Problem 1 Find the indicated limit.

\[ L = \lim_{t \to 0} \frac{\sin(t) + 1 - \cos(4t)}{3t} \]

Solution Split it up! First,

\[ \lim_{t \to 0} \frac{\sin(t)}{3t} = \lim_{t \to 0} \frac{1}{3} \lim_{t \to 0} \frac{\sin t}{t} = \frac{1}{3}. \]

Second,

\[ \lim_{t \to 0} \frac{1 - \cos(4t)}{3t} = \lim_{t \to 0} \frac{1}{3} \lim_{t \to 0} \frac{1 - \cos(4t)}{4t} = 0. \]

Altogether the answer is

\[ L = \frac{1}{3} + 0 = \frac{1}{3}. \]

Make sure you never divide by zero in problems like this! this is probably the fastest way to lose lots of points.

Problem 2 Which choice of the constant \( a \) will make the function \( f(x) \) below continuous everywhere?

\[ f(x) = \begin{cases} \ ax - 1 & \text{for } x \leq 1 \\ 3 - x^2 & \text{for } x > 1 \end{cases} \]

Solution The function \( f(x) \) is continuous everywhere except perhaps not at \( x = 1 \), because of the two different definitions meeting there. So we need to ensure

\[ \lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x). \]

For the left-hand limit, use \( ax - 1 \) and substitute \( x = 1 \). For the right-hand limit, use \( 3 - x^2 \) and substitute \( x = 1 \) again. So we need

\[ a \cdot 1 - 1 = 3 - 1^2 = 2. \]
and therefore $a = 3$ is the only choice.

Imagine that you are supposed to attach a metal rod to a curved machine part (e.g., airplane wing). The curved part has a surface with equation $y = 3 - x^2$, for $x > 1$, and your robot lowers the metal rod with equation $y = ax - 1$ (starting at $a = 1, 1.2, \ldots$) until it touches the airplane wing. If you don’t want trial and error, you need to figure out the value of $a$ when that happens.