Optimal Experimental Design for Human Thermoregulatory System Identification

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ABSTRACT

Rollins, et al. [1] proposed a block-oriented modeling (BOM) approach for obtaining human thermoregulatory models for individual subjects. The objective of this approach is to obtain a library of model structures that map input changes such as humidity, ambient temperature, exercise, etc. to dynamic thermoregulatory responses such as the core temperature, skin temperature, muscle temperature, sweat rate, etc. These model structures will be categorized by human attributes and environmental conditions. To obtain this library of model structures a huge amount of experimentation in environmental chambers will be required to determine how the structures vary over the enormous input space. Thus, it is critical that experimental time is minimized. Using a D-optimality criterion, this article presents an experimental design approach that reduced the experimental time by 70% in comparison to the design in [1].

Keywords: Dynamic Thermoregulation Modeling, Design of Experiments, D-Optimality.

1. INTRODUCTION

Models of human thermoregulation can aid in the design of military chemical suits [2], industrial protective clothing [3], space suits, and to the development of predictive control systems for environmental suits and critical environments such as space ships and space stations.
Moreover, as humans have started to work and explore in more environmentally hostile and extreme conditions, the need for understanding the human thermoregulatory system (HTS) and the usefulness of developing dynamic models for the HTS in these extreme environments has increased. These extreme environments can often have serious and detrimental affects on human health and well being. For example, in conditions of bright sunshine, high temperature, and high humidity - heat-related health problems such as heat cramps, heat exhaustion and heat stroke have a high probability of occurring. By simulating and modeling the HTS, it becomes possible to study and predict the effects of these extreme environments on the human body.

In Rollins, et al. [1], we recently proposed a model building approach from a system identification perspective (see [4] for a definition). System identification in our context is the determination of *how* input variables affect the HTS response in contrast to theoretical modeling that also seeks to understand *why*. The basic goal of our approach is to develop a mechanism to obtain a HTS model for an individual without subjecting the individual to an environmental chamber experiment. Essentially the idea is to create a library of model structures with the assistance of theoretical knowledge and extensive test subject data from environmental chambers over the domain of human attributes and experimental conditions (see [1] for more details). The basis of this approach consists of four beliefs or hypotheses. The first one is that HTS dynamic model structures can adequately be described by transfer function models of the type that can be found in any textbook on process control such as [5]. The second one is that the parameters for these model structures, which are physically interpretable, can be approximated adequately from the subject’s attributes, the environmental conditions, and the applicable physical property data. In the next section we provide support for our first two hypotheses by taking a theoretical model from literature and deriving transfer function forms while defining physical parameters that
depend on the type of variables mentioned. Thirdly, it is hypothesized that HTS structure variations can be related to physical attributes and environmental conditions. We will also discuss our reasoning for this hypothesis in the next section. Our fourth hypothesis is that complex nonlinear static and nonlinear dynamic behavior can be approximated adequately from block-oriented models (BOM). Our success in two modeling problems in [1] provides support for this hypothesis. In Section 3 we discuss BOM and present details of the BOM approach we used for the study in this article. Therefore, upon successful completion of this library and under these hypotheses, one would obtain a model for a subject by specifying their attributes and working environment as shown visually in Fig. 1 that was taken from [1].

![Diagram](image)

**Figure 1.** The library concept of [1] to identify BOM’s for individual subjects using cataloged model structures, the environmental conditions, and their attributes and physical property data. To obtain a model for a subject, their personal attributes and environmental conditions assist in selecting the model structure from the catalog, and their personal attributes and physical property data along with the environmental conditions are used to estimate model parameters.
As mentioned above, in [1] we demonstrated the ability of our BOM method to accurately model real data from literature [6] and computer generated data from a HTS semi-theoretical model with qualitatively accurate physiological behavior [7, 8]. Since the success of this approach will require extensive environmental chamber data collection to obtain the library of model structures, it is important to minimize the number of experimental runs and the run time to develop a model. Using the D-Optimality criterion [9] in this article, we demonstrate a 70% reduction in experimental time in comparison to the modeling in [1] using the model in [7, 8] as a surrogate person. The supporting work of this study is presented in Sections 4 to 7. Section 4 presents a description of the experimental approach used for process identification. Sections 5 and 6, respectively, describe the optimization of the number of experimental trials, and the optimization of the experimental time or duration. The results follow this description in Section 7 and concluding remarks are given in the last section (8).

2. A CASE FOR TRANSFER FUNCTION MODELS

As we stated in the previous section, the dynamic model structures we use are transfer function models. The model identification approach that we use can be classified as semi-empirical. That is, the transfer function model is obtained using experimental data and the function providing the best fit in this class of structures that is typically a low order form with dead time. This approach does not derive these forms using theoretical knowledge. However, the strength of these forms, as we illustrate in this section, is that they are able to represent dynamic structures consistent with HTS theory. In this illustration we also demonstrate how theory can be used to obtain equations to estimate parameter values from physical quantities.

