1. Problem 7.11, Casella & Berger, (1st or 2nd Edition) Skip the part about finding the variance and showing its limit. 
   If you’re interested in finding the variance in part (a), you should be able to show that $Y_i = -\log(X_i)$ has an exponential distribution so that $\sum_{i=1}^{n} Y_i$ is a gamma. Then, you can compute the variance.

2. Problem 7.12(a), Casella & Berger (1st or 2nd Edition)


5. Suppose someone collects a random sample $X_1, X_2, \ldots, X_n$ from an exponential $\beta = 1/\theta$ distribution with pdf $f(x|\theta) = \theta e^{-\theta x}, \ x > 0$, and a parameter $\theta > 0$. (Note: this parameterization differs slightly from the one used in the textbook, see page 624 there.) However, due to a recording mistake, only truncated integer data $Y_1, Y_2, \ldots, Y_n$ are available for analysis, where $Y_i$ represents the integer part of $X_i$ after dropping all digits after the decimal place in $X_i$’s representation. (For example, if $x_1 = 4.9854$ in reality, we would have only $y_1 = 4$ available.) Then, $Y_1, \ldots, Y_n$ represent a random sample of iid (non-negative integer-valued) random variables with pmf
   
   $$f(y|\theta) = P_\theta(Y_i = y) = P_\theta(y \leq X_i < y + 1) = e^{-\theta y} - e^{-\theta(1+y)}, \quad y = 0, 1, 2, 3, \ldots$$

   (a) Show that the likelihood equals
   
   $$L(\theta) = \left[ e^{-\theta \bar{y}_n} \left(1 - e^{-\theta}\right) \right]^n$$

   where $\bar{y}_n = \sum_{i=1}^{n} y_i/n$ denotes the observed sample mean.

   (b) If $\bar{y}_n = 0$, argue that a MLE for $\theta$ does not exist on the parameter space $(0, \infty)$.

   (c) If $0 < \bar{y}_n$, show that the MLE $\hat{\theta}$ exists and must satisfy
   
   $$\bar{y}_n = \frac{1}{e^\hat{\theta} - 1}$$

   or $\hat{\theta} = \log(1 + \bar{y}^{-1})$