C Roettger, Fall 16

Math 265 Quiz 7 – Solutions

Problem 1  Consider the function

\[ f(x, y) = x^3 + 12y - 3x^2y + 2. \]

a) Find the gradient of \( f(x, y) \) at the point \( P = (2, -3) \).
b) Find all points in the plane where the gradient of \( f(x, y) \) is parallel to the \( x \)-axis (show all work!). Sketch these points.

Solution.  a) Compute the gradient of \( f \) (as always):

\[ \nabla f(x, y) = (3x^2 - 6xy, 12 - 3x^2) \]

At \( P \), this equals

\[ \nabla f(2, -3) = (48, 0). \]

b) The gradient is parallel to the \( x \)-axis if \( f_y = 0 \), so \( 12 - 3x^2 = 0 \) which gives all points on the two vertical straight lines \( x = 2 \) and \( x = -2 \). To be very precise, we actually have to exclude the two points \((2, 1)\) and \((-2, -1)\) because there, the gradient is actually the zero vector and therefore does not have a direction, cannot be parallel to anything.