IE 361 Module 5

Gauge R&R Studies Part 1: Motivation, Data, Model and Range-Based Estimates

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Reading: Section 2.2, *Statistical Quality Assurance Methods for Engineers*

Standard R&R Data and Descriptive Statistics (Based on Ranges) for Partitioning Measurement Variation

A very common type of industrial measurement study is one where a single gauge or piece of measuring equipment is used (according to a standard protocol) by multiple operators to measure multiple parts, with the primary end goal of quantifying repeatability and reproducibility measurement variation, and comparing measurement imprecision to the basic engineering requirements that a part must satisfy in order to be functional. Remember from what we have already said in Modules 3 and 4, that

- "Repeatability" variation is variation characteristic of one operator/analyst remeasuring one specimen

- "Reproducibility" variation is variation in operator biases, i.e. variation characteristic of many operators measuring a single specimen after accounting for (or somehow mathematically eliminating) repeatability variation

In a typical (balanced data) industrial Gauge R&R study, each of I items is measured m times by each of J operators. For example, a typical data layout for $I = 2$ parts, $J = 3$ operators, and $m = 2$ repeats per "cell" might be represented as in Figure 1.

		Operator 1	Operator 2	Operator 3
Part	1	y_{111}, y_{112}	y_{121}, y_{122}	y_{131}, y_{132}
	2	y_{211}, y_{212}	y_{221}, y_{222}	y_{231}, y_{232}

Figure 1: Hypothetical Gauge R&R Data

If only *one* part/measurand were involved, the one-way model and analyses of Module 4 could be used to do inference for what we have been calling σ_β and σ_{device}. (See again Figure 3 and Example 4-3 of Module 4.) But (presumably in order to have some check on how measurement performs across a spectrum of parts) it is common in Gauge R&R studies to use multiple parts. Figure 2 shows some real R&R data collected in-class in IE 361 on $I = 4$ parts, by $J = 3$ operators, making $m = 2$ repeats per cell, and some summary statistics based on ranges (rather than standard deviations). (The data are measurements of the sizes of some Styrofoam peanuts, and range-based methods are presented here because of their connection to fairly standard industry practice.)

	Operator #1		Operator #2		Operator #3		Δ_i
Part # 1	.52 .52	$\bar{y}_{11} = .52$ $R_{11} = 0$	.54 .53	$\bar{y}_{12} = .535$ $R_{12} = .01$	.55 .55	$\bar{y}_{13} = .55$ $R_{13} = 0$	$\Delta_1 = .03$
Part # 2	.56 .55	$\bar{y}_{21} = .555$ $R_{21} = .01$	.54 .54	$\bar{y}_{22} = .54$ $R_{22} = 0$	.55 .56	$\bar{y}_{23} = .555$ $R_{23} = .01$	$\Delta_2 = .015$
Part # 3	.57 .56	$\bar{y}_{31} = .565$ $R_{31} = .01$	.55 .56	$\bar{y}_{32} = .555$ $R_{32} = .01$	.57 .57	$\bar{y}_{33} = .57$ $R_{33} = 0$	$\Delta_3 = .015$
Part # 4	.55 .55	$\bar{y}_{41} = .55$ $R_{41} = 0$	.54 .55	$\bar{y}_{42} = .545$ $R_{42} = .01$	.56 .55	$\bar{y}_{43} = .555$ $R_{43} = .01$	$\Delta_4 = .01$

$$\bar{R} = \frac{.07}{12} = .00583 \text{ and } \bar{\Delta} = \frac{.07}{4} = .0175$$

Figure 2: Styrofoam Peanut Size Measurements and Summary Statistics (Inches)

$\overline{R}$ is a very simple descriptive measure of within cell variability and is related to repeatability variation. Similarly, $\overline{\Delta}$ is a measure of between-operator variation and is related to reproducibility variation. In order to be more more precise about these (and to do statistical inference) one must adopt a probability model for the data collected in an R&R study.

The "Two-Way Random Effects" Model for Gauge R&R Data

Typical analyses of Gauge R&R studies are based on the so-called "two-way random effects" model. With

$$y_{ijk} = \text{the } k\text{th measurement made by operator } j \text{ on specimen } i$$

this model is that y_{ijk} is made up as a sum of independent contributions,

$$y_{ijk} = \mu + \alpha_i + \gamma_j + \alpha\gamma_{ij} + \epsilon_{ijk}$$

where

- μ is an (unknown) constant, an average (over all possible operators and all possible parts/specimens) measurement

- the α_i are normal with mean 0 and variance σ^2_α, (random) effects of different parts/specimens

- the γ_j are normal with mean 0 and variance σ^2_γ, (random) effects of different operators

- the $\alpha\gamma_{ij}$ are normal with mean 0 and variance $\sigma^2_{\alpha\gamma}$, (random) joint effects peculiar to particular part/operator combinations

- the ϵ_{ijk} are normal with mean 0 and variance σ^2, (random) errors that are peculiar to a particular attempt to make a measurement (they change measurement-to-measurement, even if the part and operator remain the same)

$\sigma^2_\alpha, \sigma^2_\gamma, \sigma^2_{\alpha\gamma}$, and σ^2 are called "variance components" and their sizes govern how much variability is seen in the measurements y_{ijk}.

