1996 Exam 1

1. $I = 4,\ J = 3,\ m = 2,\ \Delta_1 = .025,\ \Delta_2 = .025,\ \Delta_3 = .050,\ \Delta_4 = .075$

(a) $\overline{R} = .05,\ \widehat{\sigma} = \overline{R}/d_2(2) = .05/1.128 = .0443$
This is a measure of variation on repeat measurement of a single part by a single operator.

(b) $\overline{\Delta} = .04375,\ \widehat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \left(\frac{.04375}{1.693}\right)^2 - \frac{1}{2}(.0443)^2\right)} = 0$
This is a measure of variation associated with many operators each making a single measurement on the same part, assuming that there is no repeatability variation. (Or equivalently, it is a measure of variation of long run averages for many operators on the same part.)

(c) $G\widehat{C}R = \frac{6\widehat{\sigma}_{\text{overall}}}{U-L} = \frac{6(.0443)}{.2} = 1.33$
This figure is big. We'd like a GCR of no more than .1 at most. The measurement variation alone more than exceeds the allowable engineering "tolerance." This measurement technique is not adequate to use for checking conformance to these specifications.

2. $\overline{s} = 1.714,\ \overline{\overline{x}} = 11.84$

(a) $UCL_s = B_4\overline{s} = 2.089(1.714) = 3.58,\ LCL_s = B_3\overline{s}$ (none here)
None of the 10 sample standard deviations plots outside control limits. There is no indication here of process change/instability.

(b) $\widehat{\sigma} = \overline{s}/c_4 = 1.714/.9400 = 1.823$ (1/64 inch)

(c) $UCL_R = D_2\sigma = 4.698(1.823) = 8.564,\ LCL_R = D_1\sigma$ (none here),
$CL_R = d_2(4)\sigma = 2.059(1.823) = 3.75$

(d) $CL_{\overline{x}} = \overline{\overline{x}} = 11.84, UCL_{\overline{x}} = \overline{\overline{x}} + A_3\overline{s} = 11.84 + 1.427(1.714) = 14.29,$
$LCL_{\overline{x}} = \overline{\overline{x}} - A_3\overline{s} = 11.84 - 1.427(1.714) = 9.39$
None of the $\overline{x}$'s plot outside these control limits, in fact none even come close to the limits. There is no evidence here of process change.

(e) There is the possibility of becoming the victim of "stratification." If the blades are not positioned evenly around the drum there will be systematic differences between lengths of sheets cut on a single revolution of the drum. If there are k sheets cut per revolution, one is probably better off treating the k "positions" separately. Otherwise the systematic differences could lead to apparent "super-stability" on the charts and go undetected.

(f) $z = (10 - 11.84)/1.823 = -1.01.\ P[Z < -1.01] = .1562$, about 16%.

(g) One could reduce the mean sheet length from 11.84 (64ths above nominal) to something closer to nominal (but still far enough above to keep the fraction below 0 small). For example, one might set $\mu = 0 + 3\sigma = 3(1.823) = 5.469,$ which requires a reduction in sheet length of $11.84 - 5.469 = 6.37$ (64ths of an inch).

3. (a) One wants limits $\mu \pm z\sigma/\sqrt{n}$ where $P[-z < Z < z] = .995$, i.e. one wants z such that $P[Z > z] = .0025$. This is $z = 2.81$ and we want to use a "2.81 sigma" chart.

(b) $CL_{\widehat{u}} = .1,\ UCL_{\widehat{u}} = .1 + 3\sqrt{.1/2} = .771$ and there is no $LCL_{\widehat{u}}$ since $.1 - 3\sqrt{.1/2} < 0$. With $X =$ number of nonconformances seen on 2 units, $UCL_X = 2UCL_{\widehat{u}} = 1.54$. So (using the Poisson model with mean $\lambda = .2$ for X) $q = P[\text{first point plots outside control limits}] = P[X \geq 2] = 1 - f(0) - f(1)$ $= 1 - \text{exp}(-.2) - .2\text{exp}(-.2)/1! = .017523$. So $ARL = 1/q = 57.07$.

(c) $\sum X_i = 750$ and $\sum n_i = 75$, so $\widehat{p}_{\text{pooled}} = .1$

np chart control limits are $np \pm 3\sqrt{np(1-p)}$. So $n = 50$ gives limits $5 \pm 3\sqrt{50(.1)(.9)} = 5 \pm 6.36$ while $n = 100$ gives limits $10 \pm 3\sqrt{100(.1)(.9)}$, i.e. 10 ± 9. No points plot outside these limits. There is no evidence of process instability.

(d) This is a case where (because of tool wear) it is probably *not* fair to think of the process as physically stable, unless one takes steps to make it so. Diameters will tend to increase over time. It then makes sense to use engineering control techniques to automatically (based on measurements) compensate for tool wear *and then* to monitor for the unexpected using SPC techniques. (Such would allow one to detect abnormalities like tool-breakage, off-specification material hardness, etc.)