Using the “standard man” concept of Seagrave [10], Downey [11] developed the following differential equations from energy balances on the skin and core, respectively:
\begin{align*}
\frac{m_s C_p \, dT_s(t)}{dt} &= M_{0,s} + K_{MS} (T_M - T_s(t)) - Q_v(t) - hA(T_s(t) - T_A(t)) + Q_s \rho C_p^B (T_c(t) - T_s(t)) \\
&\quad + Q_A \rho C_p^B (T_A(t) - T_c(t)) + Q_A \rho_{air} \left( H(t) - H^E \right) \Delta H_V^{H,O} \left( \bar{T}_c \right) \\
\frac{m_c C_p \, dT_c(t)}{dt} &= M_{0,c} - K_{CM} (T_c(t) - T_M) - (Q_m - Q_s) \rho C_p^B \left( T_c(t) - \left[ \frac{Q_m T_m(t) + Q_s T_s(t)}{Q_m + Q_s} \right] \right) \\
&\quad + Q_A \rho C_p^B (T_A(t) - T_c(t)) + Q_A \rho_{air} \left( H(t) - H^E \right) \Delta H_V^{H,O} \left( \bar{T}_c \right)
\end{align*}

where \( T_s(t) \) is the skin temperature, \( T_c(t) \) is the core temperature, \( \bar{T} \) is the average core temperature (assumed constant), \( T_A(t) \) is the ambient (i.e., environmental) temperature, \( Q_v(t) \) is the sweat rate, \( H(t) \) is the humidity of the environment, and the other variables are defined in the Nomenclature Section. Note that, \( \rho C_p^B \) in the term next to the last one in Eq. 2 should perhaps be \( \rho_{air} C_{p,air} \cdot \) However, this correction will not affect the purpose of this exercise that is to show that under certain conditions this theoretical model can be converted to low order transfer functions that we will illustrate for \( T_s \). Note that the inputs are \( T_A, Q_v, \) and \( H \). From an energy balance on the muscle, [11] also derived an equation for the muscle temperature. However, for simplicity, we are assuming that the muscle temperature is constant, which represents the case of no exercise (i.e., the subject is resting). We will now derive transfer functions for \( T_s \) relative to the three inputs.

We first rearrange Eq. 1 to a form for defining gains and time constants.

\begin{align*}
\frac{m_s C_p \, dT_s(t)}{dt} &= \left( K_{MS} + hA + Q_s \rho C_p^B T_c(t) \right) T_s(t) \\
&\quad + M_{0,s} + K_{MS} T_M - Q_v(t) + hA T_A(t) + Q_s \rho C_p^B T_c(t) \\
\end{align*}

with

\begin{align*}
\tau_1 &= \frac{m_s C_p}{K_{MS} + hA + Q_s \rho C_p^B} \\
K_1 &= \frac{1}{K_{MS} + hA + Q_s \rho C_p^B}
\end{align*}
K_2 = \frac{hA}{K_{MS} + hA + Q_S \rho C_p^B} \quad (6)

K_3 = \frac{Q_S \rho C_p^B}{K_{MS} + hA + Q_S \rho C_p^B} \quad (7)

Substituting Eqs. 4-7 into Eq. 3 and putting it into deviation variable form (see [5]) gives

\tau_1 \frac{dT'_S(t)}{dt} + T'_S(t) = -K_1 Q'_V(t) + K_2 T'_A(t) + K_3 T'_C(t) \quad (8)

Taking the Laplace transform of Eq. 8 and rearranging gives

(\tau_1 + 1)T'_S(s) + T'_S(s) = -K_1 Q'_V(s) + K_2 T'_A(s) + K_3 T'_C(s) \quad (9)

Applying the same process to Eq. 2 to derive the s-domain equation for T_C gives:

\begin{align*}
\frac{dT'_C(t)}{dt} &+ \left[K_{CM} + (Q_M + Q_S + Q_A) \rho C_P^B \right] T'_C(t) = M_{0,C} + \left(K_M + \rho C_P^B Q_M \right) T_M \\
&+ \rho C_P^B Q_S T'_S(t) - Q_A \rho C_P^B T'_A(t) + Q_A \rho_{air} \left( H(t) - H^E \right) \Delta H_{V,h}^H (T_C) \\
\Rightarrow (\tau_2 s + 1)T'_C(s) + T'_C(s) &= K_4 T'_S(s) + K_5 T'_A(s) + K_6 H'(s) \quad (10)
\end{align*}

with

\tau_2 = \frac{m_C C_p}{K_{CM} + (Q_M + Q_S + Q_A) \rho C_P^B} = \frac{m_C C_p}{C_1} \quad (12)

\begin{align*}
K_4 &= \frac{Q_S \rho C_P^B}{C_1} \quad (13) \\
K_5 &= \frac{Q_A \rho C_P^B}{C_1} \quad (14) \\
K_6 &= \frac{Q_A \rho_{air} \Delta H_{V,h}^H (T)}{C_1} \quad (15)
\end{align*}

