percentage of large particles increases, *something* must decrease (either medium or small).

(c) Compute  $X^2 = 1(7.6 - 4.6, 91.0 - 86.6) \begin{pmatrix} 2.2 & -.7 \\ -.7 & 4.9 \end{pmatrix}^{-1} \begin{pmatrix} 7.6 - 4.6 \\ 91.0 - 86.6 \end{pmatrix}$ . I

expect this may produce and out-of-control signal, since both values are above their standard means, which is contrary to the standard *negative* correlation between  $x_1$  and  $x_2$ .

(d)  $\overline{MR} = 23.1/10 = 2.31$ , so  $\hat{\sigma} = \overline{MR}/d_2(2) = 2.31/1.128 = 2.05$

Thus  $UCL_x = \mu + 3\sigma = 14.8 + 3(2.05) = 20$  and

$LCL_x = \mu - 3\sigma = 14.8 - 3(2.05) = 8.7$ .

(e)  $\sigma_{\text{observed}}^2 = \sigma_{\text{measurement}}^2 + \sigma_{\text{actual}}^2$ , so  $(2.05)^2 \approx (.5)^2 + \sigma_{\text{actual}}^2$  and  $\sigma_{\text{actual}} \approx \sqrt{(2.05)^2 - (.5)^2}$ . (See the discussion leading up to display (2.3).)

(f)  $\bar{x} \approx \text{horizontal intercept} \approx 1450$  and  $s \approx 1/\text{slope} \approx (1970 - 970)/(1.5 - (1.5)) \approx 333$ .

3. (a) Use formula (5.7). The bound is

$$\frac{.02}{6(.0007)} \sqrt{\frac{14.848}{24-1}} = 3.83.$$

(b) Use formulas (5.9) and (5.10).

$$\hat{C}_{pk} = \min\left(\frac{1.30-1.2909}{3(.0007)}, \frac{1.2909-1.28}{3(.0007)}\right) = \min(4.33, 5.19) = 4.33, \text{ so the lower confidence}$$

bound is  $4.33 - 1.282\sqrt{\frac{1}{9(24)} + \frac{(4.33)^2}{2(24)-2}} = 3.51$ .

4. (a) Put feed rates 1 through 4 on the horizontal axis and plot mean numbers of cuts. Connect the points for stop delay 1 with a broken line graph and then do the same for the stop delay 2 points.

(b)  $d.f. = n - r = 3(8) - 8 = 24 - 8 = 16$  so from formula (6.9)

$$\Delta = 2.120(20)/\sqrt{3} = 24.5 \text{ cuts.}$$

(c) No, I do not expect the interactions to be detectable. There is enough uncertainty of estimation of the 8 means (indicated by  $\pm 24.5$  cut error bars) to account for the small

lack of parallelism on the plot. (One could slide the means around within the intervals indicated on the plot and produce parallelism.)

(d) From formula (6.10)  $\Delta = 2.120(20)\sqrt{\frac{1}{3} + \frac{1}{3}} = 34.6$  cuts.

(e)  $a_1 = \bar{y}_{1.} - \bar{y}_{..} = 162.5 - 167.75 = -5.25$  and  
 $b_1 = \bar{y}_{.1} - \bar{y}_{..} = 137.5 - 167.75 = -30.25$  and  
 $ab_{11} = 133 - (167.75 + (-5.25) + (-30.25)) = .75$ .

(f) Use display (6.25) and Table 6.5. The confidence bounds are then  
 $(-5.25 - (5.25)) \pm 2.120(2)\sqrt{\frac{2}{3(4)}}$  i.e.  $-10.5 \pm 17.31$ .

1997 Exam 3

1. (a)	combination	s	$\bar{y}$	cycle #1	cycle #2	cycle #3	cycle #3/8
	(1)	1.9	8.5	26.8	34.0	70.9	8.8625 = $\bar{y}_{...}$
	a	1.7	<u>18.3</u>	<u>7.2</u>	<u>36.9</u>	<u>28.9</u>	3.6125 = $a_2$
	b	1.7	1.9	25.6	13.2	- 33.9	- 4.2375 = $b_2$
	ab	3.3	<u>5.3</u>	<u>11.3</u>	<u>15.7</u>	<u>- 13.5</u>	- 1.6875 = $ab_{22}$
	c	1.6	7.1	9.8	- 19.6	2.9	.3625 = $c_2$
	ac	6.7	<u>18.5</u>	<u>3.4</u>	<u>- 14.3</u>	<u>2.5</u>	.3125 = $ac_{22}$
	bc	2.3	3.5	11.4	- 6.4	5.3	.6625 = $bc_{22}$
	abc	1.4	7.8	4.3	- 7.1	- .7	- .0875 = $abc_{222}$

(b)  $s_P^2 = \frac{(4-1)(1.9)^2 + (4-1)(1.7)^2 + \dots + (4-1)(1.4)^2}{(4-1) + (4-1) + \dots + (4-1)} = 9.3725$ , so  $s_P = \sqrt{9.3725} = 3.06$  short leads. This is (and estimate of) the standard deviation of the number of short leads that one would expect for a fixed combination of levels of A, B and C.

(c) Use formula (6.31) with  $d.f. = (4 - 1)8 = 24$ . Then  $\Delta = 1.711(3.06) \frac{1}{8} \sqrt{\frac{8}{4}} = .9255$ .

(d) The grand mean, A and B main effects and the AB two-factor interactions are all statistically detectable. (Their magnitudes exceed .9255.)

(e) Use  $\hat{y} = \bar{y}_{...} + a_1 + b_2 + ab_{12} = 8.8625 - 3.6125 + (- 4.2375) + 1.6875 = 2.7$ .

2. (a) Each effect is aliased with  $2^5 - 1 = 31$  other effects.

(b)  $A + B + C + D - E - F - G - H + J + K +$   
 $A + B - C - D - E - F - G - H - J - K +$

(c) The far left point and the 2 far right points plot off the line established by the rest of the points. The far left point represents the D main effect plus aliases, the far right point represents the G main effect plus aliases (including the ABCE 4-factor interaction) and the second to far right point represents the F main effect plus aliases (including the ABCD 4-factor interaction). It is possible that factors D, F and G all have important main effects, while all other effects are negligible.

(d) For minimum  $y$ , the above interpretation (and the signs on the fitted sums of effects) suggest that D should be set to its high level, G and F both to their low levels and all other factors can be set to any convenient (or cheap) level. (For "safety" sake it might make sense to choose one of the combinations actually run in this experiment that meets these criteria.)