1996 Exam 2

1. (a) $\widehat{C}_{pk} = \min\left(\frac{22-16.5}{3(.15)}, \frac{16.5-12}{3(.15)}\right) = \min(1.22, 1.00) = 1.00$
Then an approximate 95% lower confidence bound for C_{pk} is (from display (5.10))
$1.00 - 1.645\sqrt{\frac{1}{9(25)} + \frac{(1.00)^2}{2(25)-2}} = .738.$

(b) From display (5.5) we need $\left(6(.15)\sqrt{\frac{24}{36.415}}, 6(.15)\sqrt{\frac{24}{13.848}}\right)$. i.e. $(7.31, 11.85)$.

(c) From display (5.11) limits are $16.5 \pm 1.711(1.5)\sqrt{1 + \frac{1}{25}}$, i.e. 16.5 ± 2.62 SCUs.

(d) $\sigma_Q = \sigma_{\overline{x}} = \sigma/\sqrt{n} = 1.5/\sqrt{2} = .75$
Then from display (4.5) control limits are $17 \pm \mathcal{K}(.75)\sqrt{\frac{.1}{2-.1}}$. From Table 4.3 $\mathcal{K} = 2.15$ is appropriate. So $UCL_{EWMA} = 17 + (2.15)(.75)\sqrt{\frac{.1}{2-.1}} = 17.37$ and

$LCL_{EWMA} = 17 - (2.15)(.75)\sqrt{\frac{.1}{2-.1}} = 16.63$ and a starting value of $EWMA_0 = 17$ should be used.

2. (a) $mean = horizontal\,intercept = 12.5$,
$std\,deviation = 1/slope = run/rise = 5/4 = 1.25$

(b) From display (4.35) use $\widehat{\sigma} = \overline{MR}/1.128 = 2.22/1.128 = 1.97$.

(c) The second possibility will be preferable to the first only when
  i) there is automated computation (and data collection?), and
  ii) there is some reason to be concerned about "unusual relationships" between diameters on the different heads.

(The second condition seems especially far fetched in the present scenario. This doesn't seem like a very natural setting for multivariate process monitoring.)

(d) $X^2 = 5(3-2, 3-2)\begin{pmatrix} 9 & -4 \\ -4 & 9 \end{pmatrix}^{-1}\begin{pmatrix} 3-2 \\ 3-2 \end{pmatrix}$
I would compare this to $UCL_{X^2} = 2 + 3\sqrt{2(2)} = 8$. I expect this outcome to be "atypical" since the standard correlation is negative ($-4/\sqrt{9 \cdot 9} = -.44$) while both $\overline{x}_1$ and $\overline{x}_2$ are above their standard means.

3. (a) One assumes that one has $I \times J$ ($= 2 \times 2$ here) samples from normal populations with possibly different means μ_{ij} but a common standard deviation σ.

(b) From display (6.3) $s_{\text{pooled}} = \sqrt{\frac{(4-1)(.71)^2+(2-1)(0)^2+(3-1)(1.04)^2}{(4-1)+(2-1)+(3-1)}} = .783\,\%$ (with $n - r = 10 - 4 = 6$ associated degrees of freedom).

(c) Consider confidence limits for $\mu_{21} - \mu_{22}$ of the form in display (6.10). That is, use $(29.80 - 26.20) \pm 2.447(.738)\sqrt{\frac{1}{2} + \frac{1}{3}}$, i.e. 3.60 ± 1.75. Note that 1.75 is smaller in magnitude than 3.6. That is, there is a detectable (positive) difference between the two oven positions in terms of effect on % weight loss for PMR-II-50 polymer.

(d) Put oven positions 1 and 2 on the horizontal axis. Plot four percent losses and connect the two PMR-II-50 points and then the two Avimid-N points with line segments. The lack of parallelism on the plot is small in comparison to the apparent difference between the polymers. (The interactions look small in comparison to the polymer main effects.)

(e) Use (as in display (6.12)) " $\pm$ values" of $\kappa_2^* = s_{\text{P}}\sqrt{1/n_{ij}}$. Now for $r = 4$ means and 6 d.f. $\kappa_2^* = 3.389$ and $s_{\text{P}} = .783$ so one has

i	j	n_{ij}	"$\pm$ value"
1	1	1	2.65
2	1	2	1.88
1	2	4	1.33
2	2	3	1.53

(f) $\beta_1 - \beta_2 = (\mu_{.1} - \mu_{..}) - (\mu_{.2} - \mu_{..}) = \mu_{.1} - \mu_{.2} = \frac{1}{2}(\mu_{11} + \mu_{21}) - \frac{1}{2}(\mu_{12} + \mu_{22})$
So from display (6.25) confidence limits for this difference are
$\overline{y}_{.1} - \overline{y}_{.2} \pm ts_{\mathrm{P}}\sqrt{\frac{(\frac{1}{2})^2}{n_{11}} + \frac{(\frac{1}{2})^2}{n_{21}} + \frac{(-\frac{1}{2})^2}{n_{12}} + \frac{(-\frac{1}{2})^2}{n_{22}}}$. These are
$(19.35 - 17.715) \pm 1.943(.783)\left(\frac{1}{2}\right)\sqrt{\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{3}}$, i.e. $1.64 \pm 1.10\,\%$.