Solving Eq. 11 for T'_C(s) and substituting it into Eq. 9 gives after rearrangement
\[
T_s'(s) = \frac{\tau_2 s + 1}{(\tau_1 s + 1)(\tau_2 s + 1) + K_4 K_3}(-K_1)Q_v'(s) + \frac{K_2 (\tau_2 s + 1) + K_3 K_5}{(\tau_1 s + 1)(\tau_2 s + 1) + K_3 K_4}T_A'(s) + \frac{K_3 K_6}{(\tau_1 s + 1)(\tau_2 s + 1) + K_3 K_4}H'(s)
\]  
(16)

and the following transfer functions:

\[
\frac{T_s'(s)}{Q_v'(s)} = -K_1 \frac{\tau_2 s + 1}{(\tau_1 s + 1)(\tau_2 s + 1) + K_3 K_4} \quad \text{[with } T_A'(s) = H_s'(s) = 0]\]
(17)

\[
\frac{T_s'(s)}{T_A'(s)} = \frac{K_2 (\tau_2 s + 1) + K_3 K_5}{(\tau_1 s + 1)(\tau_2 s + 1) + K_3 K_4} \quad \text{[with } Q_v'(s) = H_s'(s) = 0]\]
(18)

\[
\frac{T_s'(s)}{H'(s)} = \frac{K_3 K_6}{(\tau_1 s + 1)(\tau_2 s + 1) + K_3 K_4} \quad \text{[with } Q_v'(s) = T_A'(s) = 0]\]
(19)

In standard form, Eqs. 17-19 become

\[
\frac{T_s'(s)}{Q_v'(s)} = -\frac{K_1}{1 + K_4 K_3} \frac{\tau_2 s + 1}{\tau^2 s^2 + 2 \tau \xi s + 1} \quad \text{[with } T_A'(s) = H_s'(s) = 0]\]
(20)

\[
\frac{T_s'(s)}{T_A'(s)} = \frac{K_2 + K_3 K_5}{1 + K_3 K_4} \frac{\tau_2 s + 1}{\tau^2 s^2 + 2 \tau \xi s + 1} \quad \text{[with } Q_v'(s) = H_s'(s) = 0]\]
(21)

\[
\frac{T_s'(s)}{H'(s)} = \frac{K_3 K_6}{1 + K_4 K_3} \frac{1}{\tau^2 s^2 + 2 \tau \xi s + 1} \quad \text{[with } Q_v'(s) = T_A'(s) = 0]\]
(22)

with

\[
\tau^2 = \frac{\tau_1 \tau_2}{1 + K_3 K_4}
\]
(23)

\[
2 \tau \xi = \frac{\tau_1 + \tau_2}{1 + K_4 K_3}
\]
(24)
Note that Eqs. 20-21 are second-order-plus-lead (SOPL) transfer functions and Eq. 22 is a second-order (SO) transfer function -- two common types of transfer functions. Note that for $T_\text{S}$, [1] obtained a SO transfer function in the real data study and a SOPL transfer function in the surrogate person data study. In the study we present in this work later, a SOPL transfer function will be used for $T_\text{S}$. Thus, based on theoretical considerations, this exercise supports our hypothesis that transfer function models have phenomenological structures that can represent thermoregulatory responses. This exercise also demonstrates how we might be able to obtain estimates for model parameters as we have derived their functional dependence on subject attributes and physical property data in support of our second hypothesis stated in the Introduction. Finally, our third hypothesis, that the structures can vary with conditions, is supported by realizing that if the model in [11] for muscle temperature became important (i.e., the subject is no longer resting but exercising), the structures for $T_\text{S}$, as derived by the process in this section, would be third order and possibly fitting well to second-order-plus-dead-time-plus-lead (SOPDTPL) structures or just SOPDT structures. In the next section we discuss block-oriented modeling and consider its ability to extend transfer function modeling to the treatment of nonlinear static and nonlinear dynamic behavior.

3. BLOCK-ORIENTED MODELING

The theoretical model equations treated in the previous section were linear with respects to static and dynamic behavior. However, as model coefficients vary more strongly with time and space, both the static and the dynamic behavior will become more nonlinear. As described in [1], we address these modeling complexities using block-oriented modeling (BOM). In BOM
static nonlinear (N) blocks (i.e., functions) and linear (L) dynamic blocks (i.e., dynamic transfer functions) are connected in series and parallel “sandwich” networks [12]. The NL network is formally called a “Hammerstein system” (see [13]) and the LN network is formally called a “Wiener system” (see [14]). Block-oriented systems are capable of representing physical processes with nonlinear static and nonlinear dynamic behavior. For example, in the Wiener system each input enters a linear dynamic block and the outputs from these blocks are inputs to the nonlinear static function. In this section we describe the Hammerstein system, shown in Fig. 2, in detail since it was used in the study we present later.

The specific BOM method we use for this study is based on a compact closed-form continuous-time exact solution to the Hammerstein system derived by [13] and validated for the surrogate subject by [1]. This approach is called the Hammerstein Block-oriented Exact Solution Technique or H-BEST and we use in this article what we called in [1] the “restricted” prediction algorithm. In [1] we used our “classical” H-BEST prediction algorithm so in this section we described the restricted algorithm in detail before applying it in this study.

The H-BEST closed-form “restricted” solution to a Hammerstein system for a series of step input changes at time $t = 0, t_1, t_2 \ldots$, (given in Eq. 26) is given in Eq. 27 below:

$$u(t) = \begin{cases} u(0) & \text{for } 0 < t \leq t_1 \\ u(t_1) & \text{for } t_1 < t \leq t_2 \\ \vdots \end{cases}$$

$$0 < t \leq t_1: \quad y_i(t) = y_i(0) + f_i(u(0); \beta) g_i(t; \tau)$$

$$t_1 < t \leq t_2: \quad y_i(t) = y_i(t_1) + \left[ f_i(u(t_1); \beta) - y_i(t_1) + y_i(0) \right] g_i(t-t_1; \tau)$$

$$t_2 < t \leq t_3: \quad y_i(t) = y_i(t_2) + \left[ f_i(u(t_2); \beta) - y_i(t_2) + y_i(0) \right] g_i(t-t_2; \tau)$$

and so on….
where \( y_i(t) \) is the output of response \( i \) at time \( t \); \( y_i(0) \) is the measured value of output \( i \) at the initial time zero; \( u(t) \) is an input vector that contains the deviation values of the process input variables at time \( t \); \( f_i(u(t)) \) is the nonlinear static gain function for response \( i \); \( \beta \) is the vector of parameters in the static gain function; and \( \tau \) is the vector of dynamic parameters. The dynamic function, \( g_i(t; \tau) \), is described by Eq. 28 below.