Example 5-1 Conduct a "Thought Experiment" generating a Gauge R&R data set, and fill in formulas for the 12 measurements in the table below. (For example, $y_{111} = \mu + \alpha_1 + \gamma_1 + \alpha\gamma_{11} + \epsilon_{111}$.)

		Operator		
		1	2	3
Part	1	$y_{111} =$ $y_{112} =$	$y_{121} =$ $y_{122} =$	$y_{131} =$ $y_{132} =$
	2	$y_{211} =$ $y_{212} =$	$y_{221} =$ $y_{222} =$	$y_{231} =$ $y_{232} =$

In this (two-way random effects) model

- σ measures within-cell/repeatability variation

- $\sigma_{\text{reproducibility}} = \sqrt{\sigma_\gamma^2 + \sigma_{\alpha\gamma}^2}$ is the standard deviation that would be experienced by many operators measuring the same specimen once each, *in the absence of repeatability variation*

- $\sigma_{\text{R\&R}} = \sqrt{\sigma_{\text{reproducibility}}^2 + \sigma^2} = \sqrt{\sigma_\gamma^2 + \sigma_{\alpha\gamma}^2 + \sigma^2}$ is the standard deviation that would be experienced by many operators measuring the same specimen once each (this is called σ_{overall} in *SQAME*)

To make connections to what we have done earlier, consider what these two-way model parameters mean if we restrict attention to part #1. The two-way random effects model says that measurements on part #1 can be thought of as

$$y_{1jk} = \mu + \alpha_1 + \gamma_j + \alpha\gamma_{1j} + \epsilon_{1jk}$$

What then varies operator-to-operator is

$$\gamma_j + \alpha\gamma_{1j}$$

This quantity thus plays the role of what we before called β_j (operator bias for operator j) and

$$\sigma^2_{\gamma_j+\alpha\gamma_{1j}} = \sigma^2_{\gamma} + \sigma^2_{\alpha\gamma}$$

plays the role of what we before called σ^2_{β} (the reproducibility variance).

The most common analyses (both those based on ranges and those based on ANOVA) (e.g. following the AIAG manual) are *wrong*, in that they purport to produce estimates of $\sigma_{\text{reproducibility}}$ and $\sigma_{\text{R\&R}}$ but fail to do so. *SQAME* presents correct range-based and ANOVA-based methods. We will use primarily the generally more effective ANOVA-based estimates and confidence intervals that can be based on them (these limits are not found in *SQAME*). (**If you**

end up doing a gauge R&R study for your project client, you will almost certainly be asked to use company standard formulas or a company spreadsheet that implements the (WRONG) AIAG formulas. If you do this you should compare those results to ones obtained using correct formulas from these modules! Failure to do so will be frowned upon when projects are graded.)

Simple Range-Based Point Estimates of Repeatability and Reproducibility Standard Deviations

For motivation sake (and because of the connection to standard formulas), first briefly consider range-based estimates. Possible estimates are:

- $\hat{\sigma} = \frac{\overline{R}}{d_2(m)}$ for $\overline{R}$ the average within-cell range and $d_2(m)$ a "control chart constant" based on "sample size" m

- $\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \left(\frac{\overline{\Delta}}{d_2(J)}\right)^2 - \frac{1}{m}(\hat{\sigma})^2\right)}$ for $\overline{\Delta}$ the average of part ranges of cell means and $d_2(m)$ a "control chart constant" based on "sample size" J

(The second of these is NOT the AIAG estimate of reproducibility standard deviation.)

Example 5-2 Below are some simple calculations based on measurements of a geometric dimension of a machined part in a study with $I = 3, J = 3$, and $m = 2$.

	Operator 1	Operator 2	Operator 3	
Part 1	$\overline{y}_{11} = .34730$ $R_{11} = 0$	$\overline{y}_{12} = .34660$ $R_{12} = .0002$	$\overline{y}_{13} = .34715$ $R_{13} = .0001$	$\Delta_1 = .00070$
Part 2	$\overline{y}_{21} = .34710$ $R_{21} = 0$	$\overline{y}_{22} = .34645$ $R_{22} = .0001$	$\overline{y}_{23} = .34710$ $R_{23} = 0$	$\Delta_2 = .00065$
Part 3	$\overline{y}_{31} = .34720$ $R_{31} = 0$	$\overline{y}_{32} = .34655$ $R_{32} = .0003$	$\overline{y}_{33} = .34710$ $R_{33} = 0$	$\Delta_3 = .00065$

So $\overline{R} = .0007/9 = .000078$ and $\overline{\Delta} = .00067$ and

$$\widehat{\sigma} = \frac{\overline{R}}{d_2(m)} = \frac{.000078}{1.128} = .000069 \text{ in}$$

and

$$\hat{\sigma}_{\text{reproducibility}} = \sqrt{\max\left(0, \left(\frac{\overline{\Delta}}{d_2(J)}\right)^2 - \frac{1}{m}(\widehat{\sigma})^2\right)}$$
$$= \sqrt{\left(\frac{.00067}{1.693}\right)^2 - \frac{1}{2}(.000069)^2} = .000391\,\text{in}$$

A natural way to estimate $\sigma_{\text{R\&R}}$ is as

$$\hat{\sigma}_{\text{R\&R}} = \sqrt{(.000069)^2 + (.00039)^2} = .000396\,\text{in}$$

and the calculations here suggest that the bulk of measurement imprecision is traceable to differences between operators.