This is a take-home examination. The exam includes 5 questions. The total mark is 100 points. Please show all the work, not only the answers.

1. [17 points] Solve Exercise 92 in Chapter 2 of the textbook.

2. [17 points]
   (a) Solve Exercise 11 in Chapter 2 of the textbook.
   (b) Solve Exercise 12 in Chapter 2 of the textbook.

3. [17 points]
   (a) Solve Exercise 78(b) in Chapter 2 of the textbook.
   (b) Solve Exercise 78(d) in Chapter 2 of the textbook.

4. [16 points] [20 points] Sets $A$ and $B$ in a metric space are called separated if $A \cap \overline{B} = \emptyset$ and $\overline{A} \cap B = \emptyset$.
   (a) If $A$ and $B$ are two disjoint closed sets in some metric space $X$, prove that they are separated.
   (b) Prove the same for disjoint open sets.
   (c) Fix $p \in X$, $\delta > 0$, and define
       
       \[ A = \{ q \in X : d(p, q) < \delta \} \quad \text{and} \quad B = \{ q \in X : d(p, q) > \delta \}. \]
       
       Prove that $A$ and $B$ are separated.
   (d) Use the result in (c), to show that every connected metric space with at least two points is uncountable (the result is Exercise 56 in Chapter 2).

5. [16 points] Solve Exercise 63 in Chapter 2 of the textbook. Use the following definition of the compact sets in $\mathbb{R}^2$: a set $A \subset \mathbb{R}^2$ is called compact if it is closed and bounded.