1996 Exam 3

1. (a) Confidence limits for any one of the 8 means represented here are $\overline{y} \pm ts_{\mathrm{P}}/\sqrt{m}$.
So $\Delta = 2.064(.118)/\sqrt{n} = .122$ (since $\nu = (m-1)r = 24$ d.f. are involved).

(b)

A	B	C	$\overline{y}$	cycle #1	cycle #2	cycle #3	cycle #3/8	estimated
−	−	−	.98	2.56	5.43	11.39	1.42	$\mu_{......} + aliases$
+	−	−	1.58	2.87	5.96	− .17	− .02	$\alpha_2 + aliases$
−	+	−	1.13	2.33	1.21	1.61	.20	$\beta_2 + aliases$
+	+	−	1.74	3.63	− 1.38	− .07	− .01	$\alpha\beta_{22} + aliases$
−	−	+	1.49	.60	.31	.53	.07	$\gamma_2 + aliases$
+	−	+	.84	.61	1.30	− 2.59	− .32	$\alpha\gamma_{22} + aliases$
−	+	+	2.18	− .65	.01	.99	.12	$\beta\gamma_{22} + aliases$
+	+	+	1.45	− .73	− .08	− .09	− .01	$\alpha\beta\gamma_{222} + aliases$

(c) Using the constant sample size version of display (6.31) one has confidence limits of the form $\widehat{E} \pm ts_{\mathrm{P}}\frac{1}{2^{p-q}}\sqrt{\frac{1}{m} + \frac{1}{m} + \cdots \frac{1}{m}}$ and therefore $\Delta = 2.064(.118)\frac{1}{8}\sqrt{\frac{8}{4}} = .043$ mm.

(d) 1.42, .20, .07, $-$ .3 and .12 are all larger in magnitude than .043 (and therefore represent statistically detectable sums of effects).

(e)

A	B	C	D (AC)	E (BC)	F (ABC)	combination
$-$	$-$	$-$	$+$	$+$	$-$	de
$+$	$-$	$-$	$-$	$+$	$+$	aef
$-$	$+$	$-$	$-$	$-$	$-$	bdf
$+$	$+$	$-$	$-$	$-$	$-$	ab
$-$	$-$	$+$	$-$	$-$	$+$	cf
$+$	$-$	$+$	$+$	$-$	$-$	acd
$-$	$+$	$+$	$-$	$+$	$-$	bce
$+$	$+$	$+$	$+$	$+$	$+$	abcdef

(f) I $\leftrightarrow$ ACD $\leftrightarrow$ BCE $\leftrightarrow$ ABCF $\leftrightarrow$ ABDE $\leftrightarrow$ BDF $\leftrightarrow$ AEF $\leftrightarrow$ CDEF

(g) There are several possible simple interpretations. Probably the simplest (involving only main effects) is that

1.42 represents the grand mean only
.20 represents the B main effect only
.07 represents the C main effect only
$-$ .32 represents the D main effect only (an alias of the AC 2-factor interaction)
.12 represents the E main effect only (an alias of the BC 2-factor interaction)

2. (a) Only the fitted A main effect plots "clearly off the line" established by the other fitted effects. Even if "most" of the effects are just "noise," a_2 is clearly representing something more than experimental variation. An "A main effects only" seems appropriate and factors B, C and D don't seem to impact y.

(b) Since a_2 is negative we see that for large y we want level 1 of A. What is done with the B, C and D "knobs" appears to be immaterial as far as mean response is concerned (and these factors may thus be set according to other considerations.

(c) $\widehat{y} = \overline{y}_{....} + a_1 = 3.594 + .806 = 4.400$. (This is the mean response we estimate for any settings of B, C and D, as long as A is set to level 1.)

3. (a) Acceptance sampling is useful for deciding lot disposal where quality varies/is unknown. If the economics dictate that we must inspect our product for purposes of sorting good from bad, sampling inspection will be of no help in this regard.

(b) For the fraction nonconforming scenario
$Pa = \binom{10}{0}(.1)^0(.9)^{10} + \binom{10}{1}(.1)^1(.9)^9 + \binom{10}{2}(.1)^2(.9)^8$ while for the mean nonconformities per unit context
$Pa = 1^0\exp(-10(.1))/0! + 1^1\exp(-10(.1))/1! + 1^2\exp(-10(.1))/2!$

(c) Use displays (8.17) and (8.18). $n \approx (-1.645)^2\left(\frac{(.02)(.98)}{(.01)^2}\right) = 530$ and $c \approx n(.01) = 530(.01) = 5$.

(d) Code letter H means that under normal inspection one uses $n = 50$ and $Ac = 1$ (with $Re = 2$), under tightened inspection one uses $n = 80$ and $Ac = 1$ (with $Re = 2$) and under reduced inspection one uses $n = 20,\ Ac = 0$ and $Re = 2$. Now employ the algorithm in Figure 8.8.