Math 165 – worksheet for 4.6, Applied Optimization

Remember the steps:

1. Give names to all relevant quantities.
2. Write the target quantity $f$ as function of the variables from 1.
3. Express $f$ as function of one single variable.
4. Find domain of $f$ (= allowed values of input).
5. Find critical points of $f$.
6. Find sign pattern of $f'$, or use just a table of values of $f$ to determine min/max.

**Problem 1 Pig Pens**
Given 100$m$ of wire fence, build two rectangular pens with a common side with maximal area.

**Problem 2 Gutter**
Given a 20cm wide strip of sheet metal, with parallel folds at 5 and at 15 cm, we want to build an open-top gutter by folding up the first and last 5 centimeters, leaving a base of 10 centimeters. At what angle should we fold up the two 5cm sides to give the cross-section maximum area?

**Problem 3 Distance**
Every 42 years, comet DXDY comes out of hyperspace on an orbit with equation

\[ y = \sqrt{5 + x^2}. \]

Mothership NEWTON wants to send a space probe to explore comet DXDY. NEWTON is positioned at (1, 0). Which point on the comet’s orbit is closest to the mothership? An earlier version of this worksheet had NEWTON’s
position as (0, 1), which gives a problem that can be solved, but the solution is rather obvious (graph the orbit!).

**Problem 4  Ladder** A ladder needs to be carried around a corner of a hallway. The hallway is 10 feet wide, all angles are right angles, and the ladder is just a line segment. How long can the ladder be so that we can still turn the corner of this hallway? (challenge problem: what if the width of the hallway is \(a\) feet on one side and \(b\) feet on the other?). The line segment below shows a ladder that is too long.

Hint: instead of maximizing the length of the ladder, minimize the length of a line segment that touches the corner and just fits into the hallway at the given angle.

![Diagram of ladder around corner](image)

**Problem 5  Dolphins** Build a dolphin enclosure for the Des Moines Zoo in the shape of a rectangle with two semicircles attached to opposite sides. The area is supposed to be 40,000 square feet. Given that you only need to build the two semicircles and the two other sides of the rectangle (not the sides where the semicircles are attached), which dimensions require the least amount of building material?
which is also one of the endpoints of the domain! The sign pattern of $L'$ is just $-\,$, meaning the minimum occurs at the right endpoint. The optimal dimensions are then given by $x$ as above and $y = 0$, the shape of the optimal enclosure is just a circle!