\[
g_i(t; \tau) = L^{-1}\left( G_i(s) \cdot \frac{1}{s} \right)
\]  

(28)

where \( L^{-1} \) is the inverse Laplace transform operator; and \( G_i(s) \) is the linear dynamic transfer function for response \( i \) of the process as described in Fig. 2.

**Figure 2.** A description of the general multiple input, multiple output (MIMO) Hammerstein model structure. The input vector \( u \) passes through static maps and produces the \( f_i(u) \)'s which can be nonlinear and they pass through the linear dynamic maps, the \( G_i(s) \)'s, and produce the outputs (i.e., the \( y_i \)'s).

**4. EXPERIMENTAL MODELING APPROACH**

In this section we apply H-BEST to develop HTS models in this study. As we stated in the Introduction, the main objective of this study is to find a practical and optimal experimental design for modeling the skin temperature and sweat rate responses for changes in the
environment by using the Wissler computer program as a surrogate subject. The input variables chosen for this study are the ambient temperature \((T)\), relative humidity \((H)\), and wind speed \((W)\).

Specifically, the approach taken in this study is to first use statistical design of experiment (SDOE) to identify the H-BEST models from a small amount of experimental trials. After that the H-BEST solution is used to further reduce the number of experimental trials, while maintaining high information content for estimating the parameters. Thirdly, the H-BEST algorithm is used to minimize the length of each experiment trial, again without loss of critical information in estimating the parameters.

The methodology for developing an H-BEST model is described in the following steps and follows the model building procedure in [1]. The first step is to use an appropriate SDOE for the chosen input variables over the input space. By using SDOE, the model parameters can be estimated from a conservative number of experimental trials. The second step is to run each experimental trial allowing the process to reach a new steady state each time and to collect data over time for each run. The third step is to use the steady state data to model the nonlinear static gain functions \(f_i(u(t); \beta), \ i = 1, 2\). The fourth and last step is to use the transient data to determine the form of the dynamic functions, the \(g_i(t; \tau)\)’s, and estimate the dynamic parameters.

The H-BEST models in this study are obtained by using a Box-Behnken experimental design (BBD) of 13 runs. There is no need to replicate the center point of the design since the experimental data come from a computer simulation without random error added. Hence, the simulation part of this study consists of executing the 13 experimental trials in Table 1 and plotting the output responses. The initial conditions simulate a man in a sitting position wearing a cotton shirt and pants, with a weight of 182 lbs., a skinfold thickness 12 mm., a resting metabolic
rate of 285.76 BTU/hr, and in an environment with a temperature of 80 °F, an air pressure of 14.969 psia, a relative humidity of 50%, and a wind speed of 3 mph. These conditions were used in [1] and will be the base case for this study.

Table 1. The 13 experimental trials of the BBD.

<table>
<thead>
<tr>
<th>Run #</th>
<th>T (°F)</th>
<th>W (mph)</th>
<th>H (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94</td>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>94</td>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>3</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>94</td>
<td>1</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>98</td>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
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<td>75</td>
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<tr>
<td>7</td>
<td>90</td>
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<td>13</td>
<td>90</td>
<td>3</td>
<td>85</td>
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</tbody>
</table>

As stated earlier, the steady state data from the 13 experimental trials in Table 1 were used to derive the nonlinear static gain functions, \( f_i(u(t), \beta) \), by regression. The dynamic transfer function model forms for the two responses were selected by visually inspecting the output responses. For the skin temperature response, a second-order-overdamped-plus-lead model was
selected. For the sweat rate response, a second-order-critically-damped-plus-lead-plus-dead-time model was selected. Estimates for the dynamic parameters in the linear dynamic models of the responses are obtained by using nonlinear regression to fit the dynamic response for each experimental trial. The dynamic parameters do not change significantly between the runs so the average values over the 13 runs were used in this study. The H-BEST static gain and dynamic models for the skin temperature and sweat rate are given in Eqs. 29-30, and Eqs. 31-32, respectively.

