Math 165 - Practice Exam 1

Problem 1 Evaluate the following limits.

a) \( \lim_{x \to 0} \cos(\pi(x + 1)) \)

b) \( \lim_{y \to 0} \frac{\tan(4y^2)}{3y^2} \)

c) \( \lim_{t \to 1} \frac{\cos(2\pi t) - 1}{t - 1} \)

In c), you may use the addition formula of the cosine,

\[
\cos(a + b) = \cos a \cos b - \sin a \sin b
\]

Problem 2 Evaluate the limits, if they exist.

a) \( \lim_{t \to \infty} \frac{5t^4 - 2t^3 + 9}{2t^4 + 100} \)

b) \( \lim_{x \to \infty} \frac{3 - 2x + 4x^3}{3x^2 + 2x} \)

c) \( \lim_{u \to 3} \frac{5 - 2u}{u^2 - 9} \)

d) \( \lim_{v \to 3^+} \frac{5 - 2v}{v^2 - 9} \)

e) \( \lim_{w \to \infty} \frac{2w^4 - \sin(w)}{w^5 + 1} \).

Problem 3 The following functions are all defined to be 0 for \( x = 0 \). The definitions below apply to \( x \neq 0 \). Which of these are continuous at \( x = 0 \), which ones are not? Give a reason for every one of your decisions.

A: \( 2|x| - 2 \)  B: \( \frac{\sin(x)}{\sqrt{|x|}} \)  C: \( \sin \left( \frac{x}{2} \right) \)  D: \( \tan(x) \)

Problem 4 Suppose the functions \( f(x) \) and \( g(x) \) are continuous everywhere and have these values: \( f(0) = 1, f(1) = 5, f(2) = 4, g(0) = 2, g(2) = 1, g(4) = 3 \). Find \( \lim_{x \to 2} f(x) + g(x) \) and \( \lim_{x \to 2} f(g(x)) \).
Problem 5  a) Find the instantaneous rate of change of
\[ f(x) = 2x^2 + 5x \]
at \( x = -2 \). Use a limit computation, no rules for derivatives that are not covered in Chapter 2.
b) Find an equation for the tangent line to the graph of \( f(x) \). Make a sketch of the graph and this tangent line, using the values
\[
\begin{array}{c|cccccc}
  x & -3 & -2 & -1 & 0 & 1 \\
  f(x) & 3 & -2 & -3 & 0 & 7 \\
\end{array}
\]

Problem 6  Evaluate the following limit.
\[
\lim_{t \to 0^+} \frac{\sqrt{\sin(4t^2)}}{t}
\]

Problem 7  Find the limit
\[
L = \lim_{x \to 0} \frac{\sqrt{x^2 + x + 1} - \sqrt{x^2 - 3x + 1}}{x}
\]

Problem 8  One night, a car drives on the road given by \( y = x^2 - 4x \) between \( x = -5 \) and \( x = 3 \). Find the point(s) \( x = a \) at which its headlights shine on the point \( P(2,0) \).

Make a sketch first! Show your working.