**Skin temperature static gain and dynamic functions:**

\[
\tilde{f}_{\text{Skin}}(T', H', W'; \beta) = 0.81 + 0.29 * T' + 0.013 * H' - 0.15 * W' - 0.0056 * T'^2 - 0.0011 * T' * H' + 0.0094 * T' * W'
\]

\[
\tilde{g}_{\text{Skin}}(t; \tau) = 1 + \frac{(\tau_a - \tau_1)}{(\tau_1 - \tau_2)} e^{-\tau_1} + \frac{(\tau_a - \tau_2)}{(\tau_2 - \tau_1)} e^{-\tau_2}
\]

\[\hat{\tau}_1 = 1.75\]

\[\hat{\tau}_2 = 40.69\]

\[\hat{\tau}_a = 14.27\]

**Sweat rate static gain and dynamic functions:**

\[
\tilde{f}_{\text{Sweat}}(T', H', W'; \beta) = 4.8 - 0.54 * T' - 0.13 * H' - 0.091 * W' + 0.017 * T'^2 + 0.011 * T' * H'
\]

\[
g_{\text{Sweat}}(t; \theta) = \left[ 1 + \left( \frac{\tau_a - \tau}{\tau_2} \right) (t - \hat{\theta}) \right] e^{-(t - \hat{\theta})/\tau}
\]

\[\hat{\theta} = 376.06 - 33.83 * T' + 16.31 * W' + 0.82 * T'^2 - 0.94 * T' * W'\]

\[\hat{\tau} = 50.94\]

\[\hat{\tau}_a = 32.81\]

where the terms \(T', H', W'\) are the deviation variables from the initial steady state; and \(\hat{\theta}\) is the dead time parameter, which is estimated by using linear regression, since the dead time varies
over the input space. The estimates for the parameters in the steady state gain and the dynamic model are all significant at the 0.05 level.

Figure 3 gives the Wissler simulated values and the H-BEST fit for the skin temperature and sweat rate data for Run 3. The next step was to use these fitted models to minimize the number of experimental trials. The approach taken for this optimization of the number of experimental trials is described next in Section 5.

![Figure 3](image_url)

**Figure 3.** The dynamic skin temperature (left) response and the dynamic sweat rate response (right) from the Wissler simulation and the fitted H-BEST model for Run 3 in Table 1.

## 5. OPTIMIZING THE NUMBER OF EXPERIMENTAL TRIALS

One of the main objectives of this study is to minimize the number of experimental trials needed to develop the H-BEST model without losing critical information for accurately estimating the parameters. When minimizing the number of trials, the number of parameters to be estimated needs to be taken into consideration. For each experimental run, only one steady state or ultimate response datum point is observed but several transient response points can be collected. Thus the number of parameters in the static gain model will determine the minimum number of runs needed. There are seven parameters in the static gain model for the skin
temperature (Eq. 29) and six parameters in the static gain model for the sweat rate (Eq. 31).

Therefore, at least seven runs are needed for estimating the parameters in the H-BEST models.

The optimal design in this study is based on the D-optimality criterion, which seeks to find the design that minimizes the volume of the confidence region of the parameter estimates. That is, the size of the volume of the confidence region reflects how well the set of parameters is estimated. For statistically linear models, the square of the volume of this confidence region is inversely proportional to the determinant of $X^T X$ (i.e., $|X^T X|$), where $X$ is the model input matrix [9]. Hence, a D-optimal design (DOD) is one in which $|X^T X|$ is maximized and therefore it is also referred to as the determinant criterion in literature. From a geometrical point of view, the D-optimal criterion implies a design in which the columns of $X$ each represent a vector that is as long as possible and are orthogonal column vectors. Thus, the DOD will be spread out as much as possible over the input space and the parameter estimates will not be correlated.

The D-optimal criterion was applied to nonlinear functions as early as 1959 by Box and Lucas [15], where in place of the $X$ matrix, the derivative matrix $V^0$ evaluated at some initial parameter estimate $\theta^0$ was used. As stated in [9], for nonlinear models, the D-optimal criterion is the design that maximizes $|V^0 X^0|$. Thus, the derivatives of the response are taken for each run with respect to the parameters in the model. Hence, if the experimental function is linear in the parameters, then $V^0 X^0$ is equal to $X^T X$.

There are many statistical software packages that can find a DOD for a linear expectation function. However, for nonlinear expectation functions there does not appear to be a commercially available software package that can generate a DOD. Hence, finding the DOD for this study will be challenging, since the H-BEST model used in this study is a nonlinear function.
in the parameters. As stated earlier, the parameters in the static gain function have a greater impact on the number of runs needed. If we choose to use a saturated DOD based only on the static gain equation for the two responses, we can use a commercially available software package to generate the DOD since these equations are linear in parameters. However, the design will not be D-optimal for dynamic parameters, but since they are considerably fewer in number than the static gain parameters, we ought to obtain accurate estimates for them as well.

The required sample size for a saturated design for the skin temperature response is seven trials and for the sweat rate response it is six trials. The DOD for the two responses were generated separately by the SAS version 8.02, SAS Institute Inc. software package. The SAS software package finds a design for which the $|XX^T|$ is maximized out of a list of candidate trials. For this study the list of candidate trials was comprised from a full 3-level factorial design with $3^3 = 27$ runs. The DOD for the skin temperature is given in Table 2 and the saturated DOD for the sweat rate is given in Table 3. The same designs were generated when executing the computer code five times in a row for different guesses of initial designs, which indicates that we are confident of having found the optimal designs.

From the SAS output, the determinant of the information matrix, i.e., $|X^T X|$, is $e^{9.7041}$ for the skin temperature response and $e^{8.3178}$ for the sweat rate response. Some of the experimental trials in Tables 2 and 3 are identical. However, we would still end up with a design with a number of runs close to 13 (7 trials for skin temperature + 6 trials for sweat rate - 3 trials in common = 10 trials) when using the two designs separately to develop the H-BEST models.
Table 2. The saturated D-optimal design for the skin temperature

<table>
<thead>
<tr>
<th>Run #</th>
<th>T (°F)</th>
<th>H (%)</th>
<th>W (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>75</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>85</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>85</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>94</td>
<td>85</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>98</td>
<td>75</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
<td>85</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>98</td>
<td>85</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3. The saturated D-optimal design for the sweat rate

<table>
<thead>
<tr>
<th>Run #</th>
<th>T (°F)</th>
<th>H (%)</th>
<th>W (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>75</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>85</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>94</td>
<td>75</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>94</td>
<td>85</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>98</td>
<td>75</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
<td>85</td>
<td>1</td>
</tr>
</tbody>
</table>

Note that the static gain models for the two outputs (Eqs. 29 and 31) differ only by the T'W' interaction term, which is not significant in the sweat rate response as given in Eq. 31. This similarity in the models suggests that we could use the design that optimizes the skin temperature for parameter estimation of both responses. Additionally we can evaluate how the information to
get parameters for sweat rate is impacted and ensure that there is not a significant loss of information in parameter estimation when developing the H-BEST model for the sweat rate response using the design in Table 2 instead of the design from Table 3. A measure of the fraction of information that is lost can be calculated using Eq. 33 below.

\[
\eta_D^* = \left( \frac{|X^T X|_{D1}}{|X^T X|_{D2}} \right)^{1/p}
\]

(33)

where \( p = 6 \) is the number of parameters in the static gain for sweat rate model. \( D1 \) is the design in Table 2 and \( D2 \) is the design in Table 3. The \( |X^T X|_{D1} \) is found manually by creating the model matrix, \( X \), for the seven runs in Table 2 based on the static gain function of the sweat rate. The \( |X^T X|_{D2} \) is the value calculated by SAS. This is a modification of the D-efficiency as stated in [16]. In this modified version of efficiency the number of experimental trials in each design is not taken into account. Recall that the objective here is to use the seven runs given by the design in Table 2 without losing significant information in estimating the parameters for the static gain of the H-BEST sweat rate prediction. Thus, \( \eta_D^* \) from Eq. 33 provides a quantitative measure of the information content without taking the design size in consideration. The S-plus Version 6.0, Lucent Technologies inc., software package was used to find these determinants. The value of \( \eta_D^* \) is almost 1 (= 0.9999944). Therefore, there is practically no loss of information. Hence, the design from Table 2 appears to be a good choice to model both the skin temperature and sweat rate.

The H-BEST models were re-evaluated by running the seven experimental trials from the DOD in Table 2, and the dynamic and static parameters were re-estimated using the same procedure as before. Recall that the linear dynamic transfer function used for skin temperature response is a second-order-overdamped-plus-lead model, and the dynamic transfer function for
the sweat rate is a second-order-critically-damped-plus-lead-plus-dead time-model. The new H-BEST models developed from for the skin temperature and sweat rate are given in Eqs. 34-35, and Eqs. 36-37, respectively.

**Skin temperature static gain and dynamic functions:**

\[ \hat{f}_{\text{Skin}}(T', H', W'; \hat{\beta}) = 1.16 + 0.27 * T' - 0.0078 * H' - 0.096 * W' \\
- 0.0076 * T'^2 + 0.0036 * T' * W' + 0.00096 * T'* H' \]  
\[ (34) \]

\[ \hat{g}_{\text{Skin}}(t; \hat{\tau}) = 1 + \left( \frac{\hat{\tau}_a - \hat{\tau}_t}{\hat{\tau}_t} \right) e^{-\hat{\tau}_t} + \left( \frac{\hat{\tau}_a - \hat{\tau}_2}{\hat{\tau}_2 - \hat{\tau}_1} \right) e^{-\hat{\tau}_2} \]
\[ (35) \]

\[ \hat{\tau}_1 = 1.80 \]
\[ \hat{\tau}_2 = 44.05 \]
\[ \hat{\tau}_a = 16.02 \]

**Sweat rate static gain and dynamic functions:**

\[ \hat{f}_{\text{Sweat}}(T', H', W'; \hat{\beta}) = 4.03 - 0.56 * T' - 0.14 * W' - 0.10 * H' \\
+ 0.018 * T'^2 + 0.011 * T' * H' \]  
\[ (36) \]

\[ \hat{g}_{\text{Sweat}}(t; \hat{\tau}) = 1 + \left( \frac{\hat{\tau} - \hat{\bar{\tau}}}{\hat{\bar{\tau}}} \right) e^{-(t - \hat{\bar{\tau}})} \]
\[ (37) \]

\[ \hat{\bar{\tau}} = 298.44 - 24.31 * T' + 11.56 * W' + 0.55 * T'^2 - 0.66 * T' * W' \]
\[ \hat{\tau} = 43.41 \]
\[ \hat{\tau}_a = 23.31 \]

The next step in this study is to use the DOD and quantitatively measure the effect of reducing the time length or the duration of the experimental trials.

**6. OPTIMIZING THE DURATION OF THE DESIGN POINTS**

As stated earlier, the third and last objective in this study is to reduce the time of each experimental trial without losing significant information for parameter estimation. When
developing H-BEST models from the BBD and the DOD in this study, the Wissler computer program simulated values of the two responses every 10 minutes for 360 minutes. The reason for using such a long experimental time was to get close to their steady state levels, and since simulated data are used for this study, the length of the experiment is not a concern. However, a more practical design using human subjects would greatly benefit by shorter experimental trials.

We will again use the D-optimality criterion to assist in this objective to obtain shorter times for trials. Since the H-BEST solution is a nonlinear function, maximizing the information matrix involves maximizing $|V^0 V^0^T|$, where $V^0$ is a derivative matrix evaluated at the parameter estimates derived from the seven run design in Table 2 (see Eqs. 34 - 37). Before the $V^0$ matrix is constructed, one assumption is made to simplify calculations. We assume that the dynamic parameters are constant. This assumption is based on the fact that the static gain parameters are estimated by using the experimental data sampled towards the end of the experiment. The dynamic parameters, on the other hand, are estimates from the data sampled during the middle of the experiment, when most of the change in the responses occurs. Hence, the static gain parameters will be more affected than the dynamic parameters when reducing the sampling time of an experiment. Thus, the $V^0$ matrix will be constructed as shown in Eq. 38 below.

$$V^0 = \begin{bmatrix}
\frac{\partial y(t)}{\partial \beta_0} & \frac{\partial y(t)}{\partial \beta_1} & \cdots & \frac{\partial y(t)}{\partial \beta_k} \\
\frac{\partial y(t)}{\partial \beta_0} & \frac{\partial y(t)}{\partial \beta_1} & \cdots & \frac{\partial y(t)}{\partial \beta_k} \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
$$

Evaluated at $u_1$ and $t = t_k$

$$\begin{bmatrix}
\frac{\partial y(t)}{\partial \beta_0} & \frac{\partial y(t)}{\partial \beta_1} & \cdots & \frac{\partial y(t)}{\partial \beta_k} \\
\frac{\partial y(t)}{\partial \beta_0} & \frac{\partial y(t)}{\partial \beta_1} & \cdots & \frac{\partial y(t)}{\partial \beta_k} \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
$$

Evaluated at $u_7$ and $t = t_k$

where $y(t)$ is the H-BEST solution from the DOD, as given in Eqs. 34 - 37; $u(t)$ is the vector of input deviation variables at time $t$ shown in Table 2; and $\beta_0, \ldots, \beta_k$ are the static gain parameters for each of the responses, where $k = 0, \ldots, 6$ for skin temperature and $k = 0, \ldots, 5$ for sweat rate.
The effect of different lengths of experimental time on the information matrix, $\|V^0 V^0\|$, can be investigated for the two responses. However, since the sweat rate response has a much larger lag time than the skin temperature response, the sweat rate response will reach steady state much later in each experiment than the skin temperature response. Thus, shortening the design time or the experimental duration will have a stronger affect on the estimation of parameters in the sweat rate model compared to the parameters in the skin temperature model. A measure of information lost by reducing the experimental time can be calculated according to D-efficiency in Eq. 39.

$$
\eta_{D_i} = \left( \frac{\|V^0 V^0\|_{t_i}^{1/p}}{\frac{N^p}{t_i}} \right) \left( \frac{\|V^0 V^0\|_{t_{max}}^{1/p}}{\frac{N^p}{t_{max}}} \right)$$

where $p$ is the number of parameters in the response models ($p = 7$ for skin temperature and $p = 6$ for sweat rate); $t_i$ represents the total experimental time investigated; $t_{max} = 360$ minutes, since this is the maximum allowed run time for the Wissler program; and $N$ is the total sample size used for building the information matrix, i.e., $N = r \times n$, where $r$ is the number of runs ($r = 7$) and $n$ is the number of points sampled for each run. The information ratio from Eq. 39 for different total experimental times when sampling every 10 minutes is given in Fig. 4.

As stated earlier, the sweat rate response is affected to a greater extent when shortening the length of the experimental trials. That is, in Fig. 4 we can see that D-efficiency for the sweat rate responses decreases faster than D-efficiency for the skin temperature response, when reducing the experimental times.
Figure 4. D-efficiency as calculated by Eq. 39 for different lengths of experimental times for both the skin temperature response and sweat rate response with a sampling rate of once every 10 minutes.

7. RESULTS

By using the closed-form H-BEST models in a DOD, the required number of trials for a two-output, three-input system is reduced from 13 to 7. The ability of H-BEST models (developed from DOD) to predict the HTS was investigated for a randomly generated input sequence also called the test sequence, as shown in Fig. 5. The simulated data by Wissler and the responses predicted by H-BEST obtained from the BBD and the DOD for the skin temperature and sweat rate are given in Fig. 6.

The predictions by H-BEST obtained by using the BBD and the DOD agree well with the simulated data. That is, there is no significant deviation between the simulated data and the predicted data. Thus, the predictive models from the DOD with almost half as many experimental trials seem to predicts almost as well as the predictive models from the BBD. This close agreement is seen in both output responses.
Figure 5. The input sequence used for evaluating the H-BEST models obtained from the BBD and the DOD.

Figure 6. The skin temperature response (left) and the sweat rate response (right) from the Wissler simulation and the H-BEST models obtained from the BBD and DOD to the test input sequence shown in Fig. 5.

The second objective was to ascertain if the experimental time for each run could be reduced without losing too much information in the parameter estimates. The H-BEST models derived for the BBD and the DOD were both developed from Wissler simulated data that are sampled every 10 minutes for total experimental time of 360 minutes. The total time of the experiments need to be reduced in order to identify models from data collected when using humans as subjects, instead of simulations. From Fig. 4 we can conclude that D-efficiency for the skin temperature response decreases slowly for experimental times above about 200 minutes. Hence, it appears that the experimental time can be reduced to 210 minutes without significant
loss of information. Figure 7 shows the output responses from the Wissler simulation and predicted responses using H-BEST derived from the DOD in Table 2 for experimental durations of 360 min. and 210 min. for the test sequence shown in Figure 5.

The skin temperature predictions by the H-BEST models derived from the DOD in Table 2 for the reduced experimental time closely follows the simulated data. In other words, there is no significant difference in the skin temperature predictions from H-BEST for the two different experimental durations. The same conclusion can be reached for the prediction of sweat shown in Fig. 7. That is, the sweat rate predictions for the reduced experimental length closely follows the simulated data. The generated sweat rate data have a lot of noise in them but H-BEST captures this response very well. Hence, it appears that the total experimental time can be significantly reduced without losing critical information needed for estimation of the H-BEST models. That is, it seems that the skin temperature and sweat rate responses can be accurately predicted from models obtained using only seven experiments with each using a total
experimental time of 210 minutes. This implies an overall reduction of about 70% when compared to the initial BBD with 13 runs and a total experimental time of 360 minutes for each run used in [1].

8. CONCLUDING REMARKS

The main objective of this study was to develop optimal dynamic models for the HTS accurately and feasibly. The skin temperature and sweat rate responses were generated using the Wissler [7, 8] computer model as a surrogate human under different environmental conditions. The optimization involved using a saturated design to minimize the number of experimental trials as well as reducing the total time of each experiment. Using this methodology, we found an efficient and practical procedure to model the HTS using human subjects.

Sweat rate and skin temperature were modeled using a methodology based on a closed-form continuous-time exact solution to a Hammerstein process called H-BEST. The study showed that H-BEST accurately predicted the responses of skin temperature and sweat rate for changes in the environmental temperature, relative humidity, and wind speed. Moreover, model identification was achieved from a design with only seven runs and a total experimental time of 210 minutes for each run, a 70% reduction from the work of Rollins, et al. [1]. Thus, our plan is to use saturated designs with a run time of about three hours in the extensive experimental program to build the library of model structures.

ACKNOWLEDGEMENT

The authors are grateful to Dr. Max Morris, Statistics Department, Iowa State University for his helpful suggestions.
**NOMENCLATURE**

\[ \begin{align*}
A &= \text{Body surface area} \\
C_p &= \text{Heat capacity of the body} \\
C_p^B &= \text{Heat capacity of the blood} \\
h &= \text{Overall convective heat transfer coefficient} \\
H &= \text{Relative humidity of the environment} \\
H^E &= \text{Relative humidity of the expired air} \\
\Delta H_{V}^{H^E,0} (\overline{T}_c) &= \text{Heat of vaporization of water at } \overline{T}_c \\
K_{CM} &= \text{Conductivity between core and muscle layers} \\
K_{MS} &= \text{Conductivity between muscle and skin layers} \\
L &= \text{Linear dynamic block} \\
m &= \text{Body mass} \\
m_j &= \text{Mass of compartment } j \ [j = (C)\text{ore}, (M)\text{uscles}, \text{or (S)kin}] \\
M_{Oj} &= \text{Metabolic oxygen generation rate in the } j\text{th compartment} \ [j = (C)\text{ore}, (M)\text{uscle, or (S)kin}] \\
\Delta M &= \text{Additional metabolic conversion rate (in the muscle compartments) due to exercise} \\
N &= \text{Static nonlinear block} \\
Q_A &= \text{Ventilation rate} \\
Q_j &= \text{Blood flow rate to the } j\text{th compartment} \ [j = (C)\text{ore}, (M)\text{uscle or (S)kin}] \\
Q_V &= \text{Sweat rate (evaporative heat loss)} \\
T_j &= \text{Temperature in compartment } j \ [j = (C)\text{ore}, (M)\text{uscle, (S)kin, or (A)mbient}] \\
T &= \text{Temperature of the environment (ambient)} \\
W &= \text{Wind speed of ambient air} \\
t &= \text{time} \\
\mathbf{u} &= \text{vector of input variables} \\
\mathbf{y} &= \text{vector of output variables} \\
\end{align*} \]

**Greek Letters**

\[ \begin{align*}
\beta &= \text{Vector of parameter estimates of the static gain function of the outputs} \\
\tau &= \text{Vector of parameter estimates of the dynamic function of the outputs} \\
\theta &= \text{Dead time for the dynamic function of the sweat rate response} \\
\rho &= \text{Density of the blood} \\
\rho_{\text{air}} &= \text{Density of the air} \\
\end{align*} \]

**Subscripts**

\[ \begin{align*}
T_{\text{skin}} &= \text{Skin Temperature} \\
\text{Sweat} &= \text{Sweat rate} \\
\end{align*} \]

**Superscripts**

\[ ^\wedge \text{ = Estimate} \]
Abbreviations

BBD = Box-Behnken Design
DOD = D-optimal Design
H-BEST = Hammerstein Block-oriented Exact Solution Technique
HTS = Human thermoregulatory system
MIMO = Multiple-input, multiple-output
SDOE = Statistical Design of Experiments
SISO = Single-input, single-output

LITERATURE